EUNOIA JUNIOR COLLEGE

JC2 Preliminary Examination 2025

General Certificate of Education Advanced Level

Higher 2

CANDIDATE NAME

CIVICS GROUP

INDEX NO.

FURTHER MATHEMATICS **9649/01**

Paper 1

02 September 2025

3 hours

Additional Materials: Printed Answer Booklet
List of Formulae (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **6** printed pages and **2** blank pages.

1 The matrices **A** and **B** are given by

$$\mathbf{A}=\begin{pmatrix}1 & \alpha & \beta\gamma\\ 1 & \beta & \gamma\alpha\\ 1 & \gamma & \alpha\beta\end{pmatrix},\quad \mathbf{B}=\begin{pmatrix}1 & \beta & \gamma\alpha\\ 1 & \alpha & \beta\gamma\\ 3 & 3\gamma & 3\alpha\beta\end{pmatrix}.$$

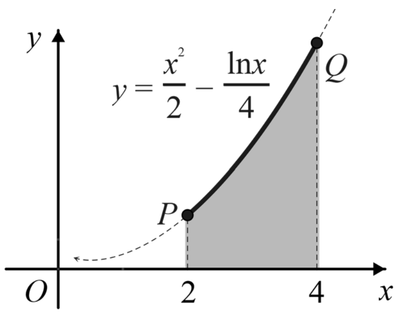
(a) Show that $\det\mathbf{A}=(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)$. [3]

(b) Hence, or otherwise, find $\det\mathbf{B}$. [2]

2 **(a)** Let $z=\cos\left(xy+\frac{\pi}{4}\right)$ where $x=\mathrm{e}^t$ and $y=\ln(1+t)$. By finding $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$, evaluate the exact value of $\frac{\mathrm{d}z}{\mathrm{d}t}$ when $t=0$. [4]

(b) Suppose the function $\mathrm{f}(x,y)$ satisfies $\mathrm{f}_{xy}(x,y)=\mathrm{f}_{yx}(x,y)=0$ for all *x* and *y*. What does this imply about how f_x and f_y depend on *x* and *y*? Hence deduce a general form of the function $\mathrm{f}(x,y)$. [2]

3 The diagram shows a shaded region bounded by an arc *PQ* having cartesian equation $y=\frac{x^2}{2}-\frac{\ln x}{4}$, vertical lines $x=2$, $x=4$ and the *x*-axis.



(a) Show that the exact length of the arc *PQ* is $6+\frac{1}{4}\ln 2$. [4]

(b) Both the arc and the shaded region are rotated completely about the *y*-axis.

(i) Find the area of the surface of revolution formed by arc *PQ*, correct to 2 decimal places. [3]

(ii) Find the volume of the solid of revolution formed by the shaded region, correct to 2 decimal places. [2]

4 **Do not use a calculator in answering this question.**

Use de Moivre's theorem to show that $\cos 5\theta = \cos\theta(16\cos^4\theta - 20\cos^2\theta + 5)$. [5]

Hence, or otherwise, evaluate $\cos^2\dfrac{\pi}{10}$ and deduce that $\cos\dfrac{\pi}{5} = \dfrac{1+\sqrt{5}}{4}$. [5]

5 The complex number z satisfies the three relations below:

- $|z-13| \le 5$
- $|z-8| \ge |z-18-10\mathrm{i}|$
- $-\tan^{-1}\dfrac{1}{2} \le \arg(z) \le \tan^{-1}\dfrac{1}{2}$

(a) **(i)** Illustrate all three relations clearly on a single Argand diagram. [6]

(ii) One of the relations is implied by another. Identify the two relations involved, justifying your answer. [2]

(b) Determine the maximum and minimum possible distances of z from the origin. [2]

6 Let $\mathbf{A} = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$ be a real symmetric matrix. The matrix $\mathbf{A}$ is said to be positive definite if $\mathbf{x}^{\mathrm{T}}\mathbf{A}\mathbf{x} > 0$ for all non-zero vectors $\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}$.

(a) Show that if $\mathbf{A}$ is symmetric and positive definite, then both the eigenvalues of $\mathbf{A}$ are positive. [2]

(b) Show that $\mathbf{x}^{\mathrm{T}}\mathbf{A}\mathbf{x} = a\left(x + \dfrac{b}{a}y\right)^2 + \left(\dfrac{ac-b^2}{a}\right)y^2$ if a is non-zero. Hence deduce the conditions for $\mathbf{A}$ to be positive definite. [5]

(c) Let $\mathrm{f}(x, y) = 2x^2 + 2xy + 3y^2$ defined for all $(x, y) \in \mathbf{R}^2$. By expressing $\mathrm{f}(x, y)$ in the form of $\mathbf{x}^{\mathrm{T}}\mathbf{A}\mathbf{x}$ for a suitable symmetric matrix $\mathbf{A}$, explain why the origin is the global minimum. [3]

7 Let u_n be a sequence defined by the recurrence relation

$$u_{n+1} = 3u_n + 2 \text{ for } n \geq 0 \text{ with } u_0 = k$$

where k is a constant.

(a) Find u_n in terms of n. [4]

Let v_n be another sequence defined by the recurrence relation

$$v_{n+1} = 3v_n + 2^n \qquad (*)$$

for $n \geq 0$ with $v_0 = 1$.

Let $\mathrm{f}(x) = \sum_{n=0}^{\infty} v_n x^n$, $|x| < \frac{1}{3}$.

(b) By multiplying x^{n+1} to both sides of $(*)$, show that $\mathrm{f}(x) = \dfrac{A}{1-3x} + \dfrac{B}{1-2x}$ where A and B are constants to be determined. [4]

(c) Hence find an expression for v_n in terms of n. Determine also the limiting behaviour of $\dfrac{v_n}{3^n}$ as $n \to \infty$. [3]

8 In polar coordinates, curve C is defined by

$$r^2 - 2r\cos\theta + \sin^2\theta = 0 \text{ for } -\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4},\ r \geq 0.$$

(a) By solving for r in terms of θ, sketch the curve C, indicating clearly the behaviour of the curve near $r = 0$ and near $\theta = \pm\frac{\pi}{4}$. [5]

(b) Show that area of the region enclosed by curve C is given by the definite integral

$$4\int_0^{\frac{\pi}{4}} \cos\theta\sqrt{1-2\sin^2\theta}\ \mathrm{d}\theta,$$

and find the exact area, using the substitution $\sin u = \sqrt{2}\sin\theta$. [7]

9 Consider the equation $f(x)=0$, where $f(x)=x^5-10x^3+45x-24\sqrt{3}$.

It is given that there is a positive root near $x=2$.

(a) Use Newton-Raphson method with $x_1=2$ to obtain a sequence of iterates until, correct to 3 decimal places, $x_n=x_{n+1}$. Write down the least value of *n* for which this is so and state the corresponding value of x_n. [3]

(b) Verify, by substitution, that $\sqrt{3}$ is a root for $f(x)=0$. Supporting working must be shown. [2]

Equations can have repeated roots. So, for example, the equation $(x+1)(x-3)^2=0$ is said to have three roots: $x=-1$ (of multiplicity 1) and $x=3$ (a repeated root of multiplicity 2). If the root has multiplicity 1, it is said to be a *simple* root.

The rate of convergence of the Newton-Raphson method is slowed if the root is not simple. One method of handling the problem of repeated roots is to define

$$h(x)=\frac{f(x)}{f'(x)}.$$

Let R be a root of $f(x)=0$ with multiplicity $m\geq 2$. Assume $f(x)$ can be written as $f(x)=(x-R)^m q(x)$, where $q(R)\neq 0$.

(c) Find $\lim_{x\to R}\frac{h(x)}{x-R}$ and hence explain why R is a simple root of $h(x)=0$. [3]

(d) By applying the Newton-Raphson method to $h(x)$, show that

$$x_{n+1}=x_n-\frac{f(x_n)f'(x_n)}{\left(f'(x_n)\right)^2-f(x_n)f''(x_n)}.$$ [2]

(e) Use the result in **(d)** with $x_1=2$ to obtain another sequence of iterates until, correct to 3 decimal places, $x_n=x_{n+1}$. Write down the least value of *n* for which this is so, state the corresponding value of x_n and comment on the efficiency. [3]

10 A pain-killing drug is injected into the bloodstream. It then diffuses into the brain, where it is absorbed.

The quantities at time t of the drug in the blood and the brain are $y(t)$ and $z(t)$ respectively, satisfying

$$\frac{\mathrm{d}y}{\mathrm{d}t} = -2(y-z), \quad \frac{\mathrm{d}z}{\mathrm{d}t} = -\frac{\mathrm{d}y}{\mathrm{d}t} - 3z.$$

(a) Obtain a second-order differential equation for y. [2]

(b) Hence, derive the general solution

$$y = A\mathrm{e}^{-t} + B\mathrm{e}^{-6t}, \quad z = \frac{1}{2}A\mathrm{e}^{-t} - 2B\mathrm{e}^{-6t},$$

where A and B are arbitrary constants. [4]

(c) When a single dose of the drug is injected into the bloodstream at initial time $t=0$ such that $y(0)=5$, given that no drug is initially in the brain $z(0)=0$, find the particular solution for the resulting quantity of the drug in the brain, denoted by $z_1(t)$. [2]

(d) In another regimen, identical doses of the drug are periodically injected at each unit time, and the system has reached a steady state: $z(0)=z(1)=c$, where c is a given constant.

Find, in terms of c, the particular solution for quantity of the drug in the brain in this regimen, denoted by $z_2(t)$. [3]

(e) In steady state, each preceding injection at times $t = \ldots -3, -2, -1, 0$ contributes its own z_1 profile.

Show that the superposition of all earlier dose responses $\sum_{n=0}^{\infty} z_1(t+n) = z_2(t)$ when c takes a particular constant value, and find it. [3]

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