Name: Solution	Register No.:	Class:

CRESCENT GIRLS' SCHOOL
SECONDARY FOUR 2025
PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS **4049/02**
Paper 2 **3 September 2025**

Candidates answer on the Question Paper. **2 hours 15 mins**
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST
Write your name, register number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90**.

For Examiner's Use

Question	1	2	3	4	5	6	7	8	9	10
Marks										

Table of Penalties		Qn. No.		
Presentation	–1			
Significant Figures/ Units	–1		Parent's/Guardian's Signature	9

This document consists of **16** printed pages.
Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-r)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} ab \sin C$$

1 **(a)** Show that the equation $3e^{2x}+7e^{x}=6$ has only one solution and find its value correct to 2 significant figures. [4]

$3e^{2x}+7e^{x}=6$

Let $u=e^{x}$

$3u^2+7u-6=0$
$(3u-2)(u+3)=0$
$u=\dfrac{2}{3}$ or $u=-3$
$e^{x}=\dfrac{2}{3}$ or $e^{x}=-3$ (no solution)
$x=\ln\dfrac{2}{3}$
$=-0.41$ (to 2 s.f.)

(b) Factorise $(3x+1)^3-8$ completely. [2]

$(3x+1)^3-2^3$
$=\left[(3x+1)-2\right]\left[(3x+1)^2+2(3x+1)+2^2\right]$
$=(3x-1)(9x^2+6x+1+6x+2+4)$
$=(3x-1)(9x^2+12x+7)$

2 **(a)** Show that $x - 1$ is a factor of $3x^3 + 8x^2 - 5x - 6$. [1]

Let $f(x) = 3x^3 + 8x^2 - 5x - 6$
$f(1) = 3 + 8 - 5 - 6 = 0$
$\therefore$ By factor theorem, $x - 1$ is a factor of $f(x)$

(b) Hence solve the equation $3x^3 + 8x^2 - 5x = 6$ completely. [4]

$f(x) = 3x^3 + 8x^2 - 5x - 6 = (x - 1)(Ax^2 + Bx + C)$
Comparing coefficients of x^3 : $A = 3$
Comparing coefficients of x^2 : $8 = B - 3$
$B = 11$
Compare constant : $C = 6$
$f(x) = (x - 1)(3x^2 + 11x + 6) = 0$
$(x - 1)(3x + 2)(x + 3) = 0$
$x = 1, x = -\frac{2}{3}, x = -3$

OR

$$\begin{array}{r}
3x^2 + 11x + 6 \\
x - 1 \enclose{longdiv}{3x^3 + 8x^2 - 5x - 6} \\
-\underline{(3x^3 - 3x^2)} \\
11x^2 - 5x \\
-\underline{(11x^2 - 11x)} \\
6x - 6 \\
-\underline{(6x - 6)} \\
\underline{0}
\end{array}$$

$f(x) = (x - 1)(3x^2 + 11x + 6) = 0$
$(x - 1)(3x + 2)(x + 3) = 0$
$x = 1, x = -\frac{2}{3}, x = -3$

(c) Hence solve the equation $3(y - 2)^3 + 8(y - 2)^2 - 5y + 4 = 0$. [2]

$3(y - 2)^3 + 8(y - 2)^2 - 5(y - 2) - 6 = 0$

$y - 2 = 1, \quad y - 2 = -\frac{2}{3}, \quad y - 2 = -3$

$y = 3, \quad y = \frac{4}{3}, \quad y = -1$

3 **(a)** Show that $\frac{d}{dx}(\tan^3 2x) = 6\sec^4 2x - 6\sec^2 2x$. [3]

$$\frac{d}{dx}(\tan^3 2x) = 6\tan^2 2x \sec^2 2x$$

$$= 6\sec^2 2x(\sec^2 2x - 1)$$

$$= 6\sec^4 2x - 6\sec^2 2x$$

(b) Hence, find the exact value of $\int_0^{\frac{\pi}{8}} \sec^4 2x \; dx$. [5]

$$\int_0^{\frac{\pi}{8}} (6\sec^4 2x - 6\sec^2 2x)\, dx = \left[\tan^3 2x\right]_0^{\frac{\pi}{8}}$$

$$\int_0^{\frac{\pi}{8}} 6\sec^4 2x \; dx = \left[\tan^3 2x\right]_0^{\frac{\pi}{8}} + \int_0^{\frac{\pi}{8}} 6\sec^2 2x\, dx$$

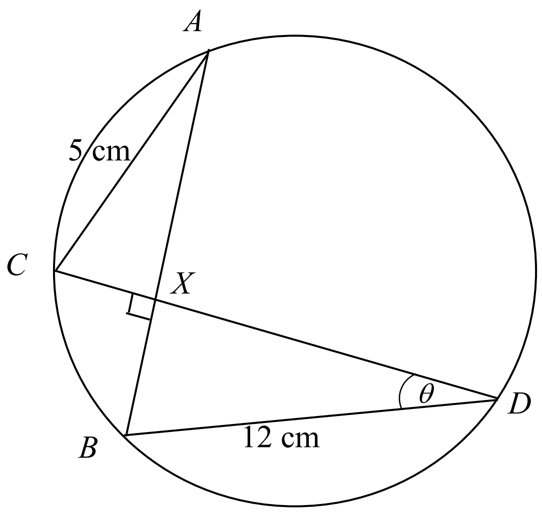
$$= \tan^3 \frac{\pi}{4} - \tan^3 0 + \frac{6}{2}\left[\tan 2x\right]_0^{\frac{\pi}{8}}$$

$$= 1 + 3\left(\tan\frac{\pi}{4} - \tan 0\right)$$

$$= 4$$

$$\int_0^{\frac{\pi}{8}} \sec^4 2x \; dx = \frac{2}{3}$$

4 The diagram shows two chords AB and CD intersect at X such that angle $CXB = 90°$.



Given that $AC = 5$ cm, $BD = 12$ cm and angle $BDC = \theta$.

(a) Prove that $AB = 5\cos\theta + 12\sin\theta$. [3]

$\angle CAX = \theta$ (can shown in diagram)

$\cos\theta = \dfrac{AX}{5}$ $\sin\theta = \dfrac{BX}{12}$

$AX = 5\cos\theta$ $BX = 12\sin\theta$

$AB = AX + BX = 5\cos\theta + 12\sin\theta$

(b) Express AB in the form $R\cos(\theta - \alpha)$, where $R > 0$ and α is an acute angle. [4]

$5\cos\theta + 12\sin\theta = R\cos(\theta - \alpha) = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

$\tan\alpha = \dfrac{12}{5}$ $R = 13$

$\alpha = 67.38^0$

$AB = 13\cos(\theta - 67.4^0)$

(c) Find the maximum length of AB and the corresponding value of θ.

Hence state the radius of the circle. [3]

Maximum $AB = 13$ cm
$\theta = 67.4^0$
Radius = 6.5 cm

5 **(a)** Find the range of values of k for which $kx^2 + 8x + k < 6$ for all real values of x. [4]

$kx^2 + 8x + k - 6 < 0$

$b^2 - 4ac < 0$

$8^2 - 4k(k - 6) < 0$

$64 - 4k^2 + 24k < 0$

$k^2 - 6k - 16 > 0$

$(k - 8)(k + 2) > 0$

$k < -2$ or $k > 8$

$\therefore k < -2$

(b) Given the curve $y = 8x^2 + mx + m - 6$, where m is a constant, find

(i) the possible values of m for which the x-axis is a tangent to the curve. [3]

$8x^2 + mx + m - 6 = 0$

$b^2 - 4ac = 0$

$m^2 - 4(8)(m - 6) = 0$

$m^2 - 32m + 192 = 0$

$(m - 8)(m - 24) = 0$

$m = 8$ or $m = 24$

(ii) the range of values of m for which the curve has a positive y-intercept. [1]

$m - 6 > 0$

$m > 6$

(c) Show that the equation $2x^2 - cx + c^2 + 6 = 0$ has no real roots for all real values of c. [2]

$$b^2 - 4ac = c^2 - 4(2)(c^2 + 6)$$
$$= -7c^2 - 48$$

Since $c^2 \geq 0$ for all real values of c, $-7c^2 \leq 0$, $b^2 - 4ac < 0$

$\therefore$ the equation has no real roots for all real values of c.

6 It is given that $\left(x+\frac{k}{x^2}\right)^n$ is a binomial expansion where k and n are positive constants.

(a) Write down the first 4 terms in the expansion $\left(x+\frac{k}{x^2}\right)^5$ in terms of k. [2]

$$\left(x+\frac{k}{x^2}\right)^5 = x^5 + \binom{5}{1}x^4\left(\frac{k}{x^2}\right)+\binom{5}{2}x^3\left(\frac{k}{x^2}\right)^2+\binom{5}{3}x^2\left(\frac{k}{x^2}\right)^3+\ldots$$

$$= x^5+5kx^2+\frac{10k^2}{x}+\frac{10k^3}{x^4}+\ldots$$

(b) Hence, find the value of k if the term independent of x in the expansion $\left(5x+\frac{3}{x^2}\right)\left(x+\frac{k}{x^2}\right)^5$ is 5. [3]

$$\left(5x+\frac{3}{x^2}\right)\left(x+\frac{k}{x^2}\right)^5 = \left(5x+\frac{3}{x^2}\right)\left(x^5+5kx^2+\frac{10k^2}{x}+\frac{10k^3}{x^4}+\ldots\right)$$

constant term = 5

$5\times 10k^2+3\times 5k=5$

$10k^2+3k-1=0$

$(5k-1)(2k+1)=0$

$k=\frac{1}{5}$ or $k=-\frac{1}{2}$ (reject)

(c) By using the formula for the general term, suggest two possible powers of n such that $\left(x+\frac{k}{x^2}\right)^n$ contains a term independent of x. [3]

$$T_{r+1}=\binom{n}{r}x^{n-r}\left(\frac{k}{x^2}\right)^r$$

$$=\binom{n}{r}k^r x^{n-3r}$$

For term independent of x, $n=3r$.

Since n must be a multiple of 3,
possible values of n are 6 and 9.

7 The equation of a circle with centre C is $x^2 + y^2 - 8x - 10y + 16 = 0$.

A point B (7, 9) lies on the circumference of the circle. P is the lowest point on the circle.

(a) Find the coordinates of C and the radius of the circle. [3]

$x^2 + y^2 - 8x - 10y + 16 = 0$
$(x - 4)^2 - 4^2 + (y - 5)^2 - 5^2 + 16 = 0$
$(x - 4)^2 + (y - 5)^2 = 25$
$C(4, 5)$ A1 radius = 5 units

(b) Find the equation of the tangent to the circle at B. [3]

Gradient of $BC = \dfrac{9-5}{7-4} = \dfrac{4}{3}$

Gradient of tangent at $B = -\dfrac{3}{4}$

Equation of tangent at B: $y - 9 = -\dfrac{3}{4}(x - 7)$

$$y = -\frac{3}{4}x + \frac{57}{4}$$

(c) Explain why P lies on the x-axis. [1]

P is vertically below centre C.
x-coordinate of P = x-coordinate of C = 4
y-coordinate of P – radius = 5 – 5 = 0
$\therefore P(4, 0)$ lies on the x-axis.

8 A particle moves in a straight line so that t seconds after leaving a fixed point O, its velocity v m/s, is given by $v = 16(2e^{-2t} - e^{-t})$. Find

(a) the time when the particle reaches its greatest distance from O, [3]

When $v = 0$, $2e^{-2t} - e^{-t} = 0$

$e^{-t}(2e^{-t} - 1) = 0$

$e^{-t} = 0$ (NA) or $e^{-t} = \frac{1}{2}$

$t = \ln 2$ or 0.693

(b) the value of the greatest distance from O. [3]

$s = \int 16(2e^{-2t} - e^{-t})\, dt$

$= 16(-e^{-2t} + e^{-t}) + c$

When $t = 0$, $s = 0$, $c = 0$

$s = 16(-e^{-2t} + e^{-t})$

when $t = \ln 2$, $s = 4$ m

(c) the minimum velocity of the particle. [4]

$a = 16(-4e^{-2t} + e^{-t})$

when $a = 0$, $-4e^{-2t} + e^{-t} = 0$

$e^{-t}(-4e^{-t} + 1) = 0$

$e^{-t} = 0$ (NA) or $4e^{-t} = 1$

$t = \ln 4$

When $t = \ln 4$, $v = -2$ m/s

9 **(a)** For the equation $py = xy + 1$ where p is an unknown constant, may be represented by a straight line, expressed in the form $Y = mX + c$, where X and Y are functions of x and/or y and m and c are constants. Using the following table, insert in it an expression for Y, X, m and c. [2]

	Y	X	m	c
$py = xy + 1$	y	xy	$\frac{1}{p}$	$\frac{1}{p}$ B1
$xy = py - 1$	OR xy	Y	P	-1
$py = xy + 1$	OR $\frac{1}{y}$	x	-1	P

(b) The variables x and y are related by the equation $x^a y = b$, where a and b are constants. The table below shows some corresponding values of x and y.

x	1.31	1.63	1.97	2.54	4.03
y	1270	398	131	35.1	2.63

(i) Plot lg y against lg x and draw a straight line graph. [3]

lg x	0.117	0.212	0.294	0.405	0.605
lg y	3.10	2.60	2.12	1.55	0.420

$a\lg x + \lg y = \lg b$

$\lg y = -a\lg x + \lg b$

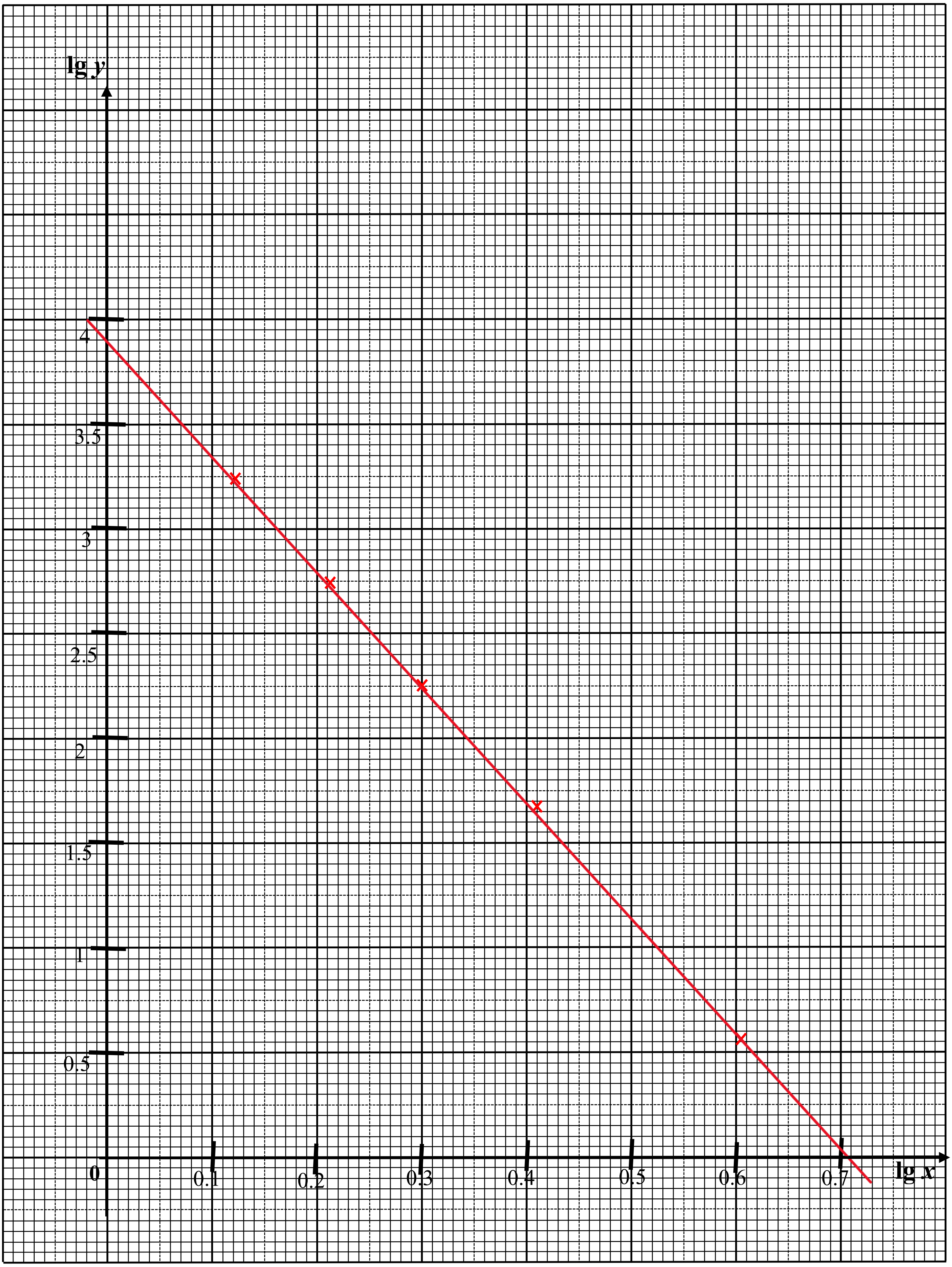
(ii) Use your graph to estimate the value of a and of b. [4]

Gradient = -5.51

$a \approx 5.51 \quad (\pm 0.1)$

Y-intercept = lg b = 3.8 ($\pm$0.1)

$b = 6310\ (5012 - 7943)$


lg y
4
3.5
3
2.5
2
1.5
1
0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
lg x

10 **(a)** **(i)** Show that $\dfrac{(\sin\theta-\cos\theta)(1+\sin\theta\cos\theta)}{\cos^3\theta}=\tan^3\theta-1$. [4]

From LHS $\dfrac{(\sin\theta-\cos\theta)(1+\sin\theta\cos\theta)}{\cos^3\theta}$

$$=\frac{\sin\theta+\sin^2\theta\cos\theta-\cos\theta-\sin\theta\cos^2\theta}{\cos^3\theta}$$

$$=\frac{\sin\theta+\cos\theta(1-\cos^2\theta)-\cos\theta-\sin\theta(1-\sin^2\theta)}{\cos^3\theta}$$

$$=\frac{\sin\theta+\cos\theta-\cos^3\theta-\cos\theta-\sin\theta+\sin^3\theta}{\cos^3\theta}$$

$$=\frac{\sin^3\theta-\cos^3\theta}{\cos^3\theta}$$

$=\tan^3\theta-1$ (shown)

(ii) Hence, for $0<\theta<2\pi$, find the reflex angle θ such that

$$(\sin\theta-\cos\theta)(1+\sin\theta\cos\theta)=\cos^3\theta\ .$$ [3]

$(\sin\theta-\cos\theta)(1+\sin\theta\cos\theta)=\cos^3\theta$

$\dfrac{(\sin\theta-\cos\theta)(1+\sin\theta\cos\theta)}{\cos^3\theta}=1$

$\tan^3\theta-1=1$

$\tan\theta=\sqrt[3]{2}$

$\theta=\tan^{-1}\left(\sqrt[3]{2}\right)$

$=4.04$

(b) Solve the equation $\sin 3A\cos 2A = \cos 3A\sin 2A$ for $0 \le A \le 180^\circ$. [2]

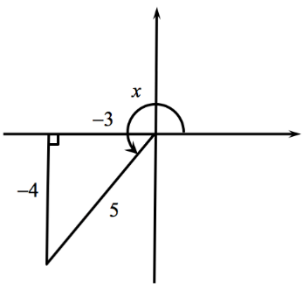
$\sin 3A\cos 2A - \cos 3A\sin 2A = 0$

$\sin(3A - 2A) = 0$

$\sin A = 0$

$A = 0^0, 180^\circ$

(c) Given that $\sin x = -\dfrac{4}{5}$ and $\tan x > 0$, find, without using a calculator, the value of $\cos 4x$. [2]



$$\cos 4x = 1 - 2\sin^2 2x$$

$$= 1 - 2\left(2\sin x\cos x\right)^2$$

$$= 1 - 2\left(2\times\left(-\frac{4}{5}\right)\times\left(-\frac{3}{5}\right)\right)^2$$

$$= -\frac{527}{625}$$

OR

$$\cos 2x = 1 - 2\sin^2 x$$

$$= 1 - 2\left(-\frac{4}{5}\right)^2$$

$$= -\frac{7}{25}$$

$$\cos 4x = 2\cos^2 2x - 1$$

$$= 2\left(-\frac{7}{25}\right)^2 - 1$$

$$= -\frac{527}{625}$$

(d) **Without using a calculator,** show that

$\tan\frac{\pi}{12} = 2-\sqrt{3}$. [4]

$$\tan\frac{\pi}{12} = \tan\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$$

$$= \frac{\tan\frac{\pi}{3}-\tan\frac{\pi}{4}}{1+\tan\frac{\pi}{3}\tan\frac{\pi}{4}}$$

$$= \frac{\sqrt{3}-1}{1+\sqrt{3}}$$

$$= \frac{\sqrt{3}-1}{1+\sqrt{3}}\times\frac{1-\sqrt{3}}{1-\sqrt{3}}$$

$$= \frac{2\sqrt{3}-4}{-2}$$

$$= 2-\sqrt{3}$$