CATHOLIC JUNIOR COLLEGE
General Certificate of Education Advanced Level
Higher 2
JC2 Preliminary Examination

CANDIDATE NAME			
CLASS		INDEX NUMBER	

MATHEMATICS

9758/02

Paper 2

17 Sep 2025

3 hours

Additional Materials: Printed Answer Booklet
List of Formulae (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** the questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

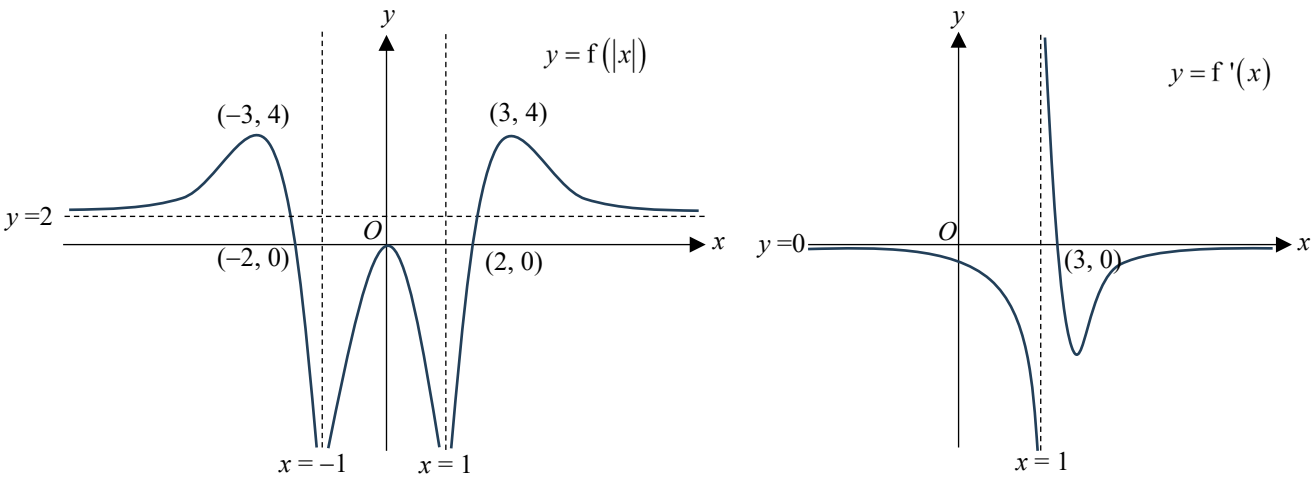
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **8** printed pages.

Section A: Pure Mathematics [40 marks]

1 The diagram shows the graphs of $y = \mathrm{f}\left(|x|\right)$ and $y = \mathrm{f}'\left(x\right)$.



The graph of $y = \mathrm{f}\left(|x|\right)$ has turning points at $(-3, 4)$ and $(3, 4)$, crosses the x-axis at $(-2, 0)$ and $(2, 0)$, and the equations of asymptotes are $x = -1$, $x = 1$ and $y = 2$. The graph of $y = \mathrm{f}'\left(x\right)$ crosses the x-axis at $(3, 0)$, and the equations of asymptotes are $x = 1$ and $y = 0$.

On separate diagrams, sketch the graphs of

(a) $y = \mathrm{f}\left(x\right)$, [2]

(b) $y = \dfrac{1}{\mathrm{f}'\left(x\right)}$, [3]

(c) $y = -\mathrm{f}\left(|x-1|\right)$, [3]

labelling clearly the equation(s) of any asymptote(s), coordinates of any axial intercept(s) and turning point(s) where applicable.

2 Functions f and g are defined by

$$\mathrm{f}: x \to \frac{4}{(x-4)^2}, \quad x \in \mathbb{R},\ x \neq 4,$$

$$\mathrm{g}: x \to \ln\left(1+\frac{1}{x}\right), \quad x \in \mathbb{R},\ x > 0.$$

(a) Sketch the graph of $y = \mathrm{f}\left(x\right)$, stating the equations of any asymptotes, the coordinates of the points where it crosses the axes and the coordinates of the turning points, if any. [2]

(b) Show that gf exists. Hence find the rule, domain and range of gf. [4]

(c) If the domain of f is further restricted to $x < k$, state the largest value of k for which the function f^{-1} exists. [1]

(d) Using the restricted domain found in part **(c)**, find f^{-1} in a similar form. [3]

3

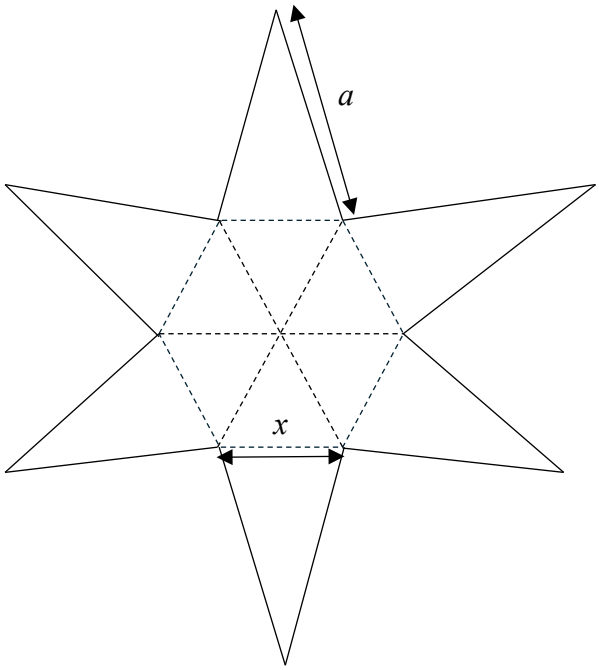


Fig. 1

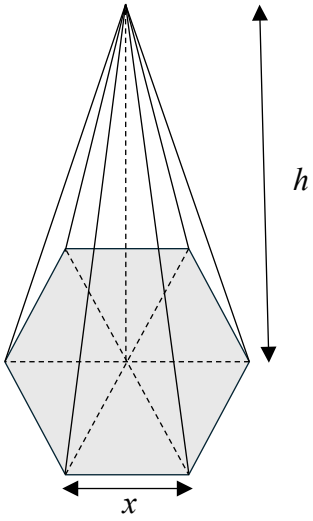


Fig. 2

Fig. 1 shows a net of a hexagonal pyramid folded from a star-shaped cardboard of equal edge length a cm. The net consists of a hexagon with equal sides of x cm and six isosceles triangles with base x cm and side a cm. The net is folded to form a right pyramid with a hexagonal base of edge length x cm and vertical height h cm, as shown in Fig. 2. The hexagonal base is made up of six equilateral triangles of side length x cm.

The volume of a right hexagonal pyramid with base edge x cm and height h cm is given by $V = \frac{\sqrt{3}}{2}x^2h$.

(a) Show that the volume of the hexagonal pyramid, V satisfies the expression given by

$$V^2 = \frac{3}{4}\left(a^2x^4 - x^6\right) \qquad [2]$$

(b) Find, in terms of a, the maximum possible volume of the hexagonal pyramid. You need not show that this value is a maximum. [4]

(c) Find, in terms of a, the total surface area of the hexagonal pyramid when the volume is a maximum. [4]

4 One of the roots of the equation

$$2z^4 - 14z^3 + 33z^2 - 26z + p = 0$$, where p is a constant

is $3+\mathrm{i}$.

(a) Based on the above information only, a student claims that the equation has a root $3-\mathrm{i}$. State, with a reason, why the student's claim may not be true. [1]

(b) Show that $p = 10$. [2]

For the rest of this question, do not use a calculator.

(c) Find the roots of the equation $2z^4 - 14z^3 + 33z^2 - 26z + 10 = 0$ and mark them clearly on a single labelled Argand diagram. [7]

(d) The points of the Argand diagram in part **(c)** form the vertices of a quadrilateral. Identify the type of quadrilateral and determine its area. [2]

Section B: Probability and Statistics [60 marks]

5 The basketball club in a college has 5 centers, 8 forwards and 7 guards. With the National School Games approaching, the coach wishes to find out the opinions of members of the club about the training programme. He gives a questionnaire to all the members of the club and receives replies from everyone.

(a) Explain whether the 20 members form a sample or a population. [1]

The coach then decides to select teams to play in the matches for National School Games. A basketball team to play in a match consists of 1 center, 2 forwards and 2 guards.

(b) Explain an advantage for choosing a random sample in each category of members for the match. [1]

(c) How many different teams can be formed? [1]

In the club, one particular forward is the classmate of one particular guard. Both classmates are injured and cannot participate in a particular match. The coach decides that one of the remaining guards can play either as a guard or as a forward.

(d) How many different teams can now be formed? [3]

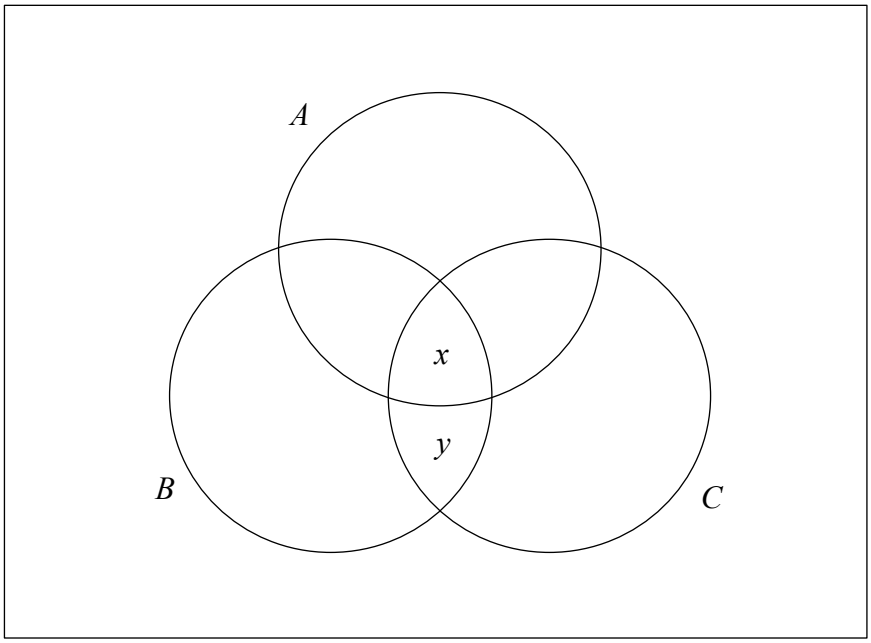
6 A group of 100 students are asked if they are student leaders, in a sports CCA, or studying in a science faculty. The number of students who are student leaders is 25, the number of students who are in a sports CCA is 30 and the number of students studying in a science faculty is 40. There are 15 student leaders who are in a sports CCA. The number of students who are student leaders, in a sports CCA and studying in a science faculty is x. The number of students who are in a sports CCA and studying in a science faculty but not a student leader is y.

One of the students is chosen at random.

A is the event that the student is a student leader.
B is the event that the student is in a sports CCA.
C is the event that the student is studying in a science faculty.

It is given that A and C are independent.

(a) Complete the Venn diagram below to represent all the above information. You are allowed to give expressions in terms of x and y. [3]



It is further given that B and C are independent.

(b) Find y in terms of x. Hence, find the greatest and least possible values of y. [4]

7 Happie, the owner of a store selling novelty items, is organizing a publicity stunt for his store. Using a stock of ultra-rare Lubaba dolls that he acquired, he sets up a game where a player opens "mystery boxes" to try to find a Lubaba doll hidden inside one of the identical boxes used for the game.

The rules of the game are:

- The player pays an initial \$5 to start the game to open one of n boxes.
- If the opened box is empty, the player can pay an additional \$3 to open a second box.
- If the second box is empty, the player can pay \$2.50 again to open a third box.
- If the third box is still empty, the player loses the game. The doll that is not won will then be donated away and the game is reset for the next player.

As the Lubaba dolls are considered rare collectors' items, it can be assumed that in every game, the players will keep opening the boxes until they win the doll and that they have the means to pay for the maximum of 3 allowable tries at opening the boxes.

Let X be the random variable denoting the number of empty boxes a player has opened in a game.

(a) Show that $\mathrm{P}(X = 2) = \frac{1}{n}$. [1]

(b) Find the probability distribution of X, leaving your answers in terms of n. [2]

(c) If $n = 10$, find $\mathrm{P}(X \leq \sigma)$, where σ is the standard deviation of X. [2]

(d) If the cost price of a Lubaba doll is \$10, find the number of boxes n, that Happie should prepare so that he will not make any profit nor incur any loss from organising this publicity stunt. [3]

8 In each batch of pralines that a chocolate factory produces, they are packed into boxes of 16. Due to the new health guidelines regarding sugar content, quality control tests are implemented and show that on average, the proportion of pralines that exceed the recommended sugar content is p. The number of pralines in a box that exceed the recommended sugar content is denoted by X.

(a)(i) State, in context of the question, two assumptions needed for X to be well-modelled by a binomial distribution. [2]

Assume now that X is modelled by a binomial distribution.

(ii) It is known that the most probable number of pralines that exceed the recommended sugar content is 2. Find the exact range of values that p can take. [3]

For each batch of pralines produced, the chocolate factory implements the following quality control system by testing the first random sample of 16 pralines in a box:

- If there are at most 1 praline that exceed the recommended sugar content in a box, the batch is accepted.
- If there are 3 or more pralines that exceed the recommended sugar content in a box, the batch is rejected.
- If there are 2 pralines that exceed the recommended sugar content in a box, a second box of 16 pralines is tested. The batch will be accepted if there are at most 1 praline that exceed the recommended sugar content in the second box, otherwise the batch will be rejected.

It is given that $p = 0.15$.

(b)(i) Find the probability that the batch of pralines is accepted under this quality control system. [2]

(ii) Given that the batch of pralines is accepted, find the probability that a second box of pralines is tested. [2]

9 A teacher conducts a survey on 7 students to investigate the relationship between the number of hours (h) of "screen-time" per week and the average scores (s) they obtained in a recently concluded examination She records her findings as shown in the following table.

h	8.5	14	20	27	10.5	17	23
s	68	61	44	12	α	58	31

(a) Given that the regression line of s on h is $s = -3.00799h + 100.27974$, show that $\alpha = 67.0$. [2]

(b) Draw a scatter diagram for the data. [1]

(c) Explain why $s = kh^2 + c$ is the better model compared to the one in part **(a),** by giving appropriate reasons. State the values of k and c. [3]

(d) Using the better model, estimate the score a student can expect to obtain if he spends 7 hours of screen-time a week. Comment on the reliability of the estimate obtained. [2]

(e) The teacher observes a trend from her findings and concludes with a statement: Increased screen-time will cause the exam scores to decrease. Comment on the validity of the statement. [1]

10 Fishing Company A has two types of fishing vessels, the "Standard" vessel and the "Large" vessel. The amount of fish caught by the respective vessels on a typical fishing trip are normally distributed with means and standard deviations as shown in the table.

	Mean (kg)	Standard deviation (kg)
Standard	300	σ
Large	540	110

(a) If a Standard vessel returns with more than 400kg of fish, it is called a Bumper catch. Bumper catches occur 8% of the time. Show that $\sigma \approx 71.2$. [2]

(b) On a particular fishing trip, Company A sends out 3 Standard vessels and 2 Large vessels, with a target to catch at least 2100kg of fish. Find the probability that the fishing vessels are able to meet their target. [2]

(c) State an assumption needed in your calculation in part **(b)**. [1]

Fishing Company B operates in the same seas as Fishing Company A.
However, Company B has the improved versions of the two types of fishing vessels that Company A has: the "Premium-Standard" vessel and the "Premium-Large" vessel. In a fishing trip, the Premium-Standard vessel is capable of catching 3 times as much fish as the Standard vessel and the Premium-Large vessel is capable of catching 2 times as much fish as Large vessel.

(d) Company B sends out 1 Premium-Standard vessel and 1 Premium-Large vessel, with the same target to catch at least 2100kg of fish. Find the probability that these two fishing vessels are able to meet their target. [2]

(e) It is found that, in a sample of n randomly chosen fishing trips, the probability that the average catch of a Large vessel being less than 552kg is at least 0.7.
Find the least possible value of n. [3]

11 EastHam is a football club in a competitive league. It is found that the average time taken by EastHam to score their first goal in a match is 50 minutes. The coach trialled a new play tactic and the time taken, x minutes, to score the first goal is recorded for a random sample of 35 matches in the season. The total time taken is found to be 1650 minutes and the variance of the time is 250 minutes^2.

The coach wants to test if the average time taken to score the first goal has decreased with the implementation of the new play tactic.

(a) Explain why the coach is able to carry out a hypothesis test without any assumption about the distribution of the time taken to score the first goal. [1]

(b) Find unbiased estimates of the population mean and variance. Give your answers correct to 3 decimal places. [2]

(c) Carry out the test at 10% level of significance. You should state your hypotheses and define any symbols you use. [4]

A fan of another football club in the same league, NewPalace, claims that the average time taken by NewPalace to score their first goal in a match is 40 minutes. The time taken by NewPalace to score the first goal in a match can be assumed to be normally distributed with variance 280 minutes^2.

(d) A random sample of 50 matches is taken and a hypothesis test is carried out on whether the fan's claim is valid. Find the range of values of the mean time of this sample for which the fan's claim would be rejected at 10% level of significance. [4]