O

JURONGVILLE SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2025
Secondary 4 Express

STUDENT NAME	**MARKING SCHEME**
CLASS	
INDEX NUMBER	

ADDITIONAL MATHEMATICS **4049/01**

Paper 1

19 AUGUST 2025

2 hours 15 minutes

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE ON ANY BARCODES.

Answer **ALL** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total marks for this paper is 90.

DO NOT OPEN THE BOOKLET UNTIL YOU ARE TOLD TO DO SO

For Examiner's Use
90

Setter: Mr Shahul Hameed

This document consists of **18** printed pages and **0** blank page.

[Turn Over

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 The curve has the equation $y = x^2 - 4x + 7$.

A line with gradient 1 is a tangent to the curve.

Find the equation of the tangent line. [3]

$\frac{dy}{dx} = 2x - 4$ [M1]

$2x - 4 = 1$

$x = 2.5, y = 3.25$ [M1]

$y - 3.25 = 1(x - 2.5)$

$y = x + 0.75$ [A1]

2 Find the set of values of the constant p for which the curve $y = px^2 + (p-2)x + 5$ lies entirely above the x-axis. [5]

$p > 0$ [B1]

$(p-2)^2 - 4(p)(5) < 0$ [M1]

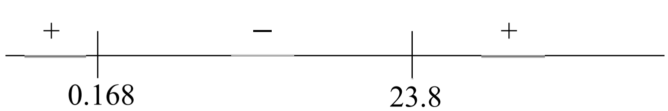
$p^2 - 24p + 4 < 0$

Use of quadratic formula [M1]

Critical Points: $p = 12 \pm 2\sqrt{35}$ [M1]

$12 - 2\sqrt{35} < p < 12 + 2\sqrt{35}$ [A1]

$0.168 < p < 23.8$



3 **(a)** Prove that $\dfrac{\cot^2\theta-\tan^2\theta}{\left(1+\tan^2\theta\right)\left(1+\cot^2\theta\right)}=\cos 2\theta$. [5]

$$LHS=\frac{\dfrac{\cos^2\theta}{\sin^2\theta}-\dfrac{\sin^2\theta}{\cos^2\theta}}{\left(\sec^2\theta\right)\left(\operatorname{cosec}^2\theta\right)}\ \text{[M1]}$$

$$=\frac{\dfrac{\cos^4\theta-\sin^4\theta}{\sin^2\theta\cos^2\theta}}{\dfrac{1}{\cos^2\theta}\times\dfrac{1}{\sin^2\theta}}\ \text{[M1]}$$

$$=\frac{\cos^4\theta-\sin^4\theta}{\sin^2\theta\cos^2\theta}\times\frac{\cos^2\theta\sin^2\theta}{1}\ \text{[M1]}$$

$$=\cos^4\theta-\sin^4\theta$$

$$=\left(\cos^2\theta-\sin^2\theta\right)\left(\cos^2\theta+\sin^2\theta\right)\ \text{[M1]}$$

$$=\left(\cos 2\theta\right)\left(1\right)$$

$$=\cos 2\theta\ \text{[A1]}$$

3 **(b)** Hence, solve the equation $\frac{\left(\cot^2\theta-\tan^2\theta\right)\left(2\sin 2\theta\right)}{\left(1+\tan^2\theta\right)\left(1+\cot^2\theta\right)}=0.75$ for $-90°\le\theta\le 90°$. [5]

$\frac{\left(\cot^2\theta-\tan^2\theta\right)\left(2\sin 2\theta\right)}{\left(1+\tan^2\theta\right)\left(1+\cot^2\theta\right)}=0.75$

$2\sin 2\theta\cos 2\theta=0.75$ [M1]

$\sin 4\theta=0.75$ [M1]

$\alpha=\sin^{-1}\left(0.75\right)=48.59037789°$ [M1]

$4\theta=48.5903,-311.4096,131.4096,-228.5903$ [M1]

$\theta=-77.9°,-57.1°,12.1°,32.9°$ [A1]

4 The line $y - x = 5$ intersects the curve $x^2 + xy + y^2 = 91$ at two points.
Find the coordinates of these two points. [5]

$y = x + 5$

$x^2 + x(x+5) + \left(x+5\right)^2 = 91$ [M1]

$x^2 + x^2 + 5x + x^2 + 10x + 25 - 91 = 0$

$3x^2 + 15x - 66 = 0$ [M1]

$x^2 + 5x - 22 = 0$

Using Quadratic Formula, [B1]

$x = 2.82$ or -7.82 [M1]

$\left(2.82,\ 7.82\right)$ or $\left(-7.82,\ -2.82\right)$ [A1]

5 The function f is defined for $x > 0$ by $f(x) = \dfrac{x^2+4}{x+1}$.

(a) Find $f'(x)$ in the form $\dfrac{ax^2+bx+c}{(x+1)^2}$. [2]

$$f'(x) = \frac{(x+1)(2x)-(x^2+4)(1)}{(x+1)^2} \text{ [M1]}$$

$$= \frac{x^2+2x-4}{(x+1)^2} \text{ [A1]}$$

(b) Hence determine the values of x for which f is increasing. [4]

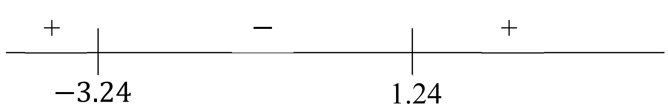
For f to be increasing, $f'(x)$ must be > 0.

Since, $(x+1)^2 > 0$ for all real values of x, then to solve $x^2+2x-4>0$. [B1]

Show use of quadratic formula [B1]

Critical Points: $x = -3.24$ or 1.24 [M1]

$x < -3.24$ or $x > 1.24$ [A1]



6 A balloon is being inflated such that its volume increases and the radius, r cm of the balloon, changes over time.

The rate of change of the radius at time t seconds is given by $\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{4}{t+2}$, $t > 0$.

Initially, at $t = 0$, the radius is 3 cm.

(a) Find an expression for r in terms of t. [2]

$r = \int \frac{4}{t+2} dt = 4\ln(t+2) + c$ [M1]

At $t = 0, r = 3, c = 3 - 4\ln 2$ or 0.227

$r = 4\ln(t+2) + 3 - 4\ln 2$ or $4\ln(t+2) + 0.227$ [A1]

(b) The volume V of the balloon is given by $V = \frac{4}{3}\pi r^3$.

Find the rate of increase of the volume of the balloon at $t = 2$. [4]

At $t = 2, r = 4\ln 4 + 3 - 4\ln 2$

$\frac{dv}{dr} = 4\pi r^2$ [M1]

At $t = 2$,

$\frac{dv}{dr} = 418.7464104,$ [M1]

$\frac{dr}{dt} = 1$

$\frac{dv}{dt} = \frac{dv}{dr} \times \frac{dr}{dt}$

$\frac{dv}{dt} = 418.7464104 \times 1$ [M1]

$\frac{dv}{dt} = 419 cm^3 / s$ [A1]

7 **(a)** Express $\dfrac{5x^2+3x+7}{(x+1)(x-2)^2}$ in partial fractions. [6]

$\dfrac{5x^2+3x+7}{(x+1)(x-2)^2}=\dfrac{A}{x+1}+\dfrac{B}{x-2}+\dfrac{C}{(x-2)^2}$ [M1]

$5x^2+3x+7=A(x-2)^2+B(x+1)(x-2)+C(x+1)$ [M1]

When $x=2, C=11$ [M1]

When $x=-1, A=1$ [M1]

When $x=0, B=4$ [M1]

$\dfrac{5x^2+3x+7}{(x+1)(x-2)^2}=\dfrac{1}{x+1}+\dfrac{4}{x-2}+\dfrac{11}{(x-2)^2}$ [A1]

(b) Hence $\displaystyle\int\frac{5x^2+3x+7}{(x+1)(x-2)^2}\,\mathrm{d}x$. [3]

$\displaystyle\int\frac{5x^2+3x+7}{(x+1)(x-2)^2}\,\mathrm{d}x$

$\displaystyle=\int\frac{1}{x+1}+\frac{4}{x-2}+\frac{11}{(x-2)^2}\,dx$ [M1]

$\displaystyle=\int\frac{1}{x+1}\,dx+\int\frac{4}{x-2}dx+\int 11(x-2)^{-2}$

$\displaystyle=\ln(x+1)+4\ln(x-2)-\frac{11}{x-2}+c$ [A2][Minus 1 for each incorrect part]

8 A curve has equation $y = x\sqrt{x+k} + 3$, where k is a positive constant.

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{3x+2k}{2\sqrt{x+k}}$. [3]

$\dfrac{dy}{dx} = x\left(\dfrac{1}{2}\right)\left(x+k\right)^{-\frac{1}{2}} + \sqrt{x+k}$ [M1]

$\dfrac{dy}{dx} = \dfrac{x}{2\sqrt{x+k}} + \sqrt{x+k}$

$\dfrac{dy}{dx} = \dfrac{x+2x+2k}{2\sqrt{x+k}}$ [M1]

$\dfrac{dy}{dx} = \dfrac{3x+2k}{2\sqrt{x+k}}$ [A1]

(b) Determine the nature of the stationary point on the curve. [6]

$$\frac{d^2y}{dx^2}=\frac{2\sqrt{x+k}\,(3)-(3x+2k)(2)\left(\frac{1}{2}\right)(x+k)^{-\frac{1}{2}}}{4(x+k)} \text{ [M1]}$$

$$\frac{d^2y}{dx^2}=\frac{(x+k)^{-\frac{1}{2}}\left[2(x+k)(3)-(3x+2k)\right]}{4(x+k)} \text{ [M1]}$$

$$\frac{d^2y}{dx^2}=\frac{6x+6k-3x-2k}{4(x+k)^{\frac{3}{2}}}$$

$$\frac{d^2y}{dx^2}=\frac{3x+4k}{4(x+k)^{\frac{3}{2}}} \text{ [M1]}$$

When $\dfrac{dy}{dx}=0, x=-\dfrac{2}{3}k$ [M1]

At $x=-\dfrac{2}{3}k, \dfrac{d^2y}{dx^2}=\dfrac{2k}{4\left(\frac{1}{3}k\right)^{\frac{3}{2}}}$

Since $k>0, \dfrac{d^2y}{dx^2}>0$. [M1]

$x=-\dfrac{2}{3}k$ is a minimum point on the curve. [A1]

9 A curve $y = \dfrac{1+3\ln x}{x^2}$ has a minimum point at P.

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [2]

$$\frac{dy}{dx} = \frac{\left(x^2\right)(3)\left(\frac{1}{x}\right) - (1+3\ln x)(2x)}{x^4} \text{ [M1]}$$

$$\frac{dy}{dx} = \frac{3x - 2x - 6x\ln x}{x^4}$$

$$\frac{dy}{dx} = \frac{1-6\ln x}{x^3} \text{ [A1]}$$

(b) Show that the x-coordinate of P is $e^{\frac{1}{6}}$. [2]

At $\dfrac{dy}{dx} = 0, 1 - 6\ln x = 0$ [M1]

$\ln x = \dfrac{1}{6}$

$x = e^{\frac{1}{6}}$ [A1]

(c) Find the equation of normal to the curve at the point $x = e$.
Leave your answer in terms of e. [5]

At $x = e, y = \frac{4}{e^2}$ [M1]

At $x = e, \frac{dy}{dx} = -\frac{5}{e^3}$ [M1]

gradient of normal $= \frac{e^3}{5}$ [M1]

$y - \frac{4}{e^2} = \frac{e^3}{5}\left(x - e\right)$ [M1]

$y = \frac{e^3}{5}x - \frac{e^4}{5} + \frac{4}{e^2}$ or $y = \frac{e^3}{5}x + \frac{20 - e^6}{5e^2}$ [A1]

10 The table shows experimental values of two variables x and y.

x	1	2	3	4	5	6	7
y	56.2	31.5	25.1	9.96	5.60	3.14	1.76

The variables x and y are related by the equation $y = \dfrac{h}{k^x}$, where h and k are constants.

It is believed that an error was made in one of the experimental values of y.

(a) On the grid, on the next page, plot lg y against x and draw a straight line graph to illustrate the information using a scale of 2 cm to represent 1 unit on the x-axis and 8 cm to represent 1 unit on the y-axis. [3]

(b) Use your graph to

(i) identify the abnormal reading of y and estimate the correct value of y, [2]

$y = 25.1$ is an abnormal reading of y. [B1]

correct value of y: $\lg y = 1.25$

$y = 10^{1.25} = 17.8$ [B1] [Accept answers between 13 to 24]

(ii) find the value of h and k. [4]

$$y = \frac{h}{k^x}$$

$\lg y = \lg h - x \lg k$

$\lg y = (-\lg k)x + \lg h$ [B1]

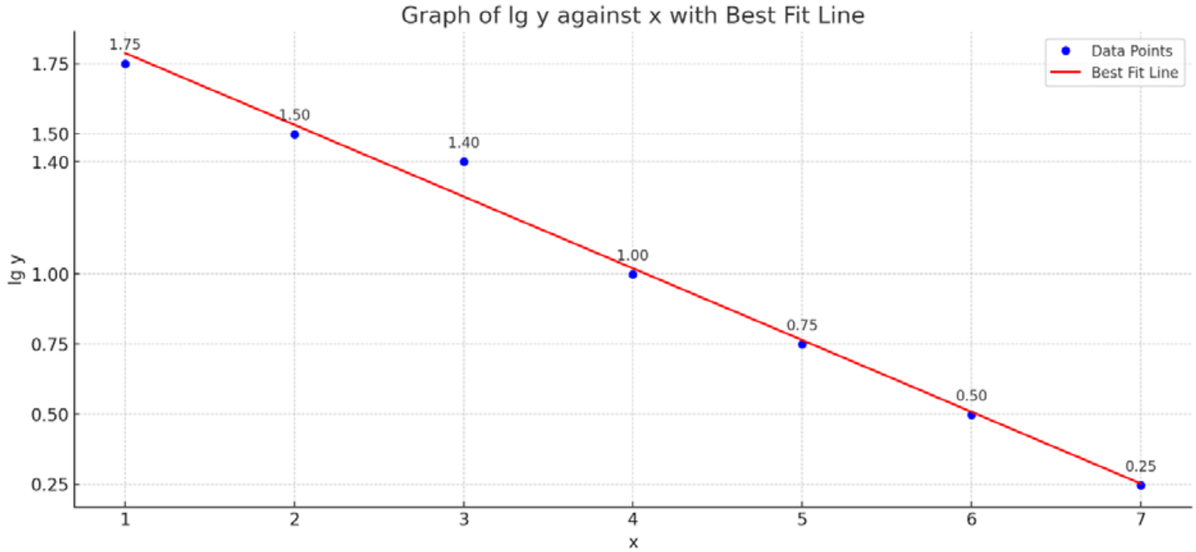
gradient = $-\lg k = \dfrac{1.75 - 1.50}{1 - 2}$ [M1]

$\lg k = 0.25$

$k = 10^{0.25} = 1.78$ [A1] [Acceptable range ± 0.1]

y-intercept $= \lg h = 2$

$h = 100$ [B1]


Graph of lg y against x with Best Fit Line
Data Points
Best Fit Line
lg y
x
1.75
1.50
1.40
1.00
0.75
0.50
0.25
1
2
3
4
5
6
7

11 A circle, C, has equation $x^2+y^2-10x+6y+9=0$.

(a) Find the coordinates of the centre and radius of C. [3]

$x^2+y^2-10x+6y+9=0$

$x^2-10x+25+y^2+6y+9-25-9+9=0$

$(x-5)^2+(y+3)^2=5^2$

Centre of Circle $=(5,\ -3)$ [B2][Minus 1 mark for each incorrect coordinate]

Radius $=5$ units [B1]

(b) Explain why the y-axis is a tangent to C. [1]

For using the correct method to find the perpendicular distance from the centre of the circle to the y-axis. Distance from centre (5, –3) to y-axis (x = 0) = 5
Distance equals radius, so y-axis is a tangent.

(c) Given that the circle C crosses the x-axis at the point $P(1,\ 0)$, find the equation of the tangent to the circle at P. [3]

Gradient of CP $=\dfrac{0-(-3)}{1-5}=-\dfrac{3}{4}$ [M1]

Gradient of tangent $=\dfrac{4}{3}$ [M1]

$y=\dfrac{4}{3}x-\dfrac{4}{3}$ [A1]

(d) Determine whether the point $A(3,\ -7)$ lies inside, outside or on the circle C? [2]

LHS $=(3-5)^2+(-7+3)^2=20$ [M1]

Since, $20<25$, $(3,\ -7)$ lies inside the circle. [A1]

12 **(a)** Find the equation of the perpendicular bisector of the points $P(a,\ 4)$ and $Q(6,\ -2)$ in terms of a. [4]

Midpoint of $PQ = \left(\dfrac{a+6}{2}, \dfrac{4-2}{2}\right) = \left(\dfrac{a+6}{2}, 1\right)$ [M1]

Gradient of $PQ = \dfrac{4-(-2)}{a-6} = \dfrac{6}{a-6}$ [M1]

Gradient of Perpendicular Bisector $PQ = -\dfrac{a-6}{6} = \dfrac{6-a}{6}$ [M1]

$y-1 = \dfrac{6-a}{6}\left(x - \dfrac{a+6}{2}\right)$ [A1]

(b) The perpendicular bisector of line segment PQ passes through the point $R(2,\ 1)$. Find the possible values of a. [2]

$$1-1=\frac{6-a}{6}\left(2-\frac{a+6}{2}\right)$$

$$0=\frac{6-a}{6}\left(\frac{4-a-6}{2}\right) \text{ [M1]}$$

$$0=(6-a)(-2-a)$$

$a=6$ or $a=-2$ [A1]

(c) S is the reflection of point Q across the perpendicular bisector of PQ. By using the positive value of a found in part (b) calculate the area of the quadrilateral $PQRS$. [4]

Midpoint of $PQ=\left(\frac{6+6}{2},1\right)=(6,1)$ [M1]

$$\frac{6+x}{2}=6$$

$$\frac{-2+y}{2}=1$$

$S=(6,4)$ [M1]

Use of shoelace method [M1]

$Area=12$ units^2 [A1]

End of Paper