TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 2025
Secondary 4

CANDIDATE NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS **4049/02**

Paper 2 **Tuesday 26 August 2025**

2 hours 15 minutes

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal in the case of angles in degree, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1).......(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

1 In an experiment, it was observed that the number of bacteria in a culture doubles every 4 hours.
It is given that N_0 is the number of bacteria present at the start of the experiment, and N is the number of bacteria t hours later.

Calculate the **exact** value of the constant k, given that $N = N_0 e^{kt}$. [4]

2 Given that $\sqrt{8^x} = \frac{2^x}{4}$, find the value of $\sqrt{8^x}$. [4]

3 Solve the simultaneous equations

$$(y+11)^2 + x^2 = 289,$$

$$2y = 7x - 48.$$ [5]

4 **(a)** State the range of values of x for $\log_x(2x-1)$ to be defined. [2]

(b) Given $\log_9 \sqrt{x} = 5\log_3 p$, express x in terms of p. [4]

5 A curve is such that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 8\sin 4x - 4\cos 2x$.

The curve passes through the point $P\left(\pi,\ -\frac{1}{6}+\pi\right)$ and has a gradient of -1 at P.

Find the equation of the curve. [7]

6 **(a)** The term containing the highest power of x in the polynomial $\mathrm{f}(x)$ is $2x^4$.

Two of the roots of the equation $\mathrm{f}(x) = 0$ are $\frac{1}{2}$ and -3.

Given that $x^2 + 1$ is a quadratic factor of $\mathrm{f}(x)$, find an expression for $\mathrm{f}(x)$ in descending powers of x. [3]

(b) Find the value of k for which $x^2 + (k-1)x + k^2 - 16$ is exactly divisible by $x - 3$ but not divisible by $x + 4$. [4]

7 An object starts from rest from a point O.

Its velocity, v cm/s, t seconds after leaving O, is such that $\frac{\mathrm{d}v}{\mathrm{d}t} = 10$.

After 5 seconds, the object reaches a point A.

(i) Find its velocity at A. [2]

(ii) Find the distance OA. [2]

On reaching A, the object then slows so that its velocity V cm/s, T seconds after leaving A, is such that $\frac{\mathrm{d}V}{\mathrm{d}T} = 10 - k\sqrt{T}$, where k is a constant.

(iii) Given that the object's velocity at a point B where $T = 4$ is 26 cm/s, show that $k = 12$. [3]

8 **(a)** Show that $(m+1)x^2+(4m+3)x+2m=0$ has real and distinct roots for all real values of m. [4]

(b) The equation of a curve is $y=4x^2-4x+3$.

(i) Find the set of values of x for which the curve lies below the line $y = 11$. [3]

The straight line L meets the curve $y=4x^2-4x+3$at one point only.

(ii) Given that L is not a tangent to the curve, what can be deduced about L? [1]

9 **(a)** The variables x and y are connected by the equation

$$x + py = qxy,$$

where p and q are constants.

Explain how a straight line graph can be drawn and state how the values of p and q could be obtained from the line. [4]

(b) The variables x and y are such that when values of $\lg(y-5)$ are plotted against $\sqrt{x}$ a straight line is obtained.
It is given that when $x = 10$, $y = 15$ and when $x = 100$, $y = 1005$.
Estimate the value of x when $y = 50$. [6]

10 A circle has centre $A(2, -3)$ and radius $\sqrt{20}$.

(a) Write the equation of this circle. [1]

(b) Find the equations of the tangents to the circle that are horizontal. [2]

The circle intersects the y-axis at points S and T.

(c) Find the length of ST. [3]

A second circle with centre B also passes through S and T.

(d) Explain why the y-coordinate of B is -3. [2]

(e) Given that the x-coordinate of B is positive and that the radius of the second circle is $\sqrt{52}$, find the x-coordinate of B. [2]

11 **(a)** Prove $\dfrac{\cos 2\theta}{\tan\theta} + \sin 2\theta = \cot\theta$. [4]

(b) Solve $3\sin x\tan x - \sec x = 1$ for $-180° \leq x \leq 180°$. [6]

12 A curve has the equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{2x+1}{x-3}$ for $x \neq k$.

(a) State the value of k. [1]

(b) Find the equation of the normal to the curve at the point $x = 4$. [4]

(c) Explain whether f is an increasing or decreasing function for $x > 3$. [2]

(d) Determine whether the **gradient** of the curve is an increasing or decreasing function for $x > 3$. [3]

(e) The line $y = c$ does not intersect the curve.

By expressing $\dfrac{2x+1}{x-3}$ as $a + \dfrac{b}{x-3}$ where a, b and c are constants, explain why $c = 2$. [2]

End of Paper

BLANK PAGE

BLANK PAGE

BLANK PAGE

Qn	Answer Key
1	$k=\frac{1}{4}\ln 2$
2	$\sqrt{8^{-4}}=\frac{1}{64}$
3	$x=8,\ -1\frac{7}{53}$ (or -1.13) $y=4,\ -27\frac{51}{53}$ (or -28.0)
4a	$\underline{\boldsymbol{x>0.5}}$ and $\underline{\boldsymbol{x\neq 1}}$
b	$x=p^{20}$
5	$y=-\frac{1}{2}\sin 4x+\cos 2x+x-\frac{7}{6}$
6a	$2x^4+5x^3-x^2+5x-3$
b	$k=-5$
7i	50 m/s
ii	125 cm
iii	Show that
8a	Show that
bi	$-1<x<2$
bii	Vertical line
9a	$y=\left(\frac{p}{q}\right)\frac{y}{x}+\left(\frac{1}{q}\right)$ $q=\frac{1}{\textbf{Vertical intercept}}$ $p=q(\textbf{Gradient})$
b	21.2
10a	$(x-2)^2+(y+3)^2=20$ OR $x^2+y^2-4x+6y-7=0$
b	$y=-3+\sqrt{20}$ & $y=-3-\sqrt{20}$
c	8 units

d	Perpendicular bisector of *ST* will pass through the centre of the circle. Since *ST* is vertical, y-coord of B = y coord of A $= \frac{1-7}{2} = -3$
e	x-coord of $B = 6$
11a	Proving
B	$x = -180°$ or $-48.2°$ or $48.2°$ or $180°$
12a	3
B	$y = \frac{1}{7}x + \frac{59}{7}$
c	f is a decreasing function
d	f’ is a decreasing function
E	$2 + \frac{7}{x-3}$ c = 2