Class/ Index Number	Centre Number/ ‘O’ Level Index Number	Name
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新加坡海星中学

MARIS STELLA HIGH SCHOOL
PRELIMINARY EXAMINATION
SECONDARY FOUR

ADDITIONAL MATHEMATICS
Paper 1

4049/1

21 August 2025

Candidates answer on the Question Paper.

2 hours 15 minutes

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is 90.

For Examiners’ Use

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9
/ 5	/ 6	/ 5	/ 6	/ 5	/ 6	/ 7	/ 7	/ 7

Q10	Q11	Q12	Q13	SUBTOTAL		
/ 8	/ 8	/ 10	/ 10	Statement		90
				Presentation		
				Units		
				Rounding Off		

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 The curve $(x-1)^2+(y-3)^2=5$ intersects the line $y-3x=5$ at two points. Find the coordinates of these two points. [5]

$(x-1)^2+(y-3)^2=5$ ------(1)
$y-3x=5$
$y=3x+5$ -----------(2)

Subst (2) to (1), $(x-1)^2+(3x+5-3)^2=5$ M1

$x^2-2x+1+9x^2+12x+4=5$

$10x^2+10x=0$ M1

$10x(x+1)=0$ M1

$x=0$ or $x=-1$

$y=5$ or $y=2$

the coordinates are (0, 5) and (–1, 2) A2

2 A stone is thrown vertically upwards such that its height, h metres from the ground at time t seconds after being thrown is given by the formula $h = -9t^2 + 12t + 1$.

(a) Explain the meaning of the constant term in the formula. [1]

The **initial height** of the stone is **1m above the ground.** B1

(b) Express h in the form $a(t+b)^2 + c$, where a, b and c are constants to be determined. [3]

$$h = -9\left(t^2 - \frac{4}{3}\right) + 1$$

$$= -9\left(t^2 - \frac{4}{3}t + \left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right)^2\right) + 1$$

$$= -9\left(t - \frac{2}{3}\right)^2 + 5$$

$a = -9$, $b = -\dfrac{2}{3}$, $c = 5$ B3 for each of the value

(c) Hence state the maximum height attained by the stone and the time at which this occurs. [2]

Maximum height = 5m when time $= \dfrac{2}{3}$ s √B2 FT from (b)

3 Find the range of values of the constant p such that $y = px^2 - 4x + p - 3$ is always positive for all real values of x. [5]

for $y = px^2 - 4x + p - 3$ to be always positive,
$p>0$ and discriminant < 0,

$(-4)^2 - 4(p)(p-3) < 0$	M1 for using discriminant < 0
$16 - 4p^2 + 12p < 0$	
$p^2 - 3p - 4 > 0$	M1 for quadratic eqn
$(p+1)(p-4) > 0$	M1 for factorising
$p < -1$ or $p > 4$	M1
since $p > 0$, $p > 4$	A1

4 Express $\dfrac{15x^2-13x+18}{(x-2)(3x^2+1)}$ in partial fractions. [6]

Let $\dfrac{15x^2-13x+18}{(x-2)(3x^2+1)} = \dfrac{A}{x-2} + \dfrac{Bx+C}{3x^2+1}$ M1

$15x^2-13x+18 = A(3x^2+1)+(Bx+C)(x-2)$ M1

when $x = 2$, $13A = 52$

$A = 4$ M1

when $x = 0$, $-2C + 4 = 18$

$C = -7$ M1

when $x = 1$, $16 - B + 7 = 20$

$B = 3$ M1

$\dfrac{15x^2-13x+18}{(x-2)(3x^2+1)} = \dfrac{4}{x-2} + \dfrac{3x-7}{3x^2+1}$ A1

5 **(a)** Find the first 4 terms in the expansion of $\left(\frac{a}{x}+3x^2\right)^6$ in ascending power of x, simplifying each term. [3]

$$\left(\frac{a}{x}+3x^2\right)^6=\left(\frac{a}{x}\right)^6+\binom{6}{1}\left(\frac{a}{x}\right)^5\left(3x^2\right)+\binom{6}{2}\left(\frac{a}{x}\right)^4\left(3x^2\right)^2+\binom{6}{3}\left(\frac{a}{x}\right)^3\left(3x^2\right)^3+\ldots$$

B1 if at least 2 terms correct and
B2 if all 4 terms correct

$$=\frac{a^6}{x^6}+\frac{18a^5}{x^3}+135a^4+540a^3x^3+\ldots$$ B1

(b) Given that there is no term in x^3 in the expansion of $(2x^3-1)\left(\frac{a}{x}+3x^2\right)^6$, find the value of the positive constant a. [2]

$$\left(2x^3-1\right)\left(\frac{a^6}{x^6}+\frac{18a^5}{x^3}+135a^4+540a^3x^3+\ldots\right)$$

since there is no x^3 term, $2\left(135a^4\right)-540a^3=0$ M1

$270a^4-540a^3=0$

$270a^3\left(a-2\right)=0$

$a=0$ (rej), $a=2$ A1

6 A particle moves along the curve $y = \dfrac{5}{(3x-1)^2}$, where $x \neq \dfrac{1}{3}$, in such a way that the x-coordinate is decreasing at a rate of 0.1 units per second.

(a) Find the rate of change of the y-coordinate when $x = 1$. [4]

$y = 5(3x-1)^{-2}$

$\dfrac{dy}{dx} = 5(-2)(3x-1)^{-3}(3)$ M1

$= \dfrac{-30}{(3x-1)^3}$ M1

when $x = 1$, $\dfrac{dy}{dx} = \dfrac{-30}{8}$

$\dfrac{dy}{dt} = \dfrac{-30}{8}(-0.1)$ M1

$= \dfrac{3}{8}$ units/s A1

(b) Find the value of x when y increases at the rate of $\dfrac{3}{125}$ units per second. [2]

$\dfrac{3}{125} = \dfrac{3}{(3x-1)^3}$

$(3x-1)^3 = 125$ M1

$3x - 1 = 5$

$x = 2$ A1

7 **A calculator must not be used in this question.**

(a) Show that $\tan 15° = 2 - \sqrt{3}$. [4]

$$\tan(60° - 45°) = \frac{\tan 60° - \tan 45°}{1 + \tan 60° \tan 45°}$$ M1

$$= \frac{\sqrt{3} - 1}{1 + \sqrt{3}} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1}$$ M1 for ratios, M1 for rationalization

$$= \frac{3 - 2\sqrt{3} + 1}{3 - 1}$$

$$= \frac{4 - 2\sqrt{3}}{2}$$

$$= 2 - \sqrt{3} \text{ (shown)}$$ A1

(b) Use the result from **part (a)** to find an expression for $\operatorname{cosec}^2 15°$, in the form $a + b\sqrt{3}$ where a and b are integers. [3]

$$\operatorname{cosec}^2 15° = 1 + \cot^2 15°$$

$$= 1 + \frac{1}{\left(2 - \sqrt{3}\right)^2}$$ M1

$$= 1 + \frac{1}{4 - 4\sqrt{3} + 3}$$

$$= 1 + \frac{1}{7 - 4\sqrt{3}} \times \frac{7 + 4\sqrt{3}}{7 + 4\sqrt{3}}$$ M1

$$= 1 + \frac{7 + 4\sqrt{3}}{49 - 48}$$

$$= 8 + 4\sqrt{3}$$ A1

8 **(a)** Prove the identity $\dfrac{\sin x}{1-\cos x}+\dfrac{1-\cos x}{\sin x}=2\operatorname{cosec} x$. [4]

$$\text{LHS}=\frac{\sin x}{1-\cos x}+\frac{1-\cos x}{\sin x}$$

$$=\frac{\sin^2 x+(1-\cos x)^2}{(1-\cos x)\sin x} \qquad \text{M1}$$

$$=\frac{\sin^2 x+1-2\cos x+\cos^2 x}{(1-\cos x)\sin x}$$

$$=\frac{1+1-2\cos x}{(1-\cos x)\sin x} \qquad \text{M1}$$

$$=\frac{2(1-\cos x)}{(1-\cos x)\sin x} \qquad \text{M1}$$

$= 2\operatorname{cosec} x$ A1

$=$ RHS

(b) Hence solve the equation $\dfrac{\sin x}{1-\cos x}+\dfrac{1-\cos x}{\sin x}=5$ for $0 \le x \le 2\pi$. [3]

$2\operatorname{cosec} x = 5$

$\operatorname{cosec} x = \dfrac{5}{2}$

$\sin x = \dfrac{2}{5}$ M1

Basic angle = 0.41151 M1

x = 0.412, 2.73 (3s.f) A1

9 A curve is such that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 3e^{2x} - e^{-x}$. The curve cuts the y-axis at P (0, 1) and has a gradient of 4 at P.

(a) Show that the curve has no stationary points. [3]

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \int (3e^{2x} - e^{-x})\mathrm{d}x$$

$$= \frac{3}{2}e^{2x} + e^{-x} + c \qquad \text{M1}$$

Gradient = 4 at (0, 1),

$$4 = \frac{3}{2} + 1 + c$$

$$c = 1.5 \qquad \text{M1}$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{2}e^{2x} + e^{-x} + \frac{3}{2}$$

since $\frac{3}{2}e^{2x} > 0,\ e^{-x} > 0,$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{2}e^{2x} + e^{-x} + \frac{3}{2} > 0, \qquad \text{A1}$$

the curve has no stationary points.

(b) Find the equation of the curve. [4]

$$y = \int \left(\frac{3}{2}e^{2x} + e^{-x} + \frac{3}{2}\right)\mathrm{d}x$$

$$= \frac{3}{4}e^{2x} - e^{-x} + \frac{3}{2}x + d \qquad \text{M2}$$

at (0, 1), $1 = \frac{3}{4} - 1 + 0 + d$

$$d = \frac{5}{4} \qquad \text{M1}$$

$$y = \frac{3}{4}e^{2x} - e^{-x} + \frac{3}{2}x + \frac{5}{4} \qquad \text{A1}$$

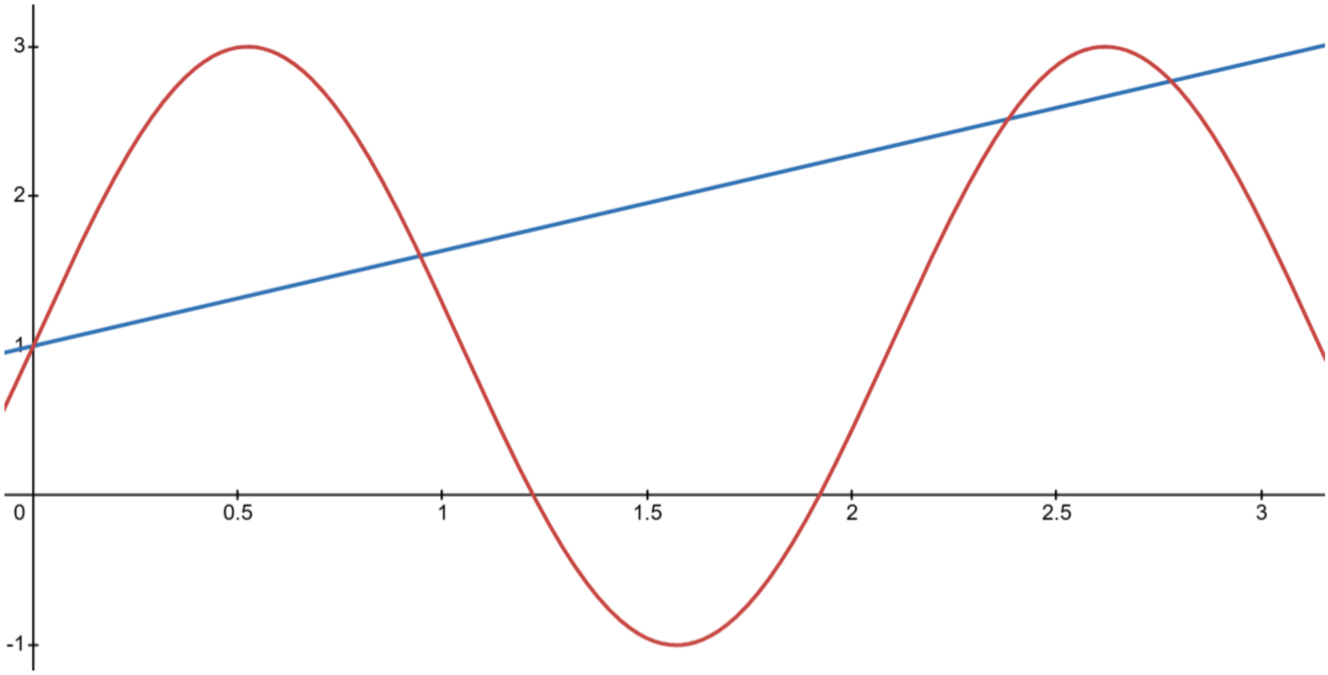
10 **(a)** State the amplitude and period of $2\sin 3x + 1$. [2]

Amplitude = 2 B1

Period = $\dfrac{2\pi}{3}$ (accept 120°) B1

(b) Sketch the graph of $y = 2\sin 3x + 1$ for $0 \le x \le \pi$.

[3]



Correct number of cycles: B1
Correct shape of curve: B1
Curve pass through critical pts (start, end and the max, min) : B1

(c) By drawing a suitable straight line on your sketch, determine the number of solutions of the equation $\pi \sin 3x - x = 0$. [3]

$\pi \sin 3x - x = 0$

$2\sin 3x - \dfrac{2x}{\pi} = 0$

$2\sin 3x + 1 = \dfrac{2x}{\pi} + 1$ M1

Draw $y = \dfrac{2x}{\pi} + 1$ M1

Number of solutions = 4 A1

11 **Solutions to this equation by accurate drawing will not be accepted.**

ABC is a triangle with vertices *A* $(0, h)$, *B* $(2h, -2)$ and *C* $(7, 5)$, where h is a positive integer. The area of the triangle is 20 units^2.

(a) Find the value of h. [3]

Area of triangle $\frac{1}{2}\begin{vmatrix} 0 & 2h & 7 & 0 \\ h & -2 & 5 & h \end{vmatrix}$

$10h + 7h - 2h^2 + 14 = 40$ M1

$2h^2 - 17h + 26 = 0$

$(h - 2)(2h - 13) = 0$ M1

$h = 2,\ h = \frac{13}{2}$ (rejected) A1

(b) Find the equation of the perpendicular bisector of *AB*. [3]

Midpoint of *AB* $= \left(\frac{0+4}{2}, \frac{2-2}{2}\right)$

$= (2, 0)$ M1

gradient of *AB* $= \frac{-2-2}{4-0}$

$= -1$ M1

Equation of perpendicular bisector of *AB*: $y = x - 2$ A1

(c) Explain why triangle *ABC* is isosceles. [2]

$AC = \sqrt{(0-7)^2 + (2-5)^2} = \sqrt{58}$

$BC = \sqrt{(4-7)^2 + (-2-5)^2} = \sqrt{58}$ M1

since $AC = BC$, triangle *ABC* is isosceles (shown) A1

Or

from part (b), when $x = 7$, $y = 5$ M1

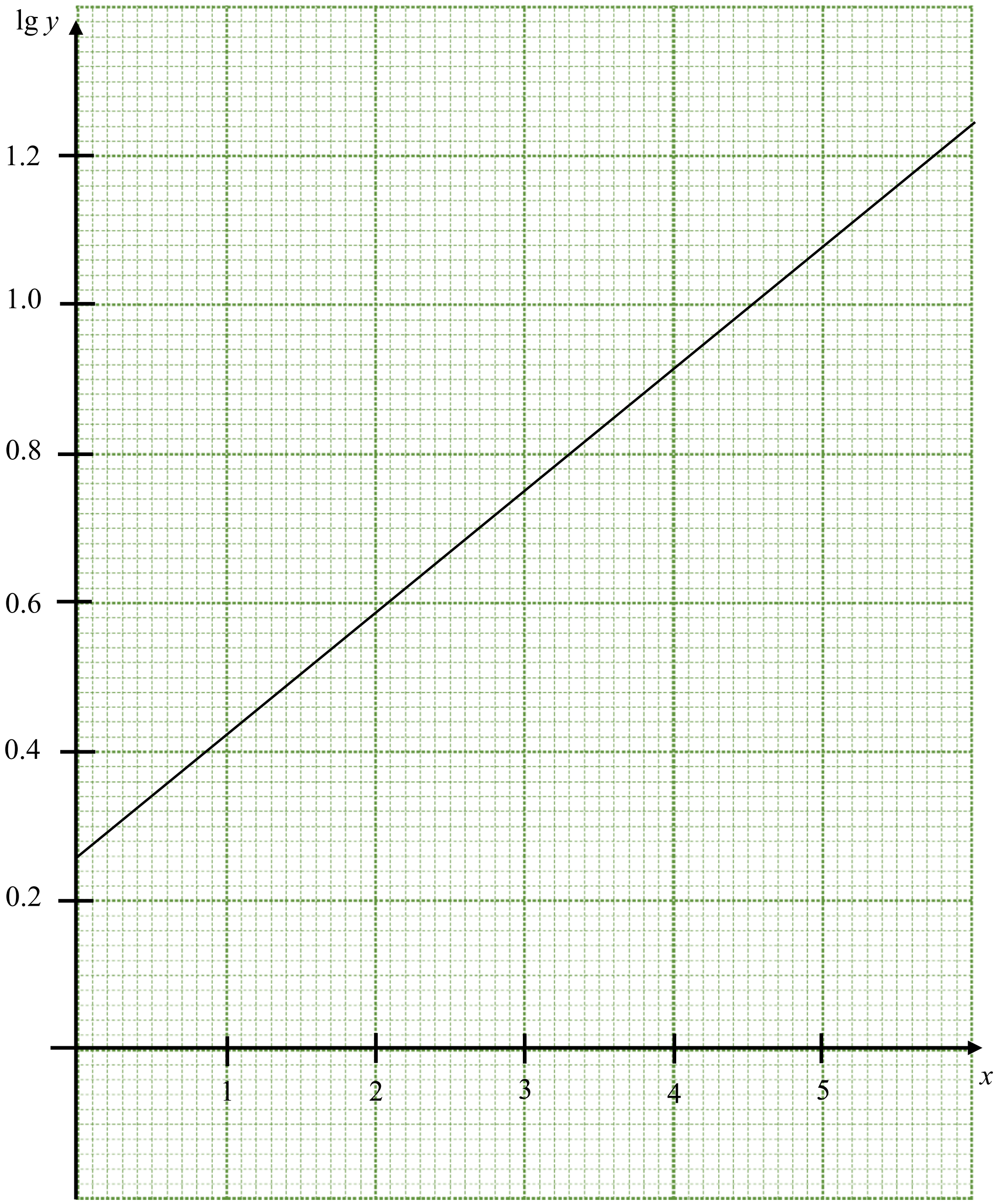
vertex *C* lies on the perpendicular bisector of *AB*, $AC = BC$ A1

triangle *ABC* is isosceles (shown)

12 **(a)** The table shows experimental values of two variables x and y. It is known that x and y are related by the equation $y = Ak^x$, where A and k are constants.

x	1	2	3	4	5
y	2.63	3.89	5.62	8.32	12.02
$\lg y$	0.420	0.590	0.750	0.920	1.08

(i) On the grid below plot $\lg y$ against x and draw a straight line graph to illustrate the information. [2]



x	1	2	3	4	5
y	2.63	3.89	5.62	8.32	12.02
$\lg y$	0.420	0.590	0.750	0.920	1.08

P1 for correct points plot. L1 for straight line drawn.

(ii) Use your graph to estimate the value of A and of k. [4]

$y = Ak^x$

$\lg y = \lg Ak^x$

$\lg y = x \lg k + \lg A$ M1

From the graph, gradient = $\dfrac{0.92 - 0.59}{4 - 2}$

$= 0.165$ M1

lg$y$-intercept = 0.26,

$\lg A = 0.26$

$A = 1.82$ (accept 1.78 to 1.86) A1

$\lg k = 0.165$

$k = 1.46$ (accept 1.43 to 1.50) A1

(b) Variables x and y are related by the formula $y = ax^3 + bx^2$, where a and b are constants. Explain clearly how a straight line graph can be drawn to represent this formula. You should state which variables should be plotted on each axis and explain how the values of a and b can be calculated. [4]

$y = x^2(ax + b)$

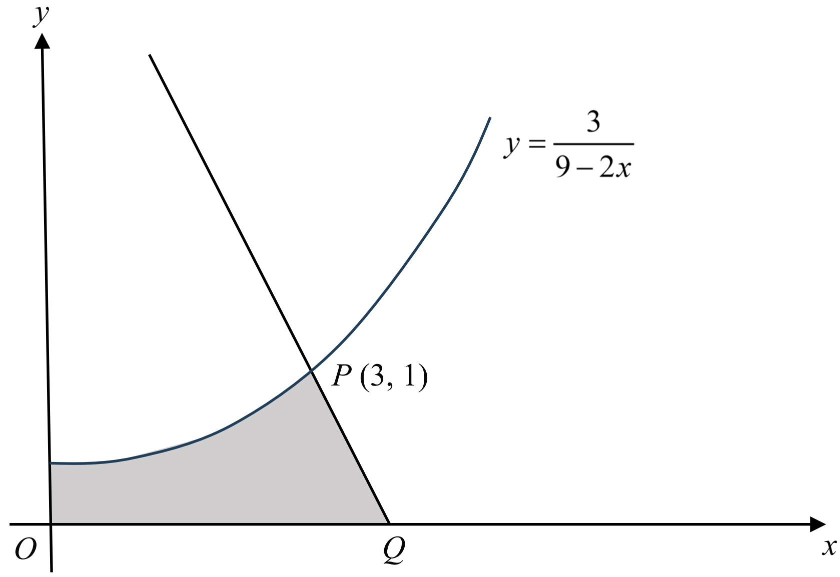
$\dfrac{y}{x^2} = ax + b$ or alternative $\dfrac{y}{x^3} = a + \dfrac{b}{x}$ M1

Plot $\dfrac{y}{x^2}$ (or $\dfrac{y}{x^3}$) on the vertical axis against x (or $\dfrac{1}{x^2}$) on the horizonal axis (M1)

Gradient of line = a (or b) A1

Vertical intercept ($\dfrac{y}{x^2}$ intercept) of line = b (or a) A1

13



The diagram shows part of the curve $y = \frac{3}{9-2x}$. The point P (3, 1) lies on the curve and the normal to the curve at P meets the x-axis at Q.

(a) Find the coordinates of Q. [5]

$$\frac{dy}{dx} = \frac{6}{(9-2x)^2} \qquad \text{M1}$$

when $x = 3$, $\frac{dy}{dx} = \frac{6}{(9-6)^2}$

$$= \frac{2}{3} \qquad \text{M1}$$

Gradient of normal = $-\frac{3}{2}$

Equation of normal: $y - 1 = -\frac{3}{2}(x-3)$ M1

when $y = 0$, $-1 = -\frac{3}{2}x + \frac{9}{2}$

$$x = \frac{11}{3} \qquad \text{M1}$$

Coordinates of Q $\left(\frac{11}{3}, 0\right)$ A1

(b) Find the area of the shaded region bounded by the curve, the normal PQ and the coordinate axes. [5]

Area of shaded region = $\int_0^3 \frac{3}{9-2x}\,dx + \frac{1}{2}(1)\left(3\frac{2}{3}-3\right)$ M2

$= 3\left[\frac{\ln(9-2x)}{-2}\right]_0^3 + \frac{1}{3}$ M1

$= 3(-0.54931 + 1.0986) + \frac{1}{3}$ M1

$= 1.98$ units^2 (3s.f) A1

End of Paper

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