Name:		Index Number:		Class:	

CATHOLIC HIGH SCHOOL
Preliminary Examination
Secondary 4 (O-Level Programme)

Additional Mathematics **SOLUTIONS** **4049/01**

Paper 1

28 August 2025

2 hours 15 minutes

Additional Materials: Answer Booklets A, B and C.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions in the space provided.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to **three significant figures**.
Give answers in **degrees to one decimal place**.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **90**.

For examiner's use:

/ 90

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n \ ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

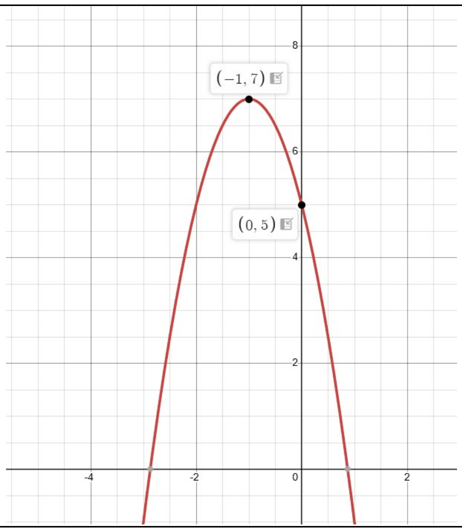
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \tfrac{1}{2} ab \sin C$$

1 **(i)** Express $5-4x-2x^2$ in the form $c-a\left(x+b\right)^2$, where a, b and c are constants and $a>0$. [2]

$$5-4x-2x^2 = -2\left[x^2+2x\right]+5$$
$$= -2\left(x+1\right)^2+7$$

(ii) Sketch the graph of $y=5-4x-2x^2$, indicating clearly the turning point and the y-intercept. [2]



(iii) Hence find the range of values of k for which the equation $5-4x-2x^2=k$ has at most one root. [1]

$$k \geq 7$$

2 The function f is defined by $\mathrm{f}(x) = 4\cos ax + b$ for $0 \le x \le 3\pi$, where a and b are constants. The maximum value of f is 6 and the period of f is 4π.

(i) State the amplitude of f. [1]

Amplitude = 4

(ii) Write down the value of a and of b. [2]

$\mathrm{f}(x) = 4\cos ax + b$

Period $= 4\pi$

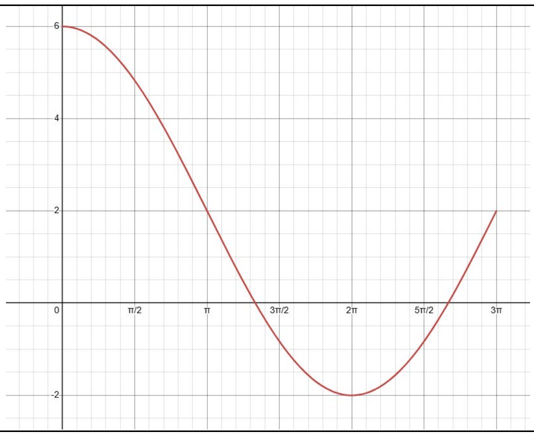
$\frac{2\pi}{a} = 4\pi$

$a = \frac{1}{2}$

$4 + b = 6$

$b = 2$

(iii) Sketch the graph of $y = \mathrm{f}(x)$ for $0 \le x \le 3\pi$. [3]



3 **(i)** Find the range of values of k for which the line $y = kx + 2$ intersects the curve $xy - y = 2x$ at two distinct points. [4]

$$x(kx+2)-(kx+2)=2x$$
$$kx^2+2x-kx-2=2x$$
$$kx^2-kx-2=0$$

$$b^2-4ac=(-k)^2-4k(-2)$$
$$>0$$

$$k^2+8k>0$$
$$k(k+8)>0$$
$$k<-8 \quad \text{or} \quad k>0$$

(ii) Hence state the possible value(s) of k for which the line $y = kx + 2$ is a tangent to the curve $xy - y = 2x$. [1]

$$k=-8$$

4 Solve the equation $8-\sqrt{11-2x}=-x$, giving your answer in the form $a+b\sqrt{c}$, where a, b and c are integers. [4]

$$
\begin{aligned}
8-\sqrt{11-2x} &= -x \\
8+x &= \sqrt{11-2x} \\
64+16x+x^2 &= 11-2x \\
x^2+18x+53 &= 0 \\
x &= \frac{-18\pm\sqrt{18^2-4(1)(53)}}{2(1)} \\
&= \frac{-18\pm\sqrt{112}}{2} \\
&= -9\pm\frac{4\sqrt{7}}{2} \\
&= -9+2\sqrt{7} \qquad \text{or} \qquad -9-2\sqrt{7}\ \text{(rej.)}
\end{aligned}
$$

5 Water is being poured, at a constant rate of 42 m^3/s, into an empty container. After t seconds, the depth of water is h m and the volume, V m^3, of the water in the container is given by

$$V = \frac{(5+2h)^3}{3\pi}.$$

(i) Find the rate of change of the depth of water in terms of h and π. [3]

$$\frac{dV}{dh} = \frac{2(5+2h)^2}{\pi}$$

$$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$$
$$= \frac{\pi}{2(5+2h)^2} \times 42$$
$$= \frac{21\pi}{(5+2h)^2}$$

(ii) As the depth of water in the container rises, determine with a reason, whether the rate of change of h will increase or decrease. [1]

Mtd 1:

As h increases, $(5+2h)^2$ increases.

Since 21π is a constant, $\frac{dh}{dt} = \frac{21\pi}{(5+2h)^2}$ will decrease.

Mtd 2:

$$\frac{d^2h}{dt^2} = -\frac{84\pi}{(5+2h)^3}$$

$h > 0$

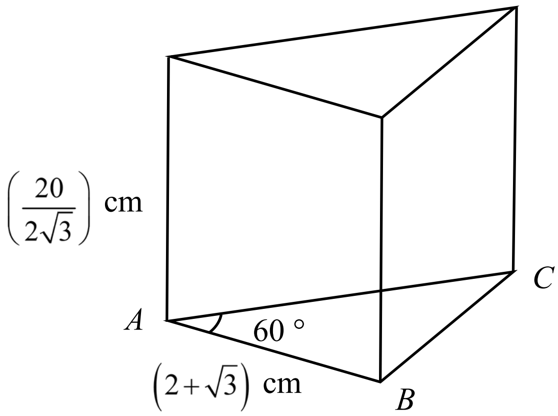
$\Rightarrow (5+2h)^3 > 0$

$\Rightarrow -(5+2h)^3 < 0$

$\Rightarrow \frac{d^2h}{dt^2} < 0$

$\Rightarrow \frac{dh}{dt}$ will decrease

6



The diagram shows a solid prism with a triangular base, ABC, with angle $BAC = 60°$ and $AB = \left(2+\sqrt{3}\right)$ cm. The height of the prism is $\left(\frac{20}{\sqrt{3}}\right)$ cm and its volume is $\left(25+10\sqrt{3}\right)$ cm^3.

Without using a calculator, express the length of AC in the form $\left(a+b\sqrt{3}\right)$ where a and b are constants. [5]

$$\frac{1}{2}\left(2+\sqrt{3}\right)(AC)\sin(60°)\times\left(\frac{20}{\sqrt{3}}\right) = 25+10\sqrt{3}$$

$$\frac{1}{2}\left(2+\sqrt{3}\right)(AC)\left(\frac{\sqrt{3}}{2}\right)\times\left(\frac{20}{\sqrt{3}}\right) = 25+10\sqrt{3}$$

$$(AC)\left(10+5\sqrt{3}\right) = 25+10\sqrt{3}$$

$$AC = \frac{25+10\sqrt{3}}{10+5\sqrt{3}}$$

$$= \frac{25+10\sqrt{3}}{10+5\sqrt{3}}\times\frac{10-5\sqrt{3}}{10-5\sqrt{3}}$$

$$= 4-\sqrt{3}$$

7 **(i)** Express $\dfrac{8-3x}{(2x+3)\left(x^2+4\right)}$ in partial fractions. [4]

Let $\dfrac{8-3x}{(2x+3)\left(x^2+4\right)}=\dfrac{A}{2x+3}+\dfrac{Bx+C}{x^2+4}$

$$8-3x=A\left(x^2+4\right)+(Bx+C)(2x+3)$$

Let $x=-\dfrac{3}{2}$: $\dfrac{25}{2}=A\left(\dfrac{25}{4}\right) \Rightarrow A=2$

Let $x=0$: $8=2(4)+C(3) \Rightarrow C=0$

Let $x=1$: $5=2(5)+B(5) \Rightarrow B=-1$

$$\therefore \quad \frac{2}{2x+3}-\frac{x}{x^2+4}$$

(ii) Differentiate $\ln\left(x^2+4\right)$ with respect to x. [1]

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\ln\left(x^2+4\right)\right]=\frac{2x}{x^2+4}$$

(iii) Use your answers to **parts (i)** and **(ii)** to find

$$\int_0^1 \frac{8-3x}{(2x+3)\left(x^2+4\right)}\,\mathrm{d}x.$$ [5]

$$\begin{aligned}
\int_0^1 \frac{8-3x}{(2x+3)\left(x^2+4\right)}\,\mathrm{d}x &= \int_0^1 \frac{2}{2x+3}-\frac{x}{x^2+4}\,\mathrm{d}x \\
&= \int_0^1 \frac{2}{2x+3}\,\mathrm{d}x-\int_0^1 \frac{x}{x^2+4}\,\mathrm{d}x \\
&= \int_0^1 \frac{2}{2x+3}\,\mathrm{d}x-\frac{1}{2}\int_0^1 \left(\frac{2x}{x^2+4}\right)\mathrm{d}x \\
&= \Big[\ln(2x+3)\Big]_0^1-\frac{1}{2}\Big[\ln\left(x^2+4\right)\Big]_0^1 \\
&= [\ln 5-\ln 3]-\frac{1}{2}[\ln 5-\ln 4] \\
&= \frac{1}{2}\ln\frac{20}{9}
\end{aligned}$$

or 0.399 (3 s.f.)

8 Prove that $\dfrac{\tan 2\theta}{\sec 2\theta - 1} = \cot\theta$. [4]

$$
\begin{aligned}
\frac{\tan 2\theta}{\sec 2\theta - 1} &= \frac{\sin 2\theta}{\cos 2\theta} \times \frac{1}{\frac{1}{\cos 2\theta} - 1} \\
&= \frac{\sin 2\theta}{\cos 2\theta} \times \frac{\cos 2\theta}{1 - \cos 2\theta} \\
&= \frac{\sin 2\theta}{1 - \cos 2\theta} \\
&= \frac{2\sin\theta\cos\theta}{1 - \left(1 - 2\sin^2\theta\right)} \\
&= \frac{2\sin\theta\cos\theta}{2\sin^2\theta} \\
&= \frac{\cos\theta}{\sin\theta} \\
&= \cot\theta
\end{aligned}
$$

9 The equation of a curve is $y = 3x\sin 2x + 1$. The point $A\ (\pi,\ 1)$ lies on the curve. The normal to the curve at A meets the x-axis at P and the y-axis at Q. Find length of PQ. [9]

$$y = 3x\sin 2x + 1$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3\sin 2x + 3x \bullet 2\cos 2x$$

$$= 3\sin 2x + 6x\cos 2x$$

$$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=\pi} = 3\sin 2\pi + 6\pi\cos 2\pi$$

$$= 6\pi$$

Grad. of normal at $A = -\dfrac{1}{6\pi}$

Eqn. of normal at A:

$$y - 1 = -\frac{1}{6\pi}(x - \pi)$$

$$y = -\frac{1}{6\pi}x + \frac{7}{6}$$

$$Q\left(0,\ \frac{7}{6}\right)$$

At P : $y = 0$

$$-\frac{1}{6\pi}x + \frac{7}{6} = 0$$

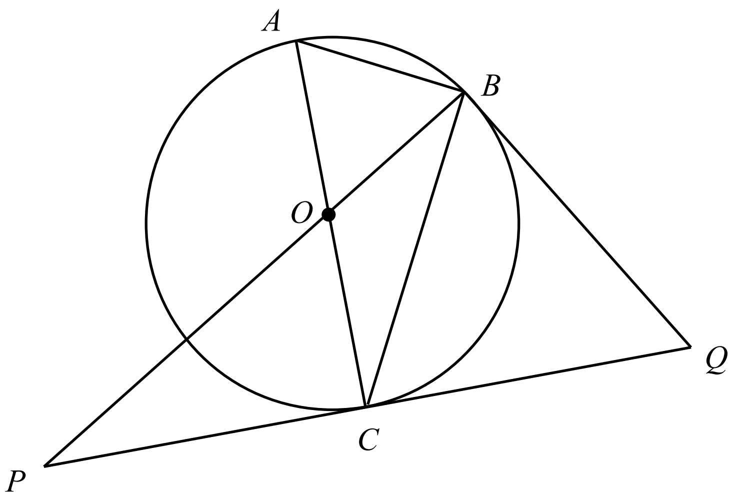
$$x = 7\pi$$

$$P(7\pi,\ 0)$$

$$PQ = \sqrt{(7\pi - 0)^2 + \left(0 - \frac{7}{6}\right)^2}$$

$= 22.0$ units (3 s.f.)

10



In the diagram, A, B and C lie on a circle where O is the centre and AC is a diameter. The tangents to the circle at B and C meet at Q. The line BO extended meets the line QC at P.

(i) Show that angle $BOC = 2\times$angle BCQ. [2]

<u>Mtd 1:</u>

$\measuredangle BAC = \measuredangle BCQ$ (Alt. Seg. Thm.)

$\measuredangle BOC = 2\left(\measuredangle BAC\right)$ ($\measuredangle$ at centre = 2$\measuredangle$s at circumference)

$= 2\measuredangle BCQ$

<u>Mtd 2:</u>

Let $\measuredangle BCQ = x$.

$\measuredangle OCB = 90° - x$ (rad. $\perp$ tan)

$\measuredangle BOC = 180° - 2\left(90° - x\right)$ (isosceles ΔOBC)

$= 180° - 180° + 2x$

$= 2x$

$= 2\measuredangle BCQ$

(ii) If C is the midpoint of PQ,

(a) prove that triangle POC is similar to triangle PQB, [2]

$\measuredangle OPC = \measuredangle QPB$ (common pt P)

$\measuredangle PCO = \measuredangle PBQ = 90°$ (rad. $\perp$ tan)

Since 2 pairs of corresponding angles are equal, triangle POC is similar to triangle PQB.

(b) show that $PO \times PB = \frac{1}{2}PQ^2$. [2]

Since triangle *POC* is similar to triangle *PQB*,

$$\frac{PO}{PQ} = \frac{PC}{PB}$$

Since *C* is the midpoint of *PQ*, $\frac{PC}{PQ} = \frac{1}{2} \Rightarrow PC = \frac{1}{2}PQ$

$$\frac{PO}{PQ} = \frac{\frac{1}{2}PQ}{PB}$$

$$PO \times PB = \frac{1}{2}PQ \times PQ$$

$$= \frac{1}{2}PQ^2$$

11 Solve the equation $2\cot^2 2x = 1 - \dfrac{5}{\sin 2x}$ for $-180° \le x \le 180°$. [6]

$$2\cot^2 2x = 1 - \frac{5}{\sin 2x}$$

$$2\cot^2 2x = 1 - 5\ \text{cosec}\ 2x$$

$$2\left(\text{cosec}^2\ 2x - 1\right) = 1 - 5\ \text{cosec}\ 2x$$

$$2\ \text{cosec}^2\ 2x + 5\ \text{cosec}\ 2x - 3 = 0$$

Let $y = \text{cosec}\ 2x$.

$$2y^2 + 5y - 3 = 0$$

$$(2y-1)(y+3) = 0$$

$$y = -3 \quad \text{or} \quad y = \frac{1}{2}$$

$$\text{cosec}\ 2x = -3 \quad \text{or} \quad \text{cosec}\ 2x = \frac{1}{2}$$

$$\sin 2x = -\frac{1}{3} \quad \text{or} \quad \sin 2x = 2\ \text{(rej.)}$$

$$\alpha = \sin^{-1}\frac{1}{3}$$

$$= 19.471°$$

$$2x = 199.471°,\quad 340.529°$$

$$x = 99.7°,\quad 170.3° \quad \text{(1 d.p)}$$

$$2x = -19.471°,\quad -160.529°$$

$$x = -9.7°,\quad -80.3° \quad \text{(1 d.p)}$$

12 **(i)** Write down the general term in the binomial expansion of $\left(px-\frac{q}{x}\right)^n$, where p and q are positive integers. [1]

$$\left(px-\frac{q}{x}\right)^n$$

$$T_{r+1}=\binom{n}{r}(px)^{n-r}\left(-\frac{q}{x}\right)^r$$

$$=\binom{n}{r}\left(p^{n-r}\right)(-q)^r\,x^{n-2r}$$

(ii) If the fifth term in the binomial expansion of $\left(px-\frac{q}{x}\right)^n$ is independent of x, show that $n=8$. [2]

5th term $\Rightarrow r=4$

$$T_5=\binom{n}{4}\left(p^{n-4}\right)(-q)^4\,x^{n-8}$$

$$n-8=0$$

$$n=8$$

(iii) Hence find the value of p and of q given that the fifth term is 1120 and $p-q=1$. [5]

$$\binom{8}{4}\left(p^4\right)(-q)^4=1120$$

$$70p^4q^4=1120$$

$$(pq)^4=16$$

$$pq=2$$

$$q=\frac{2}{p}$$

Sub $q=\frac{2}{p}$ into $p-q=1$:

$$p-\frac{2}{p}=1$$

$$p^2-2=p$$

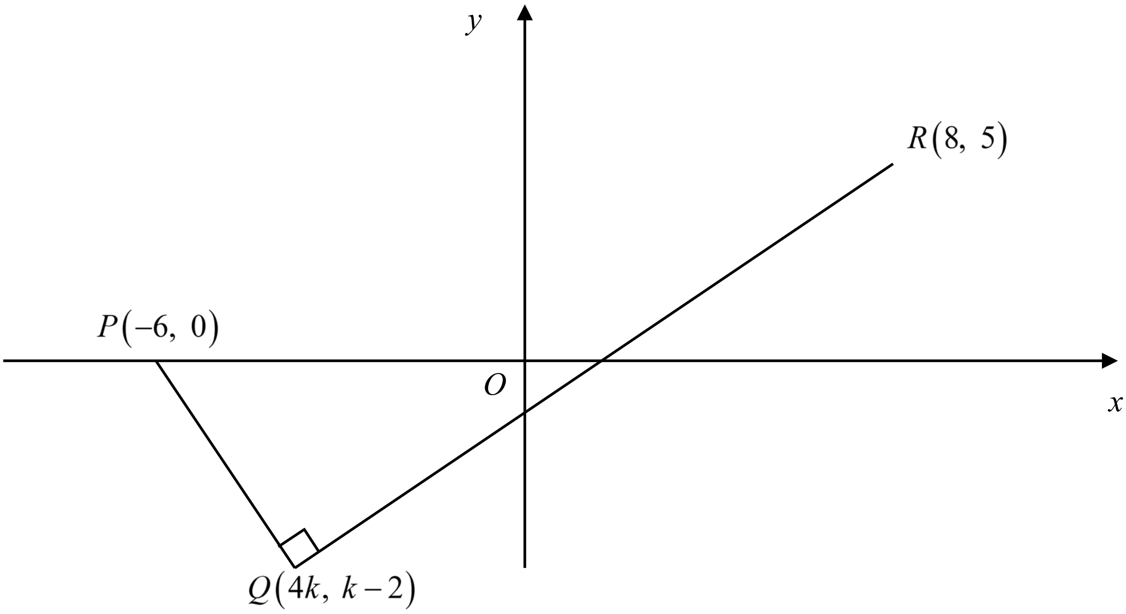
$$p^2-p-2=0$$

$$(p+1)(p-2)=0$$

$p=-1$ (rej.) or $p=2$

$\therefore$ $q=1$

13 The diagram shows three points $P(-6,\ 0)$, $Q(4k,\ k-2)$ and $R(8,\ 5)$ where PQ is perpendicular to QR and $k<0$.



(i) Show that $k=-1$. [3]

$$Grad_{PQ} = -\frac{1}{Grad_{QR}}$$

$$\frac{(k-2)-0}{4k-(-6)} = -\frac{4k-8}{(k-2)-5}$$

$$\frac{k-2}{4k+6} = \frac{-4k+8}{k-7}$$

$$(k-2)(k-7) = (-4k+8)(4k+6)$$

$$k^2-9k+14 = -16k^2+8k+48$$

$$17k^2-17k-34=0$$

$$k^2-k-2=0$$

$$(k-2)(k+1)=0$$

$$k=2 \quad \text{(N.A.) or} \quad k=-1$$

(ii) The perpendicular bisector of *PR* cuts the *y*-axis at *S*. Find the coordinates of *S*. [4]

Midpoint of $PR = \left(\frac{-6+8}{2},\ \frac{0+5}{2} \right) = \left(1,\ \frac{5}{2} \right)$

$Grad_{PR} = \frac{0-5}{-6-8} = \frac{5}{14}$

Eqn. of $\perp$ bisector of PR:

$$y - \frac{5}{2} = -\frac{14}{5}(x-1)$$

$$y = -\frac{14}{5}x + \frac{53}{10}$$

$$S\left(0,\ \frac{53}{10}\right)$$

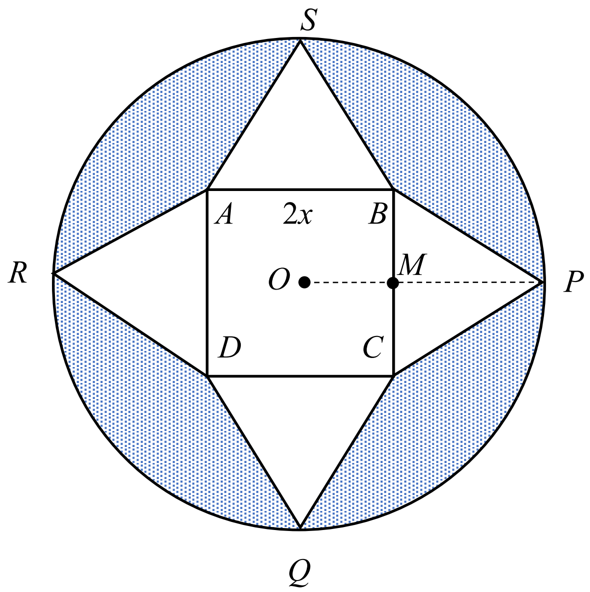
(iii) Find the area of the quadrilateral *PQRS*. [2]

$$\text{Area of } PQRS = \frac{1}{2}\begin{vmatrix} -6 & -4 & 8 & 0 & -6 \\ 0 & -3 & 5 & 5.3 & 0 \end{vmatrix}$$

$$= \frac{1}{2}\left| (18 - 20 + 42.4) - (-24 - 31.8) \right|$$

$$= 48.1 \text{ units}^2$$

14 [The volume of a pyramid of height h is $\frac{1}{3}\times(\text{Base Area})\times h$.]



The diagram shows a piece of paper in the shape of a circle centre O and radius 6 cm. $ABCD$ is a square of side $2x$ cm with the same centre O. P, Q, R and S are points on the circumference of the circle such that $AS = BS = BP = CP = CQ = DQ = DR = AR$.
The shaded regions of the paper are to be cut off and the remaining paper can be folded to form a right pyramid with base $ABCD$.

(i) M is the midpoint of BC such that OMP is a straight line. Write down an expression for MP in terms of x and hence show that the height of the pyramid is $2\sqrt{9-3x}$ cm. [2]

$$OM = x$$
$$MP = 6 - x$$
$$\text{Height of pyramid } = \sqrt{(6-x)^2 + x^2}$$
$$= \sqrt{36-12x}$$
$$= 2\sqrt{9-3x}$$

(ii) Show that the volume of the pyramid, V cm^3, is given by $V = \frac{8}{3}x^2\sqrt{9-3x}$. [1]

$$V = \frac{1}{3}(2x)^2\left(2\sqrt{9-3x}\right)$$
$$= \frac{8}{3}x^2\sqrt{9-3x}$$

(iii) Given that x can vary, find the value of x that gives the stationary value of V. [4]

$$\frac{dV}{dx} = \frac{16}{3}x \bullet (9-3x)^{\frac{1}{2}} + \frac{8}{3}x^2 \bullet \frac{1}{2}(9-3x)^{-\frac{1}{2}}(-3)$$

$$= \frac{16}{3}x(9-3x)^{\frac{1}{2}} - 4x^2(9-3x)^{-\frac{1}{2}}$$

$$= \frac{4x(12-5x)}{\sqrt{9-3x}}$$

$$\frac{dV}{dx} = 0$$

$$12 - 5x = 0$$

$$x = 2.4$$

(iv) Explain whether this value of x gives a maximum or minimum V. [2]

Mtd 1:

x	2.39	2.4	2.41
$\frac{dV}{dx}$	0.3533	0	– 0.3622
Tangent	/	–	\

V is a maximum.

Mtd 2:

$$\frac{d^2V}{dx^2} = \frac{2\sqrt{3}\left(5x^2 - 24x + 24\right)}{(3-x)^{\frac{3}{2}}}$$

$$\left.\frac{d^2V}{dx^2}\right|_{x=2.4} = -35.8 < 0$$

V is a maximum.

--- End of Paper ---