CEDAR GIRLS' SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2025
SECONDARY FOUR

CANDIDATE NAME	Mark Scheme

CLASS | | | INDEX NUMBER | | |

ADDITIONAL MATHEMATICS **4049/01**

Paper 1 **25 August 2025**

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use
90

This document consists of **21** printed pages and **1** blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{r!(n-r)!} = \dfrac{n(n-1)\ \ldots\ (n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer **all** the questions.

1 **(a)** Express $\dfrac{6\sqrt{2}-9}{\sqrt{2}-1}$ in the form of $a+b\sqrt{2}$, where a and b are integers. [2]

$\dfrac{6\sqrt{2}-9}{\sqrt{2}-1}\times\dfrac{\sqrt{2}+1}{\sqrt{2}+1}$ $=\dfrac{12+6\sqrt{2}-9\sqrt{2}-9}{2-1}$ $=3-3\sqrt{2}$	

(b) $\dfrac{6\sqrt{2}-9}{\sqrt{2}-1}$ is a root of the equation $x^2+px+q=0$ where p and q are integers.

Using the result in **part (a)**, find the value of p and of q. [4]

$(3-3\sqrt{2})^2+p(3-3\sqrt{2})+q=0$ $9-18\sqrt{2}+18+3p-3p\sqrt{2}+q=0$ $-(18+3p)\sqrt{2}+27+3p+q=0$ $18+3p=0---(1)$ $27+3p+q=0---(2)$ $p=-6$ $q=-9$	
$x=\dfrac{-p\pm\sqrt{p^2-4q}}{2}$ $3-3\sqrt{2}=\dfrac{-p\pm\sqrt{p^2-4q}}{2}$ $6-6\sqrt{2}=-p\pm\sqrt{p^2-4q}$ Comparing, $p=-6$ $-\sqrt{p^2-4q}=-6\sqrt{2}$ $p^2-4q=36\times 2$ $(-6)^2-4q=72$ $q=-9$	

2 Find the range of values of the constant m for which the curve $y=(m-3)x^2+mx+m$ lies completely above the x-axis. [5]

$y = (m-3)x^2 + mx + m$ $b^2 - 4ac < 0$ $m^2 - 4(m-3)(m) < 0$ $m^2 - 4m^2 + 12m < 0$ $-3m^2 + 12m < 0$ $-3m(m-4) < 0$ $m < 0$ or $m > 4$ $m - 3 > 0$ $m > 3$ Therefore $m > 4$	

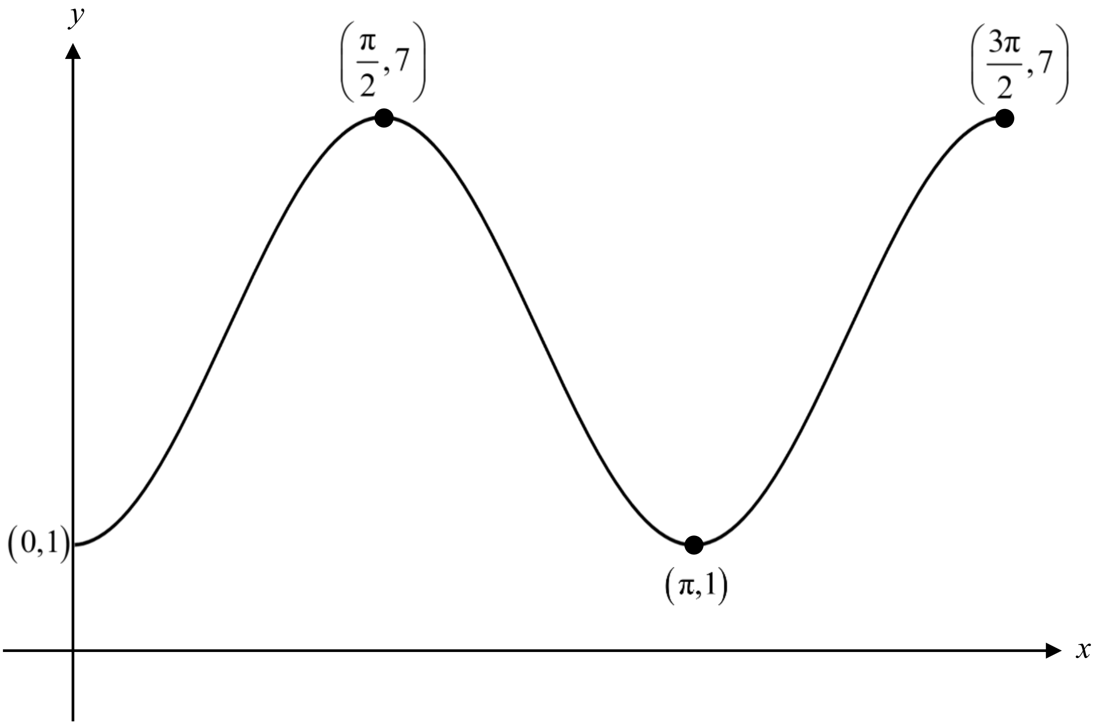
3 Express $\dfrac{2x^3-5}{x^2(x-1)}$ in partial fractions. [6]

$\dfrac{2x^3-5}{x^2(x-1)}=2+\dfrac{2x^2-5}{x^2(x-1)}$ $\dfrac{2x^2-5}{x^2(x-1)}=\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{x-1}$ $2x^2-5=Ax(x-1)+B(x-1)+Cx^2$ Let $x=1$, $-3=C$ Let $x=0$, $-5=B(-1)$ $B=5$ Let $x=-1$, $-3=-A(-2)+5(-2)-3(1)$ $A=5$ $\dfrac{2x^3-5}{x^2(x-1)}=2+\dfrac{5}{x}+\dfrac{5}{x^2}-\dfrac{3}{x-1}$	
$\dfrac{2x^3-5}{x^2(x-1)}=2+\dfrac{2x^2-5}{x^2(x-1)}$ $\dfrac{2x^2-5}{x^2(x-1)}=\dfrac{Ax+B}{x^2}+\dfrac{C}{x-1}$ $2x^2-5=(Ax+B)(x-1)+Cx^2$ Let $x=1$, $-3=C$ Let $x=0$, $-5=B(-1)$ $B=5$ Let $x=-1$, $-3=-A(-2)+5(-2)-3(1)$ $A=5$ $\dfrac{2x^3-5}{x^2(x-1)}=2+\dfrac{5x+5}{x^2}-\dfrac{3}{x-1}$ $=2+\dfrac{5}{x}+\dfrac{5}{x^2}-\dfrac{3}{x-1}$	

4 **(a)** State the range of principal values of $\tan^{-1} x$. [1]

(a)	$-\dfrac{\pi}{2} < \tan^{-1} x < \dfrac{\pi}{2}$	

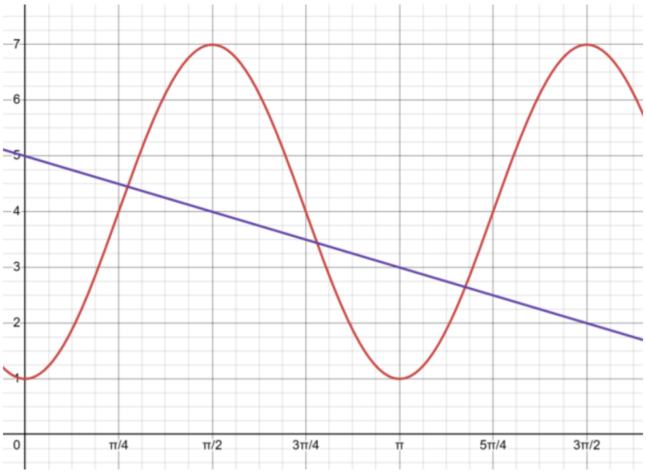
(b) The curve $y = a + b\cos cx$ is shown below for $0 \le x \le \dfrac{3\pi}{2}$ radians.



(i) Find the values of a, b and c. [3]

(b)(i)	$period = \dfrac{2\pi}{c} = \pi$ $c = 2$ $a - b = 7$ $a + b = 1$ $a = 4$ $b = -3$ $y = 4 - 3\cos 2x$	

(ii) On the same axes, draw a straight line to determine the number of solutions for the equation $a\pi + b\pi\cos cx = 5\pi - 2x$ for $0 \le x \le \frac{3\pi}{2}$. [3]

<table>
<tr><td>(b)(ii)</td><td>$a\pi + b\pi\cos cx = 5\pi - 2x$
$a + b\cos cx = 5 - \frac{2}{\pi}x$

There are 3 solutions.</td><td></td></tr>
</table>

5 When $\mathrm{f}(x)$ is divided by $x-1$, the remainder is 61.

The remainder is -123 when $\mathrm{f}(x)$ is divided by $x+3$.

Find the remainder when $\mathrm{f}(x)$ is divided by x^2+2x-3. [5]

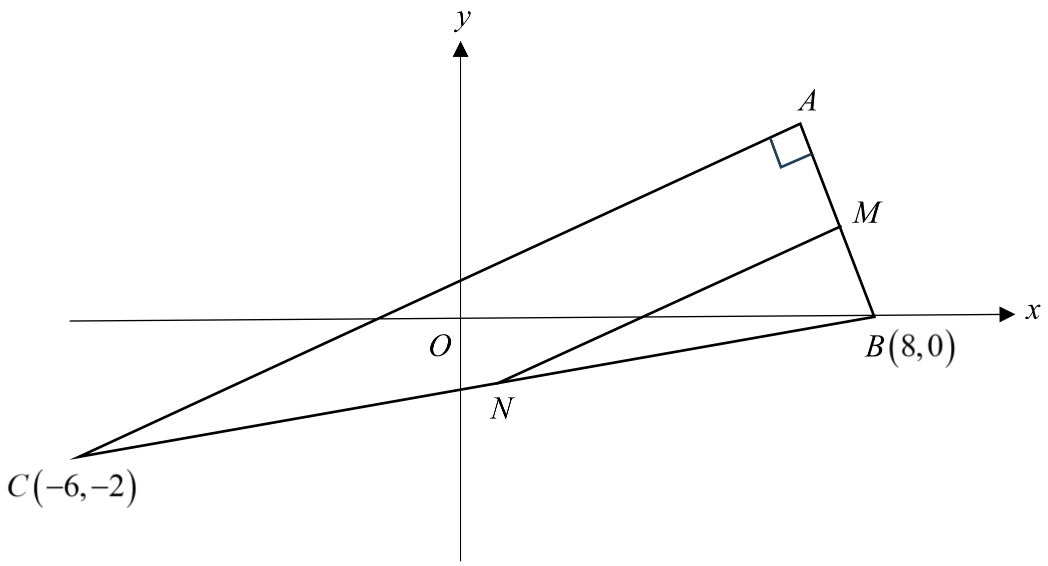
$x^2+2x-3=(x-1)(x+3)$ $f(x)=(x-1)(x+3)Q(x)+(ax+b)$ $f(1)=61$ $a+b=61$ $f(-3)=-123$ $-3a+b=-123$ $a=46$ $b=15$ The remainder is $46x+15$.	

6 A curve is defined by the equation $y = \ln\left(\dfrac{e^x}{x^2+10}\right)$.

Show that the curve has no turning points for all real values of x. [5]

$y = \ln\left(\dfrac{e^x}{x^2+10}\right)$ $y = \ln e^x - \ln(x^2+10)$ $y = x - \ln(x^2+10)$ $\dfrac{dy}{dx} = 1 - \dfrac{2x}{x^2+10}$ $\dfrac{dy}{dx} = \dfrac{x^2+10-2x}{x^2+10}$ $\dfrac{dy}{dx} = \dfrac{(x-1)^2+10-1}{x^2+10}$ $\dfrac{dy}{dx} = \dfrac{(x-1)^2+9}{x^2+10}$ Since $\left(x-1\right)^2 \geq 0$ then $\left(x-1\right)^2 + 9 > 0$ and $x^2+10>0$ therefore $\dfrac{(x-1)^2+9}{x^2+10} > 0$ and $\dfrac{dy}{dx} \neq 0$ The curve has no turning points.	
$y = \ln\left(\dfrac{e^x}{x^2+10}\right)$ $y = \ln e^x - \ln(x^2+10)$ $y = x - \ln(x^2+10)$ $\dfrac{dy}{dx} = 1 - \dfrac{2x}{x^2+10}$ $\dfrac{x^2+10-2x}{x^2+10} = 0$ $x^2+10-2x=0$ $x = \dfrac{-2(-2) \pm \sqrt{(-2)^2-4(1)(10)}}{2}$ no solutions or $b^2-4ac<0$ $\dfrac{dy}{dx} \neq 0$. The curve has no turning points.	

7 Solutions to this question by accurate drawing will not be accepted.



The diagram above shows a triangle ABC with vertices $B(7,2)$ and $C(-6,-2)$.

AB is perpendicular to AC and is parallel to the line $y=-2x$.

M and N are the mid-points of AB and BC respectively.

(a) Find the coordinates of A. [6]

gradient of $AB=-2$ gradient of $AC\times$gradient of $AB=-1$ gradient of $AC=\frac{1}{2}$ Equation of line $AB: y=-2x+c_1$ subt$(8,0)$ $c_1=16$ Equation of line $AB: y=-2x+16$ Equation of line $AC: y=\frac{1}{2}x+c_2$ subt$(-6,-2)$ $c_2=1$ Equation of line $AC: y=\frac{1}{2}x+1$ $-2x+16=\frac{1}{2}x+1$ $x=6$ $y=-2(6)+16=4$ $A\ (6\ ,4)$	

(b) Explain why the quadrilateral $ACNM$ is a trapezium. [1]

Since quadrilateral $ACNM$ has only one pair of parallel sides MN and AC by midpoint theorem. Therefore, it is a trapezium.	

(c) Find the area of the quadrilateral $ACNM$. [3]

$N=\left(\frac{-6+8}{2},\frac{-2+0}{2}\right)=(1,-1)$ $M=\left(\frac{6+8}{2},\frac{4+0}{2}\right)=(7,2)$ Area $=\frac{1}{2}\begin{vmatrix}6 & -6 & 1 & 7 & 6\\ 4 & -2 & -1 & 2 & 4\end{vmatrix}$ $=\frac{1}{2}\left[(-12+6+2+28)-(12-7-2-24)\right]$ $=\frac{45}{2}$	

8 An architect uses 40 cm of wire to design 2 window frames for a room model. He wants as much light as possible to pass through the windows for his design. One frame is to be an equilateral triangle of side x cm. The other is to be a semi-circle of radius r cm.

(a) Show that the total area, A cm^2, enclosed by the two window frames is given by

$$A=\frac{\pi(40-3x)^2}{2(\pi+2)^2}+\frac{\sqrt{3}}{4}x^2 \quad .$$

[4]

Perimeter of semicircle $=\frac{1}{2}\times 2\pi\times r+2r=\pi r+2r$ Perimeter of equilateral triangle $=3x$ $(\pi r+2r)+3x=40$ $r=\frac{40-3x}{\pi+2}$ Area of semicircle $=\frac{1}{2}\times\pi r^2$ Area of equilateral triangle $=\frac{1}{2}\times x^2\times\sin 60^\circ=\frac{\sqrt{3}}{4}x^2$ $A=\frac{\pi r^2}{2}+\frac{\sqrt{3}}{4}x^2$ $A=\frac{\pi}{2}\left(\frac{40-3x}{\pi+2}\right)^2+\frac{\sqrt{3}}{4}x^2$ $=\frac{\pi(40-3x)^2}{2(\pi+2)^2}+\frac{\sqrt{3}}{4}x^2$ (shown)	

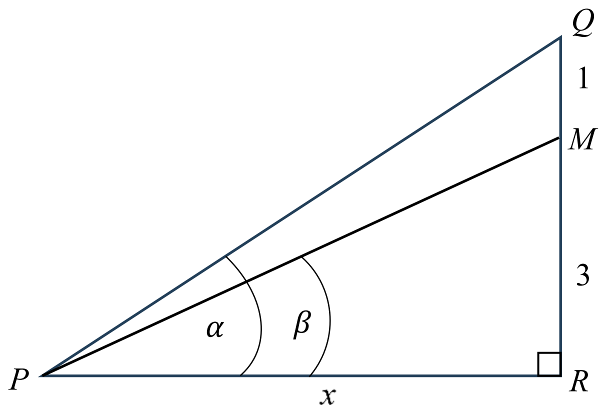
(b) Given that x can vary, find the stationary value of A. [4]

$A=\dfrac{\pi\left(40-3x\right)^2}{2\left(\pi+2\right)^2}+\dfrac{\sqrt{3}x^2}{4}$ $\dfrac{dA}{dx}=2\times\dfrac{\pi\left(40-3x\right)}{2\left(\pi+2\right)^2}\times\left(-3\right)+2\times\dfrac{\sqrt{3}x}{4}$ $\dfrac{dA}{dx}=\dfrac{-3\pi\left(40-3x\right)}{\left(\pi+2\right)^2}+\dfrac{\sqrt{3}x}{2}$ $\dfrac{-3\pi\left(40-3x\right)}{\left(\pi+2\right)^2}+\dfrac{\sqrt{3}x}{2}=0$ $\dfrac{3\pi\left(40-3x\right)}{\left(\pi+2\right)^2}=\dfrac{\sqrt{3}x}{2}$ $x=7.37\text{cm}$ $A=42.5\text{cm}^2$	

(c) Find the nature of this stationary value and explain why the architect may not be happy with his design. [2]

(c)	$\dfrac{dA}{dx}=\dfrac{-3\pi\left(40-3x\right)}{\left(\pi+2\right)^2}+\dfrac{\sqrt{3}x}{2}$ $\dfrac{dA}{dx}=\dfrac{-3\pi\left(40\right)}{\left(\pi+2\right)^2}+\dfrac{3\pi\left(3x\right)}{\left(\pi+2\right)^2}+\dfrac{\sqrt{3}x}{2}$ $\dfrac{d^2A}{dx^2}=0+\dfrac{3\pi\left(3\right)}{\left(\pi+2\right)^2}+\dfrac{\sqrt{3}}{2}$ $\dfrac{d^2A}{dx^2}=1.936$ $\dfrac{d^2A}{dx^2}>0$ A is a minimum area The architect is disappointed as he got the smallest window areas which do not allow as much light to pass through into the room.	

9



The diagram shows a triangle *PQR*.

Point *M* lies on *QR* such that *QM* = 1 unit and *MR* = 3 units.

Angle *PRQ* = 90° and *PR* is *x* units. Angles *QPR* and *MPR* are denoted by α and β respectively as shown, where $0° < \alpha < 45°$.

Given $\tan(\alpha - \beta) = \frac{1}{8}$, find the value of *x*. [5]

$\tan\alpha = \frac{4}{x}$ $\qquad \tan\beta = \frac{3}{x}$ $\tan(\alpha-\beta) = \frac{1}{8}$ $\frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta} = \frac{1}{8}$ $\frac{\frac{4}{x}-\frac{3}{x}}{1+\left(\frac{4}{x}\right)\left(\frac{3}{x}\right)} = \frac{1}{8}$ $\frac{1}{x} = \frac{1}{8}\left(1+\frac{12}{x^2}\right)$ $\frac{8}{x} = \frac{x^2+12}{x^2}$ $8x^2 = x^3 + 12x$ $x^3 - 8x^2 + 12x = 0$ $x(x^2 - 8x + 12) = 0$ $x(x-2)(x-6) = 0$ $x = 0$ (rej) $x = 2$ (rej, $\alpha < 45°, \tan\alpha < 1$) $x = 6$	

10 Solve the equation $\log_4 x - \dfrac{3}{\log_x 8} = (\log_2 x)^2$, leaving your answer in terms of $\sqrt{2}$.

[5]

$\log_4 x - \dfrac{3}{\log_x 8}$ $= \dfrac{\log_2 x}{\log_2 4} - 3 \div \left(\dfrac{\log_2 8}{\log_2 x}\right)$ $= \dfrac{\log_2 x}{2\log_2 2} - 3 \div \left(\dfrac{3\log_2 2}{\log_2 x}\right)$ $= \dfrac{\log_2 x}{2\log_2 2} - 3 \times \left(\dfrac{\log_2 x}{3\log_2 2}\right)$ $= \dfrac{p}{2} - p$ $= -\dfrac{p}{2}$ $-\dfrac{p}{2} = p^2$ $-p = 2p^2$ $2p^2 + p = 0$ $p(2p+1) = 0$ $p = 0, p = -\dfrac{1}{2}$ $\log_2 x = 0$ $x = 1(rej)$ $\log_2 x = -\dfrac{1}{2}$ $x = \dfrac{1}{\sqrt{2}}$	

11 **(a)** Prove that $\dfrac{\tan x}{1+\sec x}+\dfrac{1+\sec x}{\tan x}=2\ \text{cosec}\, x$. [4]

$\dfrac{\tan x}{1+\sec x}+\dfrac{1+\sec x}{\tan x}$ $=\dfrac{\tan^2 x+(1+\sec x)^2}{\tan x(1+\sec x)}$ $=\dfrac{\tan^2 x+1+2\sec x+\sec^2 x}{\tan x(1+\sec x)}$ $=\dfrac{\sec^2 x+2\sec x+\sec^2 x}{\tan x(1+\sec x)}$ $=\dfrac{2\sec^2 x+2\sec x}{\tan x(1+\sec x)}$ $=\dfrac{2\sec x(\sec x+1)}{\tan x(1+\sec x)}$ $=\dfrac{2}{\cos x}\div\dfrac{\sin x}{\cos x}$ $=\dfrac{2}{\sin x}$ $=2\cos ec\ x$	

(b) Hence, solve $\dfrac{\tan 2x}{1+\sec 2x}+\dfrac{1+\sec 2x}{\tan 2x}=3\sec x$ for $-\pi \le x \le \pi$. [4]

$\dfrac{\tan 2x}{1+\sec 2x}+\dfrac{1+\sec 2x}{\tan 2x}=3\sec x$ $2\operatorname{cosec} 2x=3\sec x$ $\dfrac{2}{\sin 2x}=\dfrac{3}{\cos x}$ $2\cos x=3\times 2\sin x\cos x$ $6\sin x\cos x-2\cos x=0$ $2\cos x(3\sin x-1)=0$ $\cos x\neq 0$ $\sin x=\dfrac{1}{3}$ $\alpha=0.33984$ $x=0.340,\ 2.80$	

(c) Use the result in **part (a)** to find the range of values of k for which the equation $\dfrac{\tan x}{1+\sec x}+\dfrac{1+\sec x}{\tan x}=k$ has no solutions. [2]

$2\ \operatorname{cosec} x=k$ $\dfrac{2}{\sin x}=k$ $\sin x=\dfrac{2}{k}$ Since $-1\le \sin x\le 1$, $\sin x=\dfrac{2}{k}$ has no solutions when $\dfrac{2}{k}<-1$ or $\dfrac{2}{k}>1$ therefore $2\ \operatorname{cosec} x=k$ has no solutions when $-2<k<2$	

12 **(a)** Given that $y = \mathrm{e}^x(1+\cos x)$, show $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^x(1+\cos x - \sin x)$. [1]

$y = e^x(1+\cos x)$ $\frac{dy}{dx} = e^x(1+\cos x) + e^x(-\sin x)$ $\frac{dy}{dx} = e^x(1+\cos x - \sin x)$	

(b) Hence find $\int \mathrm{e}^x(\cos x - \sin x)\,\mathrm{d}x$. [3]

$\int e^x(1+\cos x - \sin x)\ dx = \int e^x + e^x(\cos x - \sin x)\ dx = e^x(1+\cos x) + c_1$ $\int e^x dx + \int e^x(\cos x - \sin x)\ dx = e^x(1+\cos x) + c$ $e^x + \int e^x(\cos x - \sin x)\ dx = e^x(1+\cos x) + c$ $\int e^x(\cos x - \sin x)\ dx = e^x(1+\cos x) - e^x + c$ $\int e^x(\cos x - \sin x)\ dx = e^x \cos x + c$	

13 A circular patch of water of negligible thickness is evaporating. The radius of the patch, r cm is decreasing at a constant rate of 0.2 cm/s. Find the rate of change of the area of this patch, A cm^2 when the area of the patch is 225π cm^2. [5]

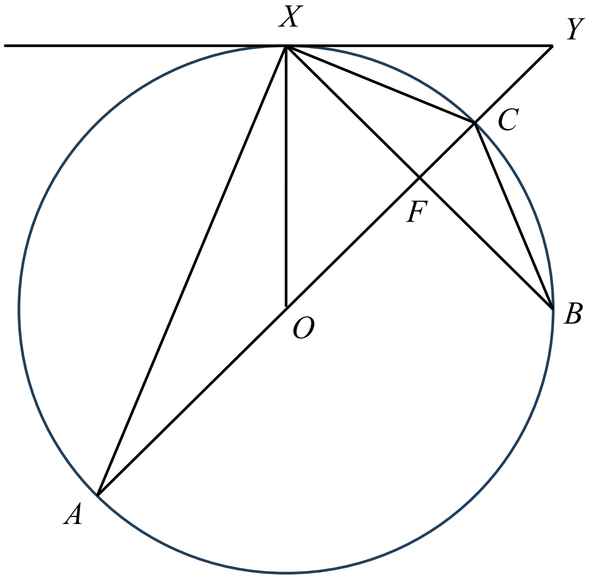
$A = \pi r^2$ $\dfrac{dA}{dr} = 2\pi r$ $A = 225\pi = \pi r^2$ $r = 15$ $\dfrac{dA}{dt} = \dfrac{dA}{dr} \times \dfrac{dr}{dt}$ $\dfrac{dA}{dt} = 2\pi(15) \times -0.2$ $\dfrac{dA}{dt} = -6\pi \text{ cm}^2/\text{s}$	

14 In the figure below, points A, B and C lie on the circle. AC is a diameter of a circle with centre O.

The tangent to the circle at X meets AC produced at Y.

Chord XB intersects AC at F.

XC bisects the angle of YXB.



(a) Prove that $CX = CB$. [2]

$\angle YXC = \angle CBX$ (tangent chord theorem) $\angle YXC = \angle CXF$ (XC is an angle bisector of YXB) $\angle CXF = \angle CBX$ ΔBCX is an isoceles triangle, $CX = CB$	

(b) Prove that triangle AXC is similar to triangle BFC. [3]

$\angle AXC = 90^o$ (right angle in a semicircle) $\angle BFC = 90^o$ (AC is a chord bisector of BX because ΔBCX is an isoc triangle (a)) $\angle AXC = \angle BFC$ $\angle XBC = \angle XAC = \angle FBC$ (angles in the same segment) *AA* similarity, triangle *AXC* is similar to triangle *BFC*.	

(c) Hence, prove that $AC \times FC = BC^2$ [2]

triangle *AXC* is similar to triangle *BFC* $\frac{AC}{BC} = \frac{XC}{FC}$ $AC \times FC = XC \times BC$ $XC = BC$ (from part (a)) $AC \times FC = BC^2$	

End of Paper

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