

JURONGVILLE SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2025
Secondary 4 Express



STUDENT
NAME

MARK SCHEME

CLASS

INDEX
NUMBER

ADDITIONAL MATHEMATICS

4049/02

Paper 2

20 AUGUST 2025

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 90.

DO NOT OPEN THE BOOKLET UNTIL YOU ARE TOLD TO DO SO

For Examiner's Use

90

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

- 1 (a) Find the first 3 terms in the expansion of $(1+3x)^8$ in ascending powers of x . [2]

$$\begin{aligned}(1+3x)^8 &= 1 + 8(3x) + \binom{8}{2}(3x)^2 + \dots && \text{M1} \\ &= 1 + 24x + 252x^2 + \dots && \text{A1}\end{aligned}$$

- (b) Hence determine the first 3 terms in the expansion of $(2+6x+3x^2)^8$. [3]

$$\begin{aligned}(2+6x+3x^2)^8 &= 2^8 \left(1+3x+\frac{3}{2}x^2\right)^8 && \text{M1} \\ &= 256 \left(1+3\left(x+\frac{1}{2}x^2\right)\right)^8 && \text{M1} \\ &= 256 \left(1+24\left(x+\frac{1}{2}x^2\right)+252\left(x+\frac{1}{2}x^2\right)^2+\dots\right) \\ &= 256(1+24x+12x^2+252x^2+\dots) \\ &= 256(1+24x+264x^2+\dots) \\ &= 256+6144x+67584x^2+\dots && \text{A1}\end{aligned}$$

- (c) Explain why there is no constant term in the expansion of $\left(2-\frac{1}{3x}\right)^2(1+3x)^8$. [3]

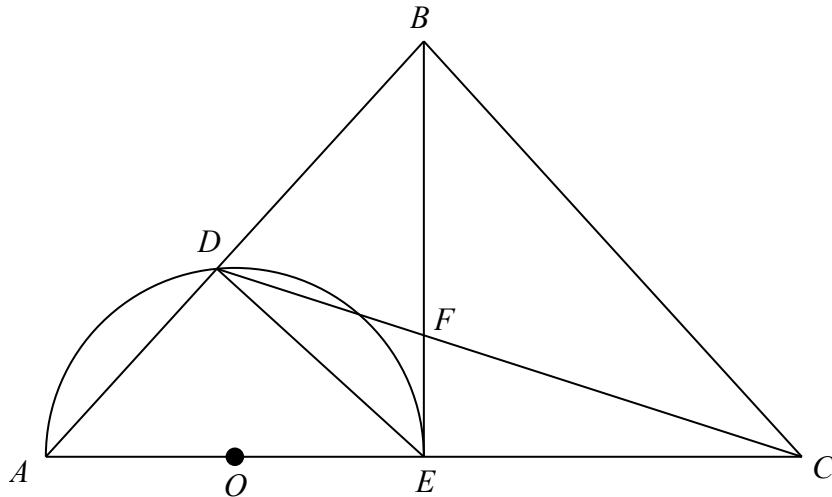
$$\begin{aligned}\left(2-\frac{1}{3x}\right)^2(1+3x)^8 &= \left(2-\frac{1}{3x}\right)^2(1+24x+252x^2+\dots) \\ &= \left(4-\frac{4}{3x}+\frac{1}{9x^2}\right)(1+24x+252x^2+\dots) && \text{M1}\end{aligned}$$

$$\begin{aligned}\text{constant term} &= 4(1) - \frac{4}{3}(24) + \frac{1}{9}(252) && \text{M1} \\ &= 0\end{aligned}$$

Hence there is no constant term in the expansion of

$$\left(2-\frac{1}{3x}\right)^2(1+3x)^8. \quad \text{A1}$$

2



ADE is a semicircle with centre O . It is given that D and E are midpoints of AB and AC respectively and $AB = BC$.

- (a) Prove that $\triangle ABE \equiv \triangle CBE$. [3]

In $\triangle ABE$ and $\triangle CBE$,

$AB = BC$ (given)

M2 for 3 conditions

$AE = EC$ (E is the midpoint of AC)

M1 for 2 conditions

BE is a common height

$\triangle ABE$ is congruent to $\triangle CBE$. (SSS)

A1

- (b) Prove that $\angle BEA = 90^\circ$. [1]

Since $\triangle ABE$ is congruent to $\triangle CBE$, $\angle BEA = \angle BEC$. (corresponding angles of congruent triangles)

$\angle BEA + \angle BEC = 180^\circ$ (angles on a straight line)

$\angle BEA + \angle BEA = 180^\circ$

$\angle BEA = 90^\circ$

} B1

- (c) Determine, with explanation, whether $\triangle ADE$ is similar to $\triangle ABC$. [3]

Since D and E are midpoints of AB and AC respectively, by midpoint theorem, $DE \parallel BC$ and $DE = \frac{1}{2}BC$. B1

In $\triangle ADE$ and $\triangle ABC$,

$\angle DAE = \angle BAC$ (common angle) M1

$\angle ADE = \angle ABC$ (corresponding angle, $DE \parallel BC$) M1

$\triangle ADE$ is similar to $\triangle ABC$. (AA similarity test)

- (d) Show that $\frac{DF}{FC} = \frac{1}{2}$. [3]

In $\triangle DFE$ and $\triangle CFB$,

$\angle EFD = \angle BFC$ (vertically opposite angle) M1

$\angle FDE = \angle FCB$ (alternate angle, $DE \parallel BC$) M1

$\triangle DFE$ is similar to $\triangle CFB$.

By midpoint theorem, $DE = \frac{1}{2}BC$.

$$\frac{DF}{FC} = \frac{DE}{BC}$$

$$= \frac{1}{2} \quad (\text{proven}) \quad \text{A1}$$

- 3 A radioactive substance decays according to the equation $A = 15e^{-bt}$, where A is the mass in grams of radioactive substance remaining and t is the time in hours after the radioactive substance starts to decay.

- (a) Given that there are 10 grams of radioactive substance remaining after 30 hours, verify that the value of b is 0.013516. [2]

$$\begin{aligned}
 A &= 15e^{-bt} \\
 10 &= 15e^{-30b} \\
 e^{-30b} &= \frac{10}{15} && \text{M1} \\
 -30b &= \ln \frac{10}{15} \\
 b &= -\frac{1}{30} \ln \frac{10}{15} \\
 &= 0.013516 && \text{A1}
 \end{aligned}$$

- (b) Find the mass of radioactive substance left at the end of 1 week. [1]

$$\begin{aligned}
 A &= 15e^{-0.013516(7 \times 24)} \\
 &= 1.55\text{g} && \text{B1}
 \end{aligned}$$

- (c) Find the number of hours for the radioactive substance to decay to half of its original mass. [3]

$$\text{When } t = 0, A = 15. \quad \text{M1}$$

$$\begin{aligned}
 A &= 15e^{-0.013516t} \\
 \frac{15}{2} &= 15e^{-0.013516t} \\
 e^{-0.013516t} &= \frac{1}{2} && \text{M1} \\
 -0.013516t &= \ln \frac{1}{2} \\
 t &= 51.3 \text{ hours} && \text{A1}
 \end{aligned}$$

- (d) Find the rate at which the substance decays at $t = 50$ hours. [2]

$$A = 15e^{-0.013516t}$$

$$\frac{dA}{dt} = -0.20274e^{-0.013516t} \quad \text{M1}$$

When $t = 50$,

$$\begin{aligned} \frac{dA}{dt} &= -0.20274e^{-0.013516(50)} \\ &= -0.103 \end{aligned}$$

The rate of radioactive decay at $t = 50$ hours is 0.103g/h. A1

- 4 (a) Prove that $\cot(45^\circ - x) = \frac{\cot x + 1}{\cot x - 1}$. [3]

$$\begin{aligned}
 \cot(45^\circ - x) &= \frac{1}{\tan(45^\circ - x)} \\
 &= \frac{1 + \tan 45^\circ \tan x}{\tan 45^\circ - \tan x} && \text{M1} \\
 &= \frac{1 + \tan x}{1 - \tan x} && \text{M1} \\
 &= \frac{1 + \frac{1}{\cot x}}{1 - \frac{1}{\cot x}} \\
 &= \frac{\cot x + 1}{\cot x - 1} \text{ (proven)} && \text{A1}
 \end{aligned}$$

$$\begin{aligned}
 \cot(45^\circ - x) &= \frac{\cos(45^\circ - x)}{\sin(45^\circ - x)} \\
 &= \frac{\cos 45^\circ \cos x + \sin 45^\circ \sin x}{\sin 45^\circ \cos x - \cos 45^\circ \sin x} && \text{M1} \\
 &= \frac{\cos x + \sin x}{\cos x - \sin x} \\
 &= \frac{\frac{\cos x}{\sin x} + 1}{\frac{\cos x}{\sin x} - 1} && \text{M1} \\
 &= \frac{\cot x + 1}{\cot x - 1} \text{ (proven)} && \text{A1}
 \end{aligned}$$

- (b) Hence express $\cot 15^\circ$ in the form $p + q\sqrt{3}$, where p and q are integers. [3]

$$\cot 15^\circ = \cot(45^\circ - 30^\circ) \quad \text{M1 - sub } x = 30^\circ$$

$$= \frac{\cot 30^\circ + 1}{\cot 30^\circ - 1}$$

$$= \frac{\frac{1}{\tan 30^\circ} + 1}{\frac{1}{\tan 30^\circ} - 1}$$

$$= \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

$$= \frac{\sqrt{3} + 1}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} \quad \text{M1}$$

$$= \frac{3 + 2\sqrt{3} + 1}{3 - 1}$$

$$= \frac{4 + 2\sqrt{3}}{2}$$

$$= 2 + \sqrt{3} \quad \text{A1}$$

5 It is given that $\cos \theta = \frac{5}{13}$, where $0^\circ < \theta < 360^\circ$.

(a) Show that the value of $2 - \sin^2 \theta$ is $\frac{194}{169}$. [2]

$$\begin{aligned}
 2 - \sin^2 \theta &= 2 - (1 - \cos^2 \theta) && \text{M1} \\
 &= 1 + \cos^2 \theta \\
 &= 1 + \left(\frac{5}{13}\right)^2 \\
 &= \frac{194}{169} && \text{A1 – with correct substitution}
 \end{aligned}$$

(b) By finding the value of $\sin^2 \theta$, find the exact value of $\tan^2(90^\circ - \theta)$. [4]

$$\begin{aligned}
 2 - \sin^2 \theta &= \frac{194}{169} \\
 \sin^2 \theta &= \frac{144}{169} && \text{B1} \\
 \tan^2(90^\circ - \theta) &= \frac{1}{\tan^2 \theta} && \text{M1} \\
 &= \frac{\cos^2 \theta}{\sin^2 \theta} \\
 &= \frac{\left(\frac{5}{13}\right)^2}{\frac{144}{169}} && \text{M1} \\
 &= \frac{25}{144} && \text{A1}
 \end{aligned}$$

- 6 The function $f(x) = x^3 + px^2 + qx - 12$, where p and q are constants, has a factor $x + 1$ and leaves a remainder of 36 when divided by $x - 3$.

- (a) Show that $p = 5$ and $q = -8$. [4]

$$f(x) = x^3 + px^2 + qx - 12$$

$$f(-1) = 0$$

$$-1 + p - q - 12 = 0$$

$$p - q = 13 \text{ ----- ①} \quad \text{M1}$$

$$f(3) = 36$$

$$27 + 9p + 3q - 12 = 36$$

$$9p + 3q = 21$$

$$3p + q = 7 \text{ ----- ②} \quad \text{M1}$$

$$\text{①} + \text{②}:$$

$$4p = 20 \quad \text{M1}$$

$$p = 5$$

$$q = 5 - 13$$

$$q = -8$$

$$\therefore p = 5 \quad \text{and} \quad q = -8 \text{ (shown)} \quad \text{A1}$$

- (b) Using these values of p and q , solve the equation $f(x) = 0$. [2]

When $p = 5$ and $q = -8$,

$$f(x) = 0$$

$$x^3 + 5x^2 - 8x - 12 = 0$$

$$(x+1)(x^2 + 4x - 12) = 0 \quad \text{M1}$$

$$(x+1)(x+6)(x-2) = 0$$

$$x = -1 \quad \text{or} \quad x = -6 \quad \text{or} \quad x = 2 \quad \text{A1}$$

- 7 (a) Given that $\log_7 x = m$ and $\log_7 y = 2n$, express xy in terms of m and n . [2]

Given that

$$\log_7 x = m \text{ and } \log_7 y = 2n$$

$$x = 7^m \qquad y = 7^{2n} \qquad \text{M1}$$

$$xy = (7^m)(7^{2n})$$

$$xy = 7^{m+2n} \qquad \text{A1}$$

- (b) Solve the equation $\log_3 3x^2 + 4 = \log_x 27$. [5]

$$\log_3 3x^2 + 4 = \log_x 27$$

$$\log_3 3 + 2\log_3 x + 4 = \log_x 27 \qquad \text{M1}$$

$$2\log_3 x + 5 = \frac{\log_3 3^3}{\log_3 x} \qquad \text{M1 – for Change of Base Law}$$

$$2\log_3 x + 5 = \frac{3}{\log_3 x}$$

$$\text{Let } y = \log_3 x,$$

$$2y + 5 = \frac{3}{y} \qquad \text{M1}$$

$$2y^2 + 5y - 3 = 0$$

$$(2y - 1)(y + 3) = 0$$

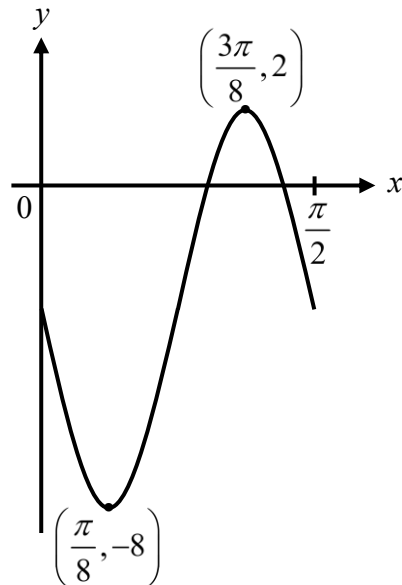
$$\therefore y = \frac{1}{2} \quad \text{or} \quad y = -3$$

$$\log_3 x = \frac{1}{2} \quad \text{or} \quad \log_3 x = -3 \qquad \text{M1}$$

$$x = 3^{\frac{1}{2}} \quad \text{or} \quad x = 3^{-3}$$

$$= \sqrt{3} \qquad = \frac{1}{27} \qquad \text{A1}$$

8



The diagram shows the curve $y = a \sin bx + c$ for $0 \leq x \leq \frac{\pi}{2}$ radians.

The curve has a minimum point at $\left(\frac{\pi}{8}, -8\right)$ and a maximum point at $\left(\frac{3\pi}{8}, 2\right)$.

- (a) Explain why $c = -3$. [2]

$y = c$ is axis of curve

$$c = \frac{\text{maximum} + \text{minimum}}{2}$$

$$= \frac{2 + (-8)}{2} \quad \text{M1}$$

$$= -3 \quad \text{A1}$$

- (b) Explain why $b = 4$. [2]

$$\frac{2\pi}{b} = \text{period}$$

$$\frac{2\pi}{b} = \frac{\pi}{2} \quad \text{M1}$$

$$b = 4 \quad \text{A1}$$

- (c) Hence find the equation of the curve. [2]

$$y = a \sin bx + c$$

Since this is a negative sine graph and the amplitude is 5, $a = -5$. M1

The equation of the curve is $y = -5 \sin 4x - 3$. A1

- 9 A boat travelling in a straight line passes a buoy, B , with a speed of v m/s. The velocity of the boat after passing B is given by $v = t^2 - 5t + 4$. The boat comes to rest momentarily, first at point C and then at point D .

(a) Find the distance CD .

[5]

$$v = t^2 - 5t + 4$$

$$s = \int t^2 - 5t + 4 \, dt$$

$$= \frac{t^3}{3} - \frac{5t^2}{2} + 4t + c$$

When $t = 0$, $s = 0$, $c = 0$.

$$s = \frac{t^3}{3} - \frac{5t^2}{2} + 4t \quad \text{M1}$$

At instantaneous rest,

$$v = 0$$

$$t^2 - 5t + 4 = 0$$

M1

$$(t-1)(t-4) = 0$$

$$t = 1 \quad \text{or} \quad 4$$

M1

When $t = 1$,

$$s = \frac{1}{3} - \frac{5}{2} + 4$$

$$= 1\frac{5}{6} \text{ m}$$

When $t = 4$,

$$s = \frac{4^3}{3} - \frac{5(4)^2}{2} + 4(4)$$

$$= -2\frac{2}{3} \text{ m}$$

$$\text{Distance between } C \text{ and } D = 1\frac{5}{6} - \left(-2\frac{2}{3}\right) \quad \text{M1}$$

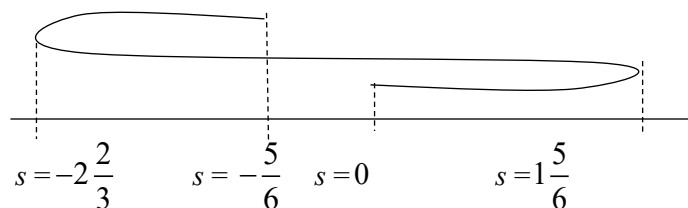
$$= 4\frac{1}{2} \text{ m} \quad \text{A1}$$

- (b) Find the total distance travelled by the boat in the first 5 seconds after passing B . [3]

When $t = 5$,

$$s = \frac{5^3}{3} - \frac{5(5)^2}{2} + 4(5) \quad \text{M1}$$

$$= -\frac{5}{6} \text{ m}$$



$$\text{Total distance travelled by boat} = 1\frac{5}{6} + 4\frac{1}{2} + \left(-\frac{5}{6} + 2\frac{2}{3}\right) \quad \text{M1}$$

$$= 8\frac{1}{6} \text{ m} \quad \text{A1}$$

- (c) Given that A is the point at which the boat has zero acceleration, determine with full working, whether A is nearer to B or to D . [3]

$$a = 2t - 5$$

When $a = 0$,

$$2t - 5 = 0 \quad \text{M1}$$

$$t = 2\frac{1}{2}$$

When $t = 2\frac{1}{2}$,

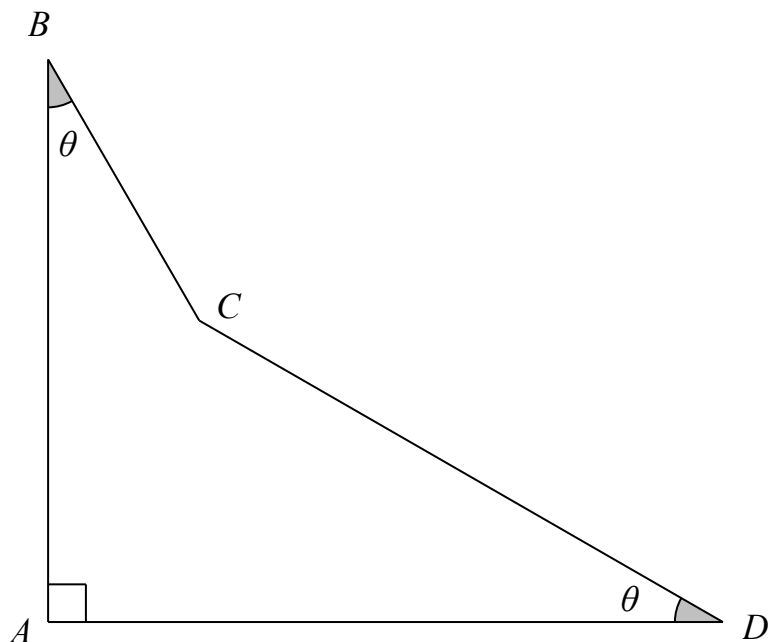
$$s = \frac{\left(2\frac{1}{2}\right)^3}{3} - \frac{5\left(2\frac{1}{2}\right)^2}{2} + 4\left(2\frac{1}{2}\right) \quad \text{M1}$$

$$= -\frac{5}{12} \text{ m}$$

$$\left. \begin{array}{l} \text{distance from } A \text{ to } B = \frac{5}{12} \text{ m} \\ \text{distance from } A \text{ to } D = \left(2\frac{2}{3} - \frac{5}{12}\right) \text{ m} \\ \qquad \qquad \qquad = 2\frac{1}{4} \text{ m} \end{array} \right\} \text{A1 - with conclusion}$$

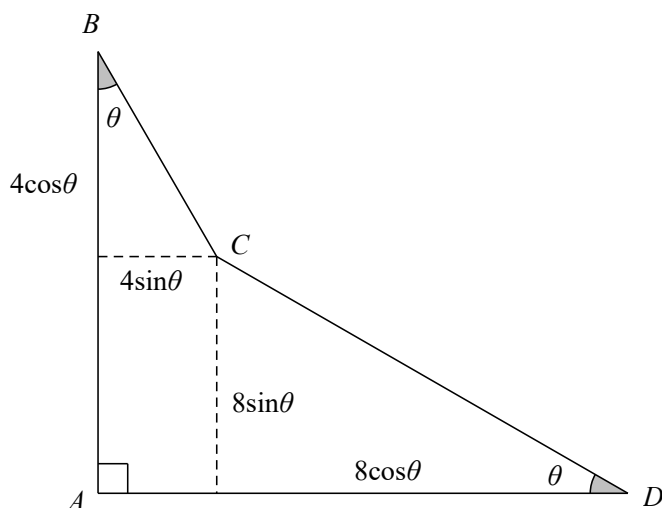
A is nearer to B .

10



The diagram shows a drawing of a mini garden $ABCD$ in the school. It is given that $BC = 4$ m, $CD = 8$ m and $\angle ABC = \angle ADC = \theta$, where θ is an acute angle measured in degrees.

- (a) Show that the area of the mini garden is $A = 20 \sin 2\theta - 16 \cos 2\theta + 16$. [4]



$$\begin{aligned}
A &= \frac{1}{2}(4 \cos \theta)(4 \sin \theta) + \frac{1}{2}(8 \cos \theta)(8 \sin \theta) + (4 \sin \theta)(8 \sin \theta) & \text{M1} \\
&= 8 \cos \theta \sin \theta + 32 \cos \theta \sin \theta + 32 \sin^2 \theta \\
&= 40 \cos \theta \sin \theta + 32 \sin^2 \theta & \text{M1} \\
&= 20 \sin 2\theta + 32 \left(\frac{1 - \cos 2\theta}{2} \right) & \text{M1} \\
&= 20 \sin 2\theta - 16 \cos 2\theta + 16 & \text{A1}
\end{aligned}$$

- (b) Express A in the form $R \sin(2\theta - \alpha) + 16$, where $R > 0$ and α is an acute angle. [3]

$$\begin{aligned}
A &= 20 \sin 2\theta - 16 \cos 2\theta + 16 \\
&= \sqrt{20^2 + 16^2} \sin \left(2\theta - \tan^{-1} \left(\frac{16}{20} \right) \right) + 16 & \begin{array}{l} \text{M1 for } R \\ \text{M1 for } \alpha \end{array} \\
&= \sqrt{656} \sin(2\theta - 38.659^\circ) + 16 \\
&= \sqrt{656} \sin(2\theta - 38.7^\circ) + 16 & \text{A1}
\end{aligned}$$

- (c) Find the value of θ when the area of the garden is 25 m^2 . [3]

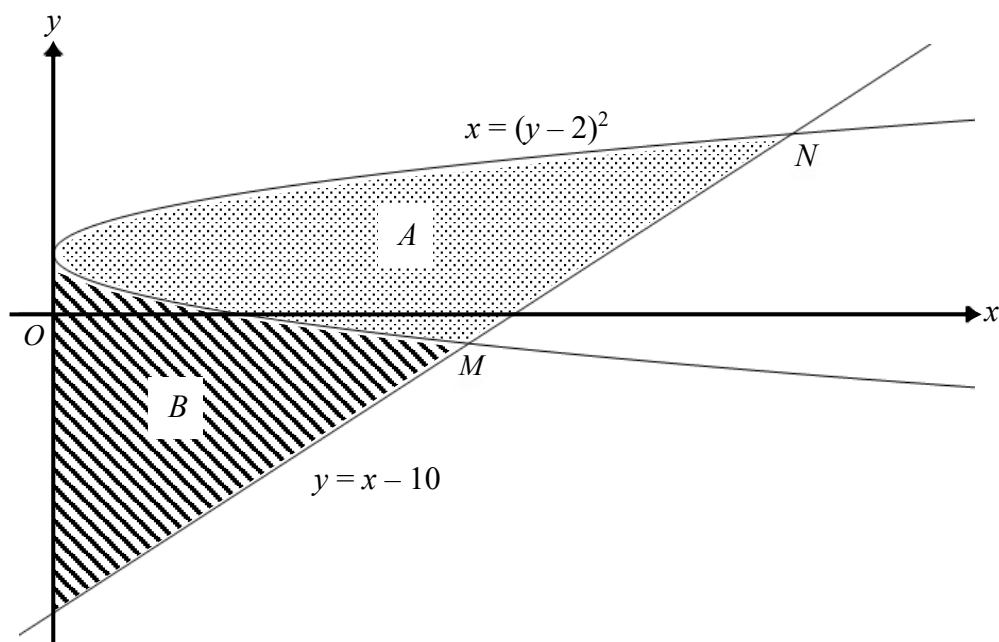
$$\begin{aligned}
\sqrt{656} \sin(2\theta - 38.659^\circ) + 16 &= 25 \\
\sin(2\theta - 38.659^\circ) &= \frac{9}{\sqrt{656}} & \text{M1} \\
2\theta - 38.659^\circ &= 20.572^\circ & \text{M1} \\
\theta &= 29.6^\circ \text{ (1 d.p.)} & \text{A1}
\end{aligned}$$

- (d) Determine the maximum value of A and the corresponding value of θ . [3]

$$\begin{aligned}
\text{Maximum value of } A &= \sqrt{656} + 16 \\
&= 41.6 \text{ m (3 s.f.)} & \text{M1}
\end{aligned}$$

$$\begin{aligned}
\text{Corresponding value of } \theta: \\
2\theta - 38.659^\circ &= 90^\circ & \text{M1} \\
\theta &= 64.3^\circ \text{ (1 d.p.)} & \text{A1}
\end{aligned}$$

11



The curve $x = (y - 2)^2$ and the line $y = x - 10$ intersect at the points M and N .

(a) Find the coordinates of M and N .

[3]

$$x = (y - 2)^2 \quad \text{---} \quad (1)$$

$$y = x - 10 \quad \text{---} \quad (2)$$

Sub (1) into (2):

$$y = (y - 2)^2 - 10 \quad \text{M1}$$

$$y = (y^2 - 4y + 4) - 10$$

$$y^2 - 5y - 6 = 0$$

$$(y + 1)(y - 6) = 0$$

$$y = -1 \quad \text{or} \quad y = 6 \quad \text{M1}$$

$$x = 9 \quad \text{or} \quad x = 16$$

Coordinates of M and N are $(9, -1)$ and $(16, 6)$ respectively. A1

- (b) Find the area of shaded region A and the area of the shaded region B . [6]

$$\begin{aligned}
 \text{area of region } A &= \frac{1}{2}(9+16)(6+1) - \int_{-1}^6 (y-2)^2 dy && \text{M2} \\
 &= 87\frac{1}{2} - \left[\frac{(y-2)^3}{3} \right]_{-1}^6 && \text{M1 – correct integration} \\
 &= 87\frac{1}{2} - \left(\frac{(6-2)^3}{3} - \frac{(-1-2)^3}{3} \right) \\
 &= 57\frac{1}{6} \text{ units}^2 && \text{A1}
 \end{aligned}$$

$$\begin{aligned}
 \text{area of region } B &= \int_{-1}^2 (y-2)^2 dy + \frac{1}{2}(10-1)(9) && \text{M1} \\
 &= \left[\frac{(y-2)^3}{3} \right]_{-1}^2 + \frac{81}{2} \\
 &= \frac{(2-2)^3}{3} - \frac{(-1-2)^3}{3} + \frac{81}{2} \\
 &= 49\frac{1}{2} \text{ units}^2 && \text{A1}
 \end{aligned}$$

END OF PAPER