2025 A Math Prelim Paper 2 Solutions

1(a)	$\frac{60}{100}A = Ae^k$ $e^k = \frac{3}{5}$ $k = \ln\frac{3}{5}$ *or* $3000000 = 5000000e^k$ $\ln 3000000 = \ln(5000000e^k)$ $\ln 3000000 = \ln 5000000 + \ln e^k$ $\ln 3000000 = \ln 5000000 + k$ $k = \ln 3000000 - \ln 5000000 = \ln\frac{3}{5}$
(b)	$2000 = 5000000e^{\left(\ln\frac{3}{5}\right)t}$ $\left(\ln\frac{3}{5}\right)t = \ln\frac{2000}{5000000}$ $t = \dfrac{\ln\left(\frac{1}{2500}\right)}{\ln\left(\frac{3}{5}\right)}$ $t = 15.3$ min
2(a)	$49u^2 - 28u - 5 = 0$ $(7u-5)(7u+1) = 0$ $u = \frac{5}{7}$ or $u = -\frac{1}{7}$ $7^x = \frac{5}{7}$ or $7^x = -\frac{1}{7}$ (rej) $x = \log_7\frac{5}{7}$ $= \log_7 5 - \log_7 7$ $= \log_7 5 - 1$ Or

	$7^x = \frac{5}{7}$ $x\lg 7 = \lg\frac{5}{7}$ $x = \frac{\lg\left(\frac{5}{7}\right)}{\lg 7} = \log_7 \frac{5}{7} \quad [\log_a b = \frac{\log_c b}{\log_c a}]$ $Note: \frac{\lg A}{\lg B} \neq \lg(A-B)$
2 **(b)**	$\log_3(2x-1) - \log_9(x^2+2) = \log_{25} 5$ $\log_3(2x-1) - \frac{\log_3(x^2+2)}{\log_3 9} = \frac{1}{2}$ $2\log_3(2x-1) - \log_3(x^2+2) = 1$ $\log_3 \frac{(2x-1)^2}{x^2+2} = \log_3 3$ $\frac{(2x-1)^2}{x^2+2} = 3$ $4x^2 - 4x + 1 = 3x^2 + 6$ $x^2 - 4x - 5 = 0$ $(x-5)(x+1) = 0$ $x = 5$ or $x = -1$ (reject)

3(a)	$\frac{dy}{dx} = 3x^2 - 6x + 3$ $\frac{dy}{dx} = 0$ $\Rightarrow 3x^2 - 6x + 3 = 0$ $\Rightarrow 3(x-1)^2 = 0$ $x = 1$, $y = -6$ Coordinates of the stationary point are $(1, -6)$.
(b)	(table below)

x	1^-	1	1^+
$\frac{dy}{dx}$	$+$	0	$+$

	$(1,-6)$ is a point of inflexion.
4(a)	$h = 2 + \frac{4}{5}t - \frac{1}{250}t^2$ $= -\frac{1}{250}\left(t^2 - 200t\right) + 2$ $= -\frac{1}{250}\left[\left(t-100\right)^2 - 100^2\right] + 2$ $= -\frac{1}{250}\left(t-100\right)^2 + 40 + 2$ $= 42 - \frac{1}{250}\left(t-100\right)^2$
(b)	Max height = 42 m Time = 100 s
(c)	$42 - \frac{1}{250}\left(t-100\right)^2 = 32$ $\frac{1}{250}\left(t-100\right)^2 = 10$ $\left(t-100\right)^2 = 2500$ $t = 100 \pm 50$ t = 50 or 150 $\therefore$ length of time = 100s Or $42 - \frac{1}{250}\left(t-100\right)^2 \geq 32$ $\left(t-100\right)^2 \leq 2500$ $\left[\left(t-100\right)^2 - 50^2\right] \leq 0$ $\left[\left(t-100\right)+50\right]\left[\left(t-100\right)-50\right] \leq 0$ $(t-50)(t-150) \leq 0$ $50 \leq t \leq 150$ Duration=100s

5**(a)**	$f(x)=x^3+2kx+2$ $f(-2)=(-2)^3+2k(-2)+2$ $\quad=-6-4k$ $f(1)=(1)^3+2k(1)+2$ $\quad=2k+3$ $-6-4k=2k+3-3$ $6k=-6$ $k=-1$
(b)	$f(x)=2x^3-5x^2-3x+10$ $f(2)=2(2)^3-5(2)^2-3(2)+10$ $\quad=0$ $\therefore$ $(x-2)$ is a factor $2x^3-5x^2-3x+10=(x-2)(2x^2+bx-5)$ Compare x^2, $-5=-4+b$ $b=-1$ $f(x)=(x-2)(2x^2-x-5)$ When $f(x)=0$. $x-2=0$ or $2x^2-x-5=0$ $x=2$ or $x=\dfrac{1\pm\sqrt{1-4(2)(-5)}}{4}$ $=\dfrac{1\pm\sqrt{41}}{4}$

6(a)	$T_{r+1} = \binom{6}{r}\left(x^4\right)^{6-r}\left(-\frac{1}{kx^2}\right)^r$ $= \binom{6}{r}\left(-\frac{1}{k}\right)^r\left(x^{24-4r}\right)x^{-2r}$ $= \binom{6}{r}\left(-\frac{1}{k}\right)^r x^{24-6r}$ Powers of $x = 24 - 6r$ Since 24 and $6r$ are even numbers, subtraction between even numbers will give an even number. $\therefore$ there are only even powers of x in this expansion.
(b)	$24-6r=0$ $r=4$ $\binom{6}{4}\left(-\frac{1}{k}\right)^4 = \frac{5}{27}$ $\left(-\frac{1}{k}\right)^4 = \frac{1}{81}$ $k=3$
(c)	$24-6r=-6$ $r=5$ Coefficient of $x^{-6} = \binom{6}{5}\left(-\frac{1}{3}\right)^5 = -\frac{6}{243} = -\frac{2}{81}$ For the expansion $\left(2-3x^6\right)\left(x^4-\frac{1}{kx^2}\right)^6$ Term independent of $x = 2\left(\frac{5}{27}\right)-3\left(-\frac{6}{243}\right)$ $=\frac{10}{27}+\frac{2}{27}$ $=\frac{12}{27}$ $=\frac{4}{9}$

7(a)	When $t = 0$, $v = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ m/s
(b)	When $v = 0$, $\cos(\frac{\pi}{4}t + \frac{\pi}{6}) = 0$ $\frac{\pi}{4}t + \frac{\pi}{6} = \frac{\pi}{2}$ or $\frac{\pi}{4}t + \frac{\pi}{6} = \frac{3\pi}{2}$ $t = \frac{4}{3}$ or $\frac{16}{3} = 1.33$ or 5.33 $\cos(\frac{\pi}{4}t + \frac{\pi}{6}) = 0$ $\frac{\pi}{4}t + \frac{\pi}{6} = \frac{\pi}{2}$ or $\frac{\pi}{4}t + \frac{\pi}{6} = \frac{3\pi}{2}$ $t = \frac{4}{3}$ or $\frac{16}{3} = 1.33$ or 5.33
(c)	$s = \int \cos(\frac{\pi}{4}t + \frac{\pi}{6})\ \mathrm{d}t$ $s = \frac{4}{\pi}\sin(\frac{\pi}{4}t + \frac{\pi}{6}) + c$ When $t = 0$, $s = \frac{1}{\pi}$, $\frac{1}{\pi} = \frac{4}{\pi}\left(\sin\frac{\pi}{6}\right) + c$ $\frac{1}{\pi} = \frac{4}{\pi}\left(\frac{1}{2}\right) + c$ $c = \frac{1}{\pi} - \frac{2}{\pi} = -\frac{1}{\pi}$ $\therefore s = \frac{4}{\pi}\left[\sin(\frac{\pi}{4}t + \frac{\pi}{6})\right] - \frac{1}{\pi}$ When $t = 0$ $\quad s = \frac{1}{\pi}$ m = 0.31830 m When $t = \frac{4}{3}$ $\quad s = \frac{4}{\pi}\left(\sin\frac{\pi}{2}\right) - \frac{1}{\pi} = \frac{3}{\pi}$ m or 0.95492 m When $t = 4$ $\quad s = \frac{4}{\pi}\left(\sin\frac{7\pi}{6}\right) - \frac{1}{\pi} = -\frac{3}{\pi}$ or -0.95492 m Total distance travelled = $\frac{2}{\pi} + \frac{3}{\pi} + \frac{3}{\pi} = \frac{8}{\pi}$ or 2.55 m

8(a)	$\text{LHS} = \sin^3 x \sec^2 x + \sin x$ $= \sin^3 x\left(\dfrac{1}{\cos^2 x}\right) + \sin x$ $= \tan^2 x \sin x + \sin x$ $= \sin x\left(\tan^2 x + 1\right)$ $= \sin x \sec^2 x$ $= \sin x\left(\dfrac{1}{\cos^2 x}\right)$ $= \tan x \sec x$ = RHS (shown)
(b)	$\tan x \sec x = 5$ $\dfrac{\sin x}{\cos x}\left(\dfrac{1}{\cos x}\right) = 5$ $\dfrac{\sin x}{\cos^2 x} = 5$ $\sin x = 5\left(1 - \sin^2 x\right)$ $5\sin^2 x + \sin x - 5 = 0$ $\sin x = \dfrac{-1 \pm \sqrt{(1)^2 - 4(5)(-5)}}{2(5)}$ $\sin x = 0.90498$ or -1.10498 (NA) Basic angle = 64.8215° $x = 64.8^\circ,\ 115.2^\circ$

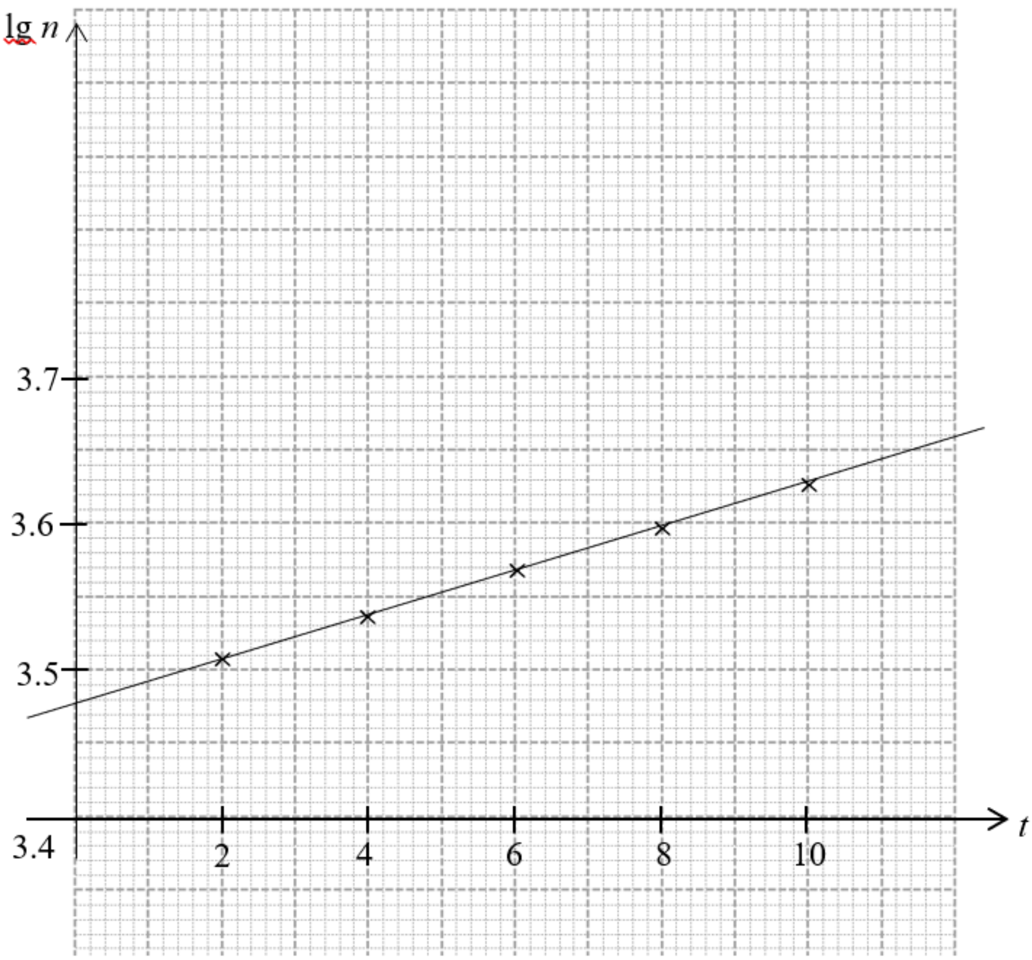
9(a)(i)	$y=\dfrac{2x-6}{x-2}$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{(x-2)(2)-(2x-6)(1)}{(x-2)^2}$ $=\dfrac{2}{(x-2)^2}$ When the curve meets the y-axis, $x = 0$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{2}{(0-2)^2}=\dfrac{1}{2}$ Gradient of normal = -2 $x=0, y=3$ Equation of normal is $y-3=-2(x-0)$ $y=-2x+3$
9(a)(ii)	For $x>2,\ x-2>0,$ $\Rightarrow (x-2)^2>0$ Since $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{2}{(x-2)^2}$>0 for x >2, y is an increasing function.

9b	$\dfrac{d^2y}{dx^2}=(3x+1)^2$ $\dfrac{dy}{dx}=\int(3x+1)^2\,dx$ $=\dfrac{(3x+1)^3}{(3)(3)}+C$ $=\dfrac{(3x+1)^3}{9}+C$ $\dfrac{dy}{dx}=45,\ x=2,$ $45=\dfrac{343}{9}+C$ $C=\dfrac{62}{9}$ $\dfrac{dy}{dx}=\dfrac{(3x+1)^3}{9}+\dfrac{62}{9}$ $y=\int\left[\dfrac{(3x+1)^3}{9}+\dfrac{62}{9}\right]dx$ $y=\dfrac{(3x+1)^4}{108}+\dfrac{62}{9}x+D$ $36=\dfrac{2401}{108}+\dfrac{124}{9}+D$ $D=-\dfrac{1}{108}$ $\therefore y=\dfrac{(3x+1)^4}{108}+\dfrac{62}{9}x-\dfrac{1}{108}$

Solution 1

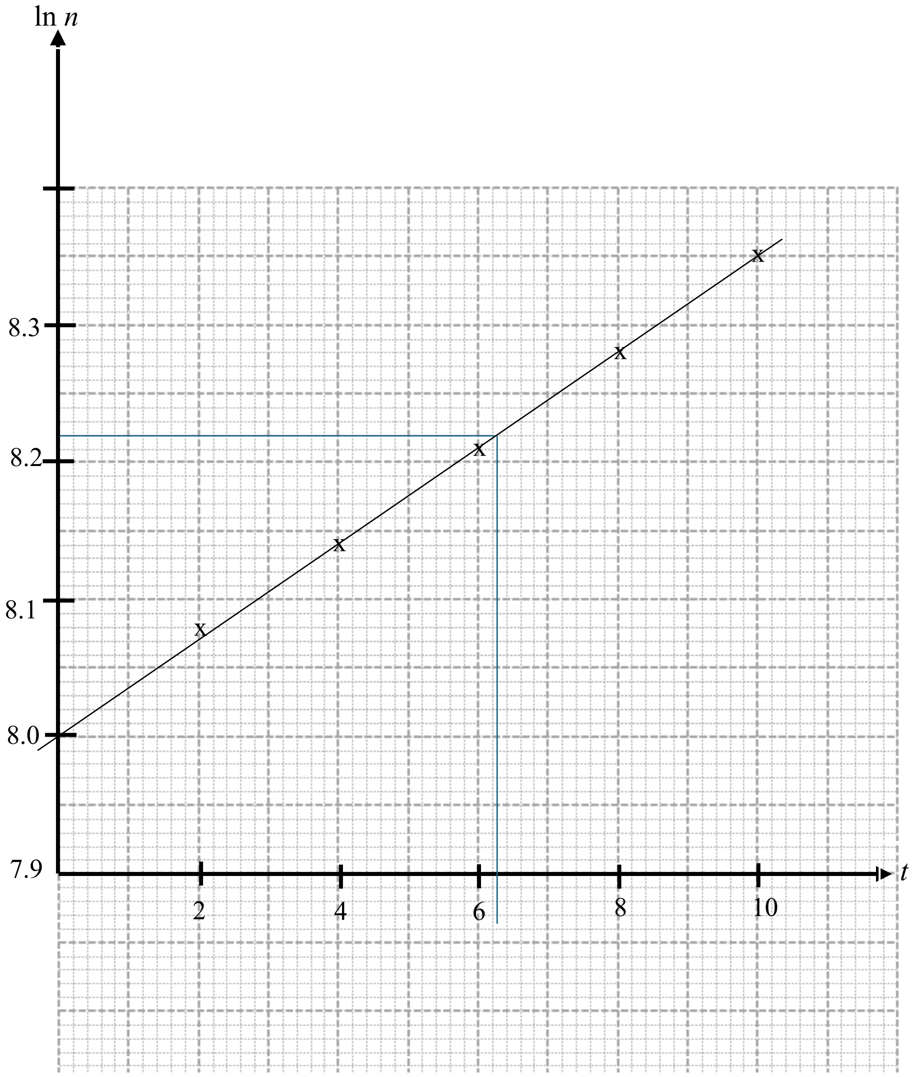
10(a)	$n = n_0(2)^{kt}$ $\lg n = \lg n_0 + (k \lg 2)t$ Draw a straight line of $\lg n$ against *t*. *t*: 2, 4, 6, 8, 10 lg *n*: 3.51, 3.54, 3.57, 3.60, 3.63 Best fit line
(b)	$\lg n_o = 3.48$ $n_o = 10^{3.48}$ $= 3020$
(c)	$k \lg 2 = \dfrac{3.584 - 3.48}{7}$ $k = 0.0494$
(d)	$\lg\left[1.25\left(10^{3.48}\right)\right] = 3.58$ $t = 6.7$ (6.6 - 7.2)

t	2	4	6	8	10
lg *n*	3.51	3.54	3.57	3.60	3.63


lg n
3.7
3.6
3.5
3.4
2
4
6
8
10
t

Solution 2

10(a)	$n = n_0 (2)^{kt}$ $\ln n = \ln(n_o) + (k \ln 2) t$ Draw a straight line of $\ln n$ against *t*. <table><tr><td>*t*</td><td>2</td><td>4</td><td>6</td><td>8</td><td>10</td></tr><tr><td>ln *n*</td><td>8.08</td><td>8.14</td><td>8.21</td><td>8.28</td><td>8.35</td></tr></table> Best fit line
(b)	$\ln n_o = 8.00$ $n_o = e^8$ $= 2980.957$ $= 2980$
(c) **(d)**	$k \ln 2 = \dfrac{8.35 - 8.0}{10}$ $k = 0.050494$ $= 0.0505$ $\ln\left[1.25\left(e^8\right)\right] = 8.223$ $t = 6.3$ (6.0 - 6.6)



ln n
8.3
8.2
8.1
8.0
7.9
2
4
6
8
10
t

11(a)	Let $B = (x,\ y)$ Distance AB = $\sqrt{(x-6)^2+(y-8)^2}=15$ $(x-6)^2+(y-8)^2=225$ ------ (1) Grad AB = $\frac{4}{3}$ Equation of AB: $y=\frac{4}{3}x$ ------ (2) Solving, $(x-6)^2+\left(\frac{4}{3}x-8\right)^2=225$ $\frac{25}{9}x^2-\frac{100}{3}x-125=0$ $x^2-12x-45=0$ $(x+3)(x-15)=0$ $x=-3$ or $x=15$ (rej) $y=-4$ $\therefore B=(-3,\ -4)$ Or Distance OA = $\sqrt{6^2+8^2}=10$ units Distance OB = 5 units Let $B = (x,\ y)$ By ratio theorem, $\frac{5(6)+10x}{15}=0,\quad \frac{5(8)+10y}{15}=0$ $\therefore B=(-3,\ -4)$
(b)	Grad BC = $-\frac{3}{4}$ $y=-\frac{3}{4}x+c$ $-4=-\frac{3}{4}(-3)+c \Rightarrow c=-\frac{25}{4}$ Coordinates of C = $\left(0,-\frac{25}{4}\right)$ Let D = $(x,\ 0)$ $\frac{\frac{25}{4}}{x}=\frac{4}{3}$ $x=\frac{75}{16}$ Coordinates of D = $\left(\frac{75}{16},0\right)$

(c)	$\text{Area} = \frac{1}{2}\begin{vmatrix} 6 & -3 & 0 & \frac{75}{16} & 6 \\ 8 & -4 & -\frac{25}{4} & 0 & 8 \end{vmatrix}$ $= \frac{1}{2}\left\| -24 + \frac{75}{4} + \frac{75}{2} - \left(-24 - \frac{1875}{64}\right) \right\|$ $= \frac{5475}{128}$ or $42\frac{99}{128}$ or 42.8 units^2 or $\text{Area} = \frac{1}{2}(AB + CD)(BC)$ $= \frac{1}{2}\left(15 + \frac{125}{16}\right)\left(\frac{15}{4}\right) = \frac{5475}{128}$