EUNOIA JUNIOR COLLEGE

JC2 Preliminary Examination 2025

General Certificate of Education Advanced Level

Higher 2

CANDIDATE NAME

CIVICS GROUP | INDEX NO.

FURTHER MATHEMATICS 9649/01

Paper 1

02 September 2025

3 hours

Additional Materials: Printed Answer Booklet
List of Formulae (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

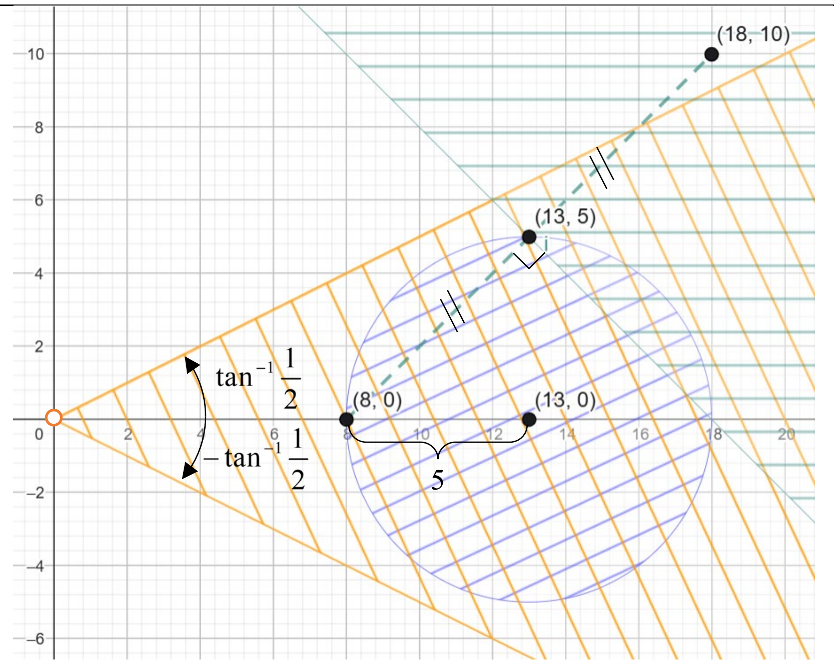
This document consists of **6** printed pages and **2** blank pages.

1	**Solution**
(a)	Cofactor expansion along the first column: $\det\mathbf{A} = \beta(\alpha\beta) - \gamma(\gamma\alpha) - \alpha(\alpha\beta) + \gamma(\beta\gamma) + \alpha(\gamma\alpha) - \beta(\beta\gamma)$ $= -\alpha\beta(\alpha-\beta) - \gamma^2(\alpha-\beta) + (\alpha-\beta)(\alpha\gamma+\beta\gamma)$ $= (\alpha-\beta)(-\alpha\beta-\gamma^2+\alpha\gamma+\beta\gamma)$ $= (\alpha-\beta)(\gamma(\beta-\gamma)-\alpha(\beta-\gamma))$ $= (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)$
(b)	$\det\mathbf{B} = -3(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)$

2	**Solution**
(a)	$\dfrac{\partial z}{\partial x} = -y\sin\left(xy+\dfrac{\pi}{4}\right)$, $\dfrac{\partial z}{\partial y} = -x\sin\left(xy+\dfrac{\pi}{4}\right)$ $\dfrac{\mathrm{d}z}{\mathrm{d}t} = \dfrac{\partial z}{\partial x}\dfrac{\mathrm{d}x}{\mathrm{d}t} + \dfrac{\partial z}{\partial y}\dfrac{\mathrm{d}y}{\mathrm{d}t}$ $= -y\sin\left(xy+\dfrac{\pi}{4}\right)\cdot \mathrm{e}^t - x\sin\left(xy+\dfrac{\pi}{4}\right)\cdot\dfrac{1}{1+t}$ When $t=0$, $x=1, y=0$ Therefore $\dfrac{\mathrm{d}z}{\mathrm{d}t} = -\sin\dfrac{\pi}{4} = -\dfrac{\sqrt{2}}{2}$
(b)	$\mathrm{f}_{xy}(x,y)=0$ implies f_x is independent of y $\mathrm{f}_{yx}(x,y)=0$ implies f_y is independent of x So f must take the form of $\mathrm{f}(x,y)=\mathrm{g}(x)+\mathrm{h}(y)$ for some functions g and h

3	**Solution**
(a)	Length of arc PQ $=\int_2^4 \sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x = \int_2^4 \sqrt{1+\left(x-\frac{1}{4x}\right)^2}\,\mathrm{d}x$ $=\int_2^4 \sqrt{1+x^2-\frac{1}{2}+\left(\frac{1}{4x}\right)^2}\,\mathrm{d}x = \int_2^4 \sqrt{x^2+\frac{1}{2}+\left(\frac{1}{4x}\right)^2}\,\mathrm{d}x$ $=\int_2^4 \sqrt{x^2+2x\left(\frac{1}{4x}\right)+\left(\frac{1}{4x}\right)^2}\,\mathrm{d}x$ $=\int_2^4 \sqrt{\left(x+\frac{1}{4x}\right)^2}\,\mathrm{d}x$ $=\int_2^4 \left(x+\frac{1}{4x}\right)\mathrm{d}x,\quad \because \sqrt{\left(x+\frac{1}{4x}\right)^2}=\left\lvert x+\frac{1}{4x}\right\rvert = x+\frac{1}{4x},$ when $2\le x\le 4$, $=\left[\frac{x^2}{2}\right]_2^4+\left[\frac{1}{4}\ln x\right]_2^4$ $=\frac{1}{2}\left(4^2-2^2\right)+\frac{1}{4}\left(\ln 4-\ln 2\right) = 6+\frac{1}{4}(\ln\frac{4}{2}) = 6+\frac{1}{4}\ln 2.$ (shown)
(bi)	Area of surface of revolution generated by arc PQ about the y-axis $=2\pi\int_2^4 x\sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x$ $=2\pi\int_2^4 x\sqrt{1+\left(x-\frac{1}{4x}\right)^2}\,\mathrm{d}x$ $=120.4277...$ $=120.43$ (2 d.p.)
(bii)	Volume of solid of revolution generated by shaded region abt. y-axis $=\int_2^4 2\pi xy\,\mathrm{d}x$ $=2\pi\int_2^4 x\left(\frac{x^2}{2}-\frac{\ln x}{4}\right)\mathrm{d}x,$ $=177.9648...$ $=177.96$ (2 d.p.)

4	**Solution**
	By de Moivre's Theorem, $(\cos\theta+\mathrm{i}\sin\theta)^5=\cos5\theta+\mathrm{i}\sin5\theta$ $(\cos\theta+\mathrm{i}\sin\theta)^5$ $=\cos^5\theta+5\cos^4\theta(\mathrm{i}\sin\theta)+10\cos^3\theta(\mathrm{i}\sin\theta)^2$ $+10\cos^2\theta(\mathrm{i}\sin\theta)^3+5\cos\theta(\mathrm{i}\sin\theta)^4+(\mathrm{i}\sin\theta)^5$ $=\cos^5\theta-10\cos^3\theta\sin^2\theta+5\cos\theta\sin^4\theta$ $+\mathrm{i}(5\cos^4\theta\sin\theta-10\cos^2\theta\sin^3\theta+\sin^5\theta)$ $\cos5\theta=\cos^5\theta-10\cos^3\theta\sin^2\theta+5\cos\theta\sin^4\theta$ $=\cos\theta(\cos^4\theta-10\cos^2\theta\sin^2\theta+5\sin^4\theta)$ Hence $=\cos\theta\left(\cos^4\theta-10\cos^2\theta(1-\cos^2\theta)+5(1-\cos^2\theta)^2\right)$ $=\cos\theta\left(\cos^4\theta-10\cos^2\theta+10\cos^4\theta+5-10\cos^2\theta+5\cos^4\theta\right)$ $=\cos\theta(16\cos^4\theta-20\cos^2\theta+5)$ Substitute $\theta=\frac{\pi}{10}$: $\cos\frac{\pi}{2}=\cos\frac{\pi}{10}\left(16\cos^4\frac{\pi}{10}-20\cos^2\frac{\pi}{10}+5\right)$ $0=\cos\frac{\pi}{10}\left(16\cos^4\frac{\pi}{10}-20\cos^2\frac{\pi}{10}+5\right)$ $16\cos^4\frac{\pi}{10}-20\cos^2\frac{\pi}{10}+5=0$ since $\cos\frac{\pi}{10}\neq0$ $\cos^2\frac{\pi}{10}=\frac{20\pm\sqrt{80}}{32}=\frac{5\pm\sqrt{5}}{8}$ Note that the cosine function is decreasing from 0 to $\frac{\pi}{2}$, so $\cos^2\frac{\pi}{10}>\cos^2\frac{\pi}{6}=\frac{3}{4}$. Since $\frac{5-\sqrt{5}}{8}<\frac{3}{4}$, $\cos^2\frac{\pi}{10}=\frac{5+\sqrt{5}}{8}$ $\cos\frac{\pi}{5}=2\cos^2\frac{\pi}{10}-1=2\left(\frac{5+\sqrt{5}}{8}\right)-1=\frac{1+\sqrt{5}}{4}$ (shown)

5(a)(i)	Solution
	
(a)(ii)	The circle's tangents through the origin have gradients $\pm\frac{5}{12}$, both smaller in magnitude than the half lines' gradients $\pm\frac{1}{2}$. So the points satisfying $\lvert z-13\rvert\le 5$ also satisfy $-\tan^{-1}\frac{1}{2}\le\arg(z)\le\tan^{-1}\frac{1}{2}$. Hence the inequality $-\tan^{-1}\frac{1}{2}\le\arg(z)\le\tan^{-1}\frac{1}{2}$ is implied by $\lvert z-13\rvert\le 5$.
(b)	Maximum distance of z from the origin $=18$ Minimum distance of z from the origin $=\sqrt{5^2+13^2}$ $=\sqrt{194}$

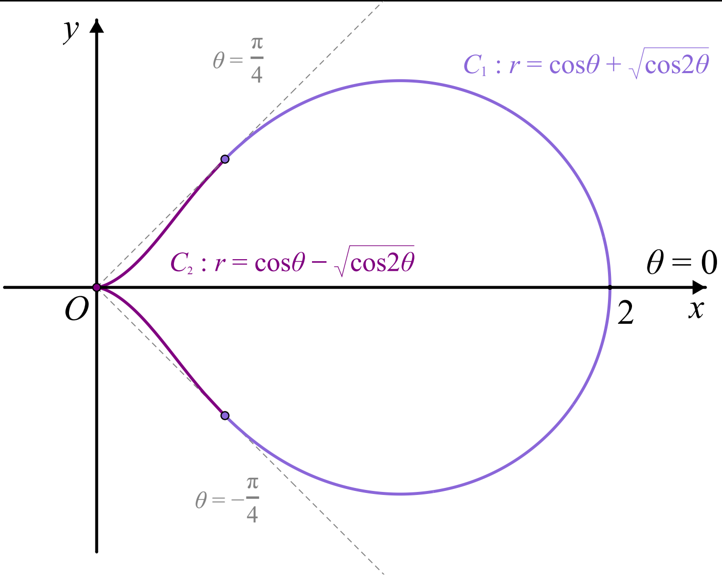
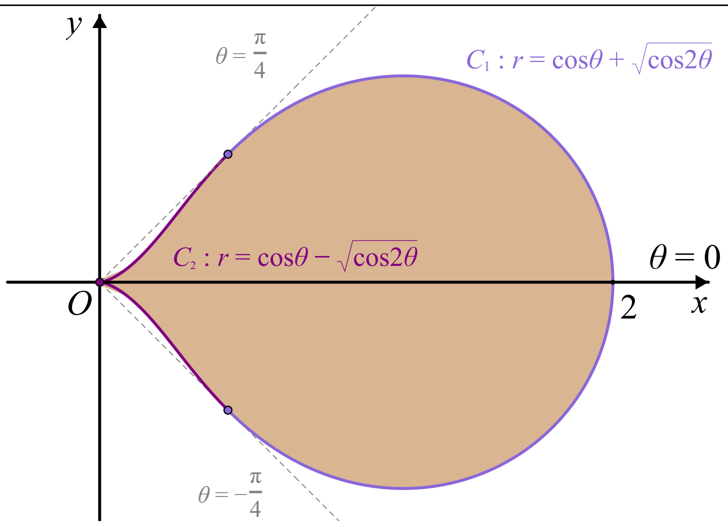
6	Solution
(a)	Consider $\mathbf{Ax}=\lambda\mathbf{x}$ where λ is an eigenvalue and $\mathbf{x}$ is the corresponding non-zero eigenvector. Pre-multiplying both sides by $\mathbf{x}^T$: $\mathbf{x}^{\mathrm{T}}(\mathbf{Ax})=\mathbf{x}^{\mathrm{T}}(\lambda\mathbf{x})$ $\mathbf{x}^{\mathrm{T}}\mathbf{Ax}=\lambda\mathbf{x}^{\mathrm{T}}\mathbf{x}$ $\mathbf{x}^{\mathrm{T}}\mathbf{Ax}=\lambda\lvert\mathbf{x}\rvert^2$ Since $\mathbf{A}$ is positive definite, $\mathbf{x}^{\mathrm{T}}\mathbf{Ax}>0$ for all non-zero $\mathbf{x}$. Also, $\lvert\mathbf{x}\rvert^2>0$ since $\mathbf{x}$ is a non-zero. Therefore, $\lambda=\dfrac{\mathbf{x}^{\mathrm{T}}\mathbf{Ax}}{\lvert\mathbf{x}\rvert^2}>0$ (shown)

(b)	$\mathbf{x}^{\mathrm{T}}\mathbf{A}\mathbf{x} = \begin{pmatrix} x & y \end{pmatrix}\begin{pmatrix} a & b \\ b & c \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$ $= \begin{pmatrix} ax+by & bx+cy \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$ $= ax^2 + bxy + bxy + cy^2$ $= ax^2 + 2bxy + cy^2$ $= a\left(x^2 + \frac{2b}{a}xy\right) + cy^2 \quad (a \neq 0)$ $= a\left(x + \frac{b}{a}y\right)^2 - \frac{b^2}{a}y^2 + cy^2$ $= a\left(x + \frac{b}{a}y\right)^2 + \left(\frac{ac-b^2}{a}\right)y^2$ For **A** to be positive definite, we need $a > 0$ and $ac - b^2 > 0$.
(c)	$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$ Since $a = 2 > 0$ and $ac - b^2 = 5 > 0$, **A** is positive definite $\therefore \mathrm{f}(x, y) = \mathbf{x}^{\mathrm{T}}\mathbf{A}\mathbf{x} > 0$ for all (x, y) except the origin where $\mathrm{f}(0,0) = 0$ Hence the origin must be the global minimum.

7	**Solution**
(a)	$u_{n+1}=3u_n+2$ for $n\geq 0$ with $u_0=k$ $u_n=3u_{n-1}+2$ $=3(3u_{n-2}+2)+2$ $=3^2u_{n-2}+3\cdot 2+2$ $=3^2(3u_{n-3}+2)+3\cdot 2+2$ $=3^3u_{n-3}+3^2\cdot 2+3\cdot 2+2$ $\vdots$ $=3^n u_0+3^{n-1}\cdot 2+3^{n-2}\cdot 2+\ldots+3\cdot 2+2$ $=k\cdot 3^n+\dfrac{2(1-3^n)}{1-3}$ $=(k+1)\cdot 3^n-1$ Alternatively $u_n=3u_{n-1}+2$ $u_n+1=3(u_{n-1}+1)$ $=3^2(u_{n-2}+1)$ $\vdots$ $=3^n(u_0+1)$ $\therefore u_n=3^n(k+1)-1$
(b)	$v_{n+1}=3v_n+2^n$ Multiplying x^{n+1} to both sides, $v_{n+1}x^{n+1}=3v_nx^{n+1}+2^n x^{n+1}$ $\sum_{n=0}^{\infty}v_{n+1}x^{n+1}=\sum_{n=0}^{\infty}3v_nx^{n+1}+\sum_{n=0}^{\infty}2^n x^{n+1}$ $\mathrm{f}(x)-v_0=3x\mathrm{f}(x)+x\cdot\dfrac{1}{1-2x}$ $(1-3x)\mathrm{f}(x)=1+\dfrac{x}{1-2x}$ $(1-3x)\mathrm{f}(x)=\dfrac{1-x}{1-2x}$ $\mathrm{f}(x)=\dfrac{1-x}{(1-3x)(1-2x)}$ $\dfrac{1-x}{(1-3x)(1-2x)}=\dfrac{A}{1-3x}+\dfrac{B}{1-2x}$ Let $x=\dfrac{1}{3}$, $A=\dfrac{1-\frac{1}{3}}{1-2\left(\frac{1}{3}\right)}=2$ Let $x=\dfrac{1}{2}$, $B=\dfrac{1-\frac{1}{2}}{1-3\left(\frac{1}{2}\right)}=-1$

	Hence, $\mathrm{f}(x)=\dfrac{2}{1-3x}+\dfrac{-1}{1-2x}$
(c)	$\mathrm{f}(x)=\dfrac{2}{1-3x}+\dfrac{-1}{1-2x}$ $=2\sum_{n=0}^{\infty}(3x)^n-\sum_{n=0}^{\infty}(2x)^n$ $=\sum_{n=0}^{\infty}(2\cdot 3^n-2^n)x^n$ Hence $v_n=2\cdot 3^n-2^n$ $\dfrac{v_n}{3^n}=2-\left(\dfrac{2}{3}\right)^n\to 2$ as $n\to\infty$ <u>Further note:</u> $v_{n+1}=3v_n+2^n$ $v_n=3v_{n-1}+2^{n-1}$ By eliminating 2^n, $v_n=3v_{n-1}+\dfrac{1}{2}(v_{n+1}-3v_n)$ $v_{n+1}=5v_n-6v_{n-1}$ Therefore, the sequence is also a solution of another second order linear recurrence relation.

8	**Solution**
(a)	Given that $r^2-2r\cos\theta+\sin^2\theta=0$, $r=\dfrac{-(-2\cos\theta)\pm\sqrt{(-2\cos\theta)^2-4(1)(\sin^2\theta)}}{2(1)}$ $=\dfrac{2\cos\theta\pm\sqrt{4(\cos^2\theta-\sin^2\theta)}}{2}$ $=\cos\theta\pm\sqrt{\cos^2\theta-\sin^2\theta}$ $\Rightarrow\ r=\cos\theta\pm\sqrt{\cos 2\theta}$

	 Curve C is a combination of an outer branch C_1 & an inner branch C_2.
(b)	

	Required area of region enclosed by C (shaded) $= 2\cdot$ Area of region enclosed by C and above horizontal axis $\theta=0$ due to reflective symmetry about θ=0 $=\cancel{2}\cdot\left(\frac{1}{\cancel{2}}\int_0^{\frac{\pi}{4}} r_{\text{outer}}{}^2\,\mathrm{d}\theta-\frac{1}{\cancel{2}}\int_0^{\frac{\pi}{4}} r_{\text{inner}}{}^2\,\mathrm{d}\theta\right)$ $=\int_0^{\frac{\pi}{4}}\left[\left(\cos\theta+\sqrt{\cos 2\theta}\right)^2-\left(\cos\theta-\sqrt{\cos 2\theta}\right)^2\right]\mathrm{d}\theta$ $=\int_0^{\frac{\pi}{4}}\left[\begin{array}{r}\left(\cos^2\theta+2\cos\theta\sqrt{\cos 2\theta}+\cos 2\theta\right)\\ -\left(\cos^2\theta-2\cos\theta\sqrt{\cos 2\theta}+\cos 2\theta\right)\end{array}\right]\mathrm{d}\theta$ $=4\int_0^{\frac{\pi}{4}}\cos\theta\sqrt{\cos 2\theta}\ \mathrm{d}\theta$ $=4\int_0^{\frac{\pi}{4}}\cos\theta\sqrt{1-2\sin^2\theta}\ \mathrm{d}\theta$(shown). $\therefore$ Required area $=4\int_0^{\frac{\pi}{4}}\cos\theta\sqrt{1-2\sin^2\theta}\ \mathrm{d}\theta$ $=4\int_{\sin^{-1}(\sqrt{2}\sin 0)}^{\sin^{-1}(\sqrt{2}\sin\frac{\pi}{4})}\cos\theta\sqrt{1-(\sqrt{2}\sin\theta)^2}\,\frac{\mathrm{d}\theta}{\mathrm{d}u}\mathrm{d}u$ $\sin u=\sqrt{2}\sin\theta$ $\cos u=\sqrt{2}\cos\theta\frac{\mathrm{d}\theta}{\mathrm{d}u}$ $u=\sin^{-1}(\sqrt{2}\sin\theta)$ $=4\int_0^{\frac{\pi}{2}}\frac{1}{\sqrt{2}}\cos u\sqrt{1-(\sin u)^2}\,\mathrm{d}u$ $=2\sqrt{2}\int_0^{\frac{\pi}{2}}\cos^2 u\,\mathrm{d}u$ $=\cancel{2}\sqrt{2}\int_0^{\frac{\pi}{2}}\frac{1+\cos 2u}{\cancel{2}}\mathrm{d}u$ $=\sqrt{2}\left[u+\frac{1}{2}\sin 2u\right]_0^{\frac{\pi}{2}}=\sqrt{2}\left[\frac{\pi}{2}+\frac{1}{2}\underbrace{\sin\left(\cancel{2}\frac{\pi}{\cancel{2}}\right)}_{=0}\right]_0^{\frac{\pi}{2}}=\sqrt{2}\frac{\pi}{2}=\frac{\pi}{\sqrt{2}}$

9	**Solution**
(a)	$x_{n+1}=x_n-\frac{x_n{}^5-10x_n{}^3+45x_n-24\sqrt{3}}{5x_n{}^4-30x_n{}^2+45}$ By GC, $x_{12}=1.73532$, $x_{13}=1.73423$, $x_{14}=1.73351$ Least $n=13$ and $x_{13}=1.734$ correct to 3 decimal places
(b)	$\mathrm{f}\left(\sqrt{3}\right)=\left(\sqrt{3}\right)^5-10\left(\sqrt{3}\right)^3+45\sqrt{3}-24\sqrt{3}$ $=9\sqrt{3}-30\sqrt{3}+45\sqrt{3}-24\sqrt{3}$ $=0$

(c)	$\lim_{x\to R}\frac{\mathrm{h}(x)}{x-R}=\lim_{x\to R}\frac{(x-R)^{m-1}\mathrm{q}(x)}{m(x-R)^{m-1}\mathrm{q}(x)+(x-R)^m\mathrm{q}'(x)}$ $=\lim_{x\to R}\frac{\mathrm{q}(x)}{m\mathrm{q}(x)+(x-R)\mathrm{q}'(x)}$ $=\frac{\mathrm{q}(R)}{m\mathrm{q}(R)}$ $=\frac{1}{m}\neq 0$ Since $\mathrm{h}(R)=0$ and $\mathrm{h}'(R)=\lim_{x\to R}\frac{\mathrm{h}(x)-\mathrm{h}(R)}{x-R}=\frac{1}{m}\neq 0$, h crosses the x-axis with non-zero gradient at $x=R$, so R is a simple root of $\mathrm{h}(x)$
(d)	$x_{n+1}=x_n-\frac{\mathrm{h}(x_n)}{\mathrm{h}'(x_n)}$ $=x_n-\frac{\frac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}}{\frac{\mathrm{f}'(x_n)\mathrm{f}'(x_n)-\mathrm{f}(x_n)\mathrm{f}''(x_n)}{\left(\mathrm{f}'(x_n)\right)^2}}$ $=x_n-\frac{\mathrm{f}(x_n)\mathrm{f}'(x_n)}{\left(\mathrm{f}'(x_n)\right)^2-\mathrm{f}(x_n)\mathrm{f}''(x_n)}$
(e)	The least n for $x_n=x_{n+1}$ correct to 3 decimal places 1.732 is 3 and thus the sequence converges much faster than in **(b)**

10	**Solution**
(a)	Given $\frac{\mathrm{d}y}{\mathrm{d}t}=-2(y-z)$, $z=\frac{1}{2}\frac{\mathrm{d}y}{\mathrm{d}t}+y$. Differentiating w.r.t. time t, $\frac{\mathrm{d}z}{\mathrm{d}t}=\frac{1}{2}\frac{\mathrm{d}^2y}{\mathrm{d}t^2}+\frac{\mathrm{d}y}{\mathrm{d}t}$. Also given that $\frac{\mathrm{d}z}{\mathrm{d}t}=-\frac{\mathrm{d}y}{\mathrm{d}t}-3z$, $\frac{1}{2}\frac{\mathrm{d}^2y}{\mathrm{d}t^2}+\frac{\mathrm{d}y}{\mathrm{d}t}=-\frac{\mathrm{d}y}{\mathrm{d}t}-3\left(\frac{1}{2}\frac{\mathrm{d}y}{\mathrm{d}t}+y\right)$ $\frac{\mathrm{d}^2y}{\mathrm{d}t^2}+2\frac{\mathrm{d}y}{\mathrm{d}t}=-2\frac{\mathrm{d}y}{\mathrm{d}t}-3\cdot 2\left(\frac{1}{2}\frac{\mathrm{d}y}{\mathrm{d}t}+y\right)$ $\therefore\ \frac{\mathrm{d}^2y}{\mathrm{d}t^2}+7\frac{\mathrm{d}y}{\mathrm{d}t}+6y=0$
(b)	Characteristic / Auxiliary Equation for this 2nd-order ODE of y :

	$x^2+7x+6=0$ $(x+1)(x+6)=0$ $\Rightarrow x=-1$ or $x=-6$ $\therefore\ y(t)=A\mathrm{e}^{(-1)t}+B\mathrm{e}^{(-6)t}$, where $A,\ B\in\mathbb{R}$ are arbitrary constants, $= A\mathrm{e}^{-t}+B\mathrm{e}^{-6t}$ (shown) Then, $z(t)=\frac{1}{2}\frac{\mathrm{d}y}{\mathrm{d}t}+y$ $=\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left(A\mathrm{e}^{-t}+B\mathrm{e}^{-6t}\right)+\left(A\mathrm{e}^{-t}+B\mathrm{e}^{-6t}\right)$ $=\frac{1}{2}\left(-A\mathrm{e}^{-t}-6B\mathrm{e}^{-6t}\right)+\left(A\mathrm{e}^{-t}+B\mathrm{e}^{-6t}\right)$ $\Rightarrow z(t)=\frac{1}{2}A\mathrm{e}^{-t}-2B\mathrm{e}^{-6t}$ (shown)
(c)	Given the initial conditions, $y(0)=A+B=5$ $z(0)=\frac{1}{2}A-2B=0$ Solving this pair of simultaneous equations, $A=4,\ B=1$ $\therefore\ z_1(t)=\frac{1}{2}(4)\mathrm{e}^{-t}-2(1)\mathrm{e}^{-6t}$ $=2\mathrm{e}^{-t}-2\mathrm{e}^{-6t}$
(d)	Given the boundary conditions, $z(0)=z(1)=c$ $z(0)=\frac{1}{2}A-2B=c \quad\Rightarrow\quad A=4B+2c$ $z(1)=\frac{1}{2}A\mathrm{e}^{-1}-2B\mathrm{e}^{-6}=c \quad\Rightarrow\quad \frac{1}{2}(4B+2c)\mathrm{e}^{-1}-2B\mathrm{e}^{-6}=c$ $(2B+c)-2B\mathrm{e}^{-5}=c\mathrm{e}$ $2B(1-\mathrm{e}^{-5})=c(\mathrm{e}-1)$ $B=\frac{c(\mathrm{e}-1)}{2(1-\mathrm{e}^{-5})}$ Then, $A=4B+2c$ $=4\left(\frac{c(\mathrm{e}-1)}{2(1-\mathrm{e}^{-5})}\right)+2c \quad =2c\left(\frac{\mathrm{e}-1}{1-\mathrm{e}^{-5}}+1\right) \quad =2c\left(\frac{\mathrm{e}^6-1}{\mathrm{e}^5-1}\right)$ $\therefore\ z_2(t)=\frac{1}{2}\left(2c\frac{\mathrm{e}^6-1}{\mathrm{e}^5-1}\right)\mathrm{e}^{-t}-2\left(\frac{1}{2}c\frac{\mathrm{e}-1}{1-\mathrm{e}^{-5}}\right)\mathrm{e}^{-6t}$ $=c\left(\frac{\mathrm{e}^6-1}{\mathrm{e}^5-1}\right)\mathrm{e}^{-t}-c\left(\frac{\mathrm{e}^6-\mathrm{e}^5}{\mathrm{e}^5-1}\right)\mathrm{e}^{-6t}$

(e)	The expression $\sum_{n=0}^{\infty} z_1(t+n)$ $= \sum_{n=0}^{\infty}\left(2\mathrm{e}^{-(t+n)} - 2\mathrm{e}^{-6(t+n)}\right)$ $= 2\mathrm{e}^{-t}\sum_{n=0}^{\infty}\mathrm{e}^{-n} - 2\mathrm{e}^{-6t}\sum_{n=0}^{\infty}\mathrm{e}^{-6n}$ $= 2\mathrm{e}^{-t}\frac{1}{1-\mathrm{e}^{-1}} - 2\mathrm{e}^{-6t}\frac{1}{1-\mathrm{e}^{-6}}$ $= 2\frac{\mathrm{e}}{\mathrm{e}-1}\mathrm{e}^{-t} - 2\frac{\mathrm{e}^6}{\mathrm{e}^6-1}\mathrm{e}^{-6t}$ $= 2\frac{\mathrm{e}}{\mathrm{e}-1}\left(\frac{\mathrm{e}^5-1}{\mathrm{e}^6-1}\right)\left(\frac{\mathrm{e}^6-1}{\mathrm{e}^5-1}\right)\mathrm{e}^{-t} - \underbrace{2\frac{\mathrm{e}^6}{\mathrm{e}^6-1}\left(\frac{\mathrm{e}^5-1}{\mathrm{e}^6-\mathrm{e}^5}\right)}_{=2\frac{\mathrm{e}^6}{\mathrm{e}^6-1}\left(\frac{\mathrm{e}^5-1}{\mathrm{e}^5(\mathrm{e}-1)}\right)}\left(\frac{\mathrm{e}^6-\mathrm{e}^5}{\mathrm{e}^5-1}\right)\mathrm{e}^{-6t}$ $= \underbrace{\left(\frac{2\mathrm{e}(\mathrm{e}^5-1)}{(\mathrm{e}-1)(\mathrm{e}^6-1)}\right)}_{:=c}\left(\frac{\mathrm{e}^6-1}{\mathrm{e}^5-1}\right)\mathrm{e}^{-t} - \underbrace{\left(\frac{2\mathrm{e}(\mathrm{e}^5-1)}{(\mathrm{e}-1)(\mathrm{e}^6-1)}\right)}_{:=c}\left(\frac{\mathrm{e}^6-\mathrm{e}^5}{\mathrm{e}^5-1}\right)\mathrm{e}^{-6t}$ $= c\left(\frac{\mathrm{e}^6-1}{\mathrm{e}^5-1}\right)\mathrm{e}^{-t} - c\left(\frac{\mathrm{e}^6-\mathrm{e}^5}{\mathrm{e}^5-1}\right)\mathrm{e}^{-6t}$ $= z_2(t)$ $\therefore \sum_{n=0}^{\infty} z_1(t+n) = z_2(t)$, where $c = \frac{2\mathrm{e}(\mathrm{e}^5-1)}{(\mathrm{e}-1)(\mathrm{e}^6-1)}$ (shown)