Ngee Ann Secondary School
Secondary 4 Additional Mathematics-O
2025 Preliminary Examination Paper 2
Marking Scheme

Qn No.	Qn Part	Solutions	Marks (Remarks)	Total
1	**(a)**	$5\sqrt{108}-\sqrt{243}+\dfrac{\sqrt{192}}{2}$		
		$=5\sqrt{36\times 3}-\sqrt{81\times 3}+\dfrac{\sqrt{64\times 3}}{2}$	M1: At least 2 correct	
		$=30\sqrt{3}-9\sqrt{3}+4\sqrt{3}$		
		$=25\sqrt{3}$	A1	2
	(b)	$\dfrac{p+2}{p-1}=\dfrac{\sqrt{5}+2+2}{\sqrt{5}+2-1}$		
		$=\dfrac{\sqrt{5}+4}{\sqrt{5}+1}$		
		$=\dfrac{\sqrt{5}+4}{\sqrt{5}+1}\times\dfrac{\sqrt{5}-1}{\sqrt{5}-1}$	M1: Attempt to rationalise	
		$=\dfrac{5+3\sqrt{5}-4}{5-1}$	M1: Expansion of surds	
		$=\dfrac{3}{4}\sqrt{5}+\dfrac{1}{4}$	A1	3

Qn No.	Qn Part	Solutions	Marks (Remarks)	Total
2	**(a)**	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{\cos 2x}{1+\sin 2x}\right)$		
		$=\dfrac{-2\sin 2x(1+\sin 2x)-(\cos 2x)(2\cos 2x)}{(1+\sin 2x)^2}$	B1: numerator B1 for denominator	
		$=\dfrac{-2\sin 2x-2\sin^2 2x-2\cos^2 2x}{(1+\sin 2x)^2}$		
		$=\dfrac{-2\sin 2x-2(\sin^2 2x+\cos^2 2x)}{(1+\sin 2x)^2}$		
		$=-\dfrac{-2\sin 2x-2}{(1+\sin 2x)^2}$	M1: Realising $\sin^2 2x+\cos^2 2x=1$	
		$=\dfrac{-2(\sin 2x+1)}{(1+\sin 2x)^2}$	A1	
		$=\dfrac{-2}{1+\sin 2x}$	AG	4
	(b)	$\displaystyle\int_0^{\frac{\pi}{4}}\frac{-2}{1+\sin 2x}\,\mathrm{d}x=\left[\frac{\cos 2x}{1+\sin 2x}\right]_0^{\frac{\pi}{4}}$	M1: Reverse statement	
		$\displaystyle\frac{1}{-2\times 8}\int_0^{\frac{\pi}{4}}\frac{-2}{1+\sin 2x}\,\mathrm{d}x=-\frac{1}{16}\left[\frac{\cos 2x}{1+\sin 2x}\right]_0^{\frac{\pi}{4}}$	M1: Multiply by $-\dfrac{1}{16}$	
		$\displaystyle\int_0^{\frac{\pi}{4}}\frac{1}{8(1+\sin 2x)}\,\mathrm{d}x=-\frac{1}{16}\left(\frac{0}{2}-\frac{1}{1}\right)$	M1: Correct use of limits	
		$=\dfrac{1}{16}$	A1	4

Qn No.	Qn Part	Solutions	Marks (Remarks)	Total
3	**(a)**	$x(x+1)^2 = x^3+2x^2+x$ $\begin{array}{r} 2 \\ x^3+2x^2+x\;\overline{)\;2x^3+12x^2+14x+5} \\ -(2x^3+4x^2+2x) \\ \hline 8x^2+12x+5 \\ \hline \end{array}$ $\dfrac{2x^3+12x^2+14x+5}{x(x+1)^2} = 2+\dfrac{8x^2+12x+5}{x(x+1)^2}$	B1	1
	(b)	$\dfrac{8x^2+12x+5}{x(x+1)^2} = \dfrac{A}{x}+\dfrac{B}{x+1}+\dfrac{C}{(x+1)^2}$ $8x^2+12x+5 = A(x+1)^2+Bx(x+1)+Cx$ When $x=0$, $A=5$. When $x=-1$, $C=-1$. When $x=1$, $2B=6$ $B=3$ $\dfrac{2x^3+12x^2+14x+5}{x(x+1)^2} = 2+\dfrac{5}{x}+\dfrac{3}{x+1}-\dfrac{1}{(x+1)^2}$	M1: Correct form of partial fractions M1: Remove denominator A1: Value of *A* A1: Value of *C* A1: Value of *B*	5

<table>
<tr><th>Qn No.</th><th>Qn Part</th><th>Solutions</th><th>Marks (Remarks)</th><th>Total</th></tr>
<tr><td>4</td><td>(a)</td><td>$10x^2 - nx + 2m = m - 6x$
$10x^2 + (6-n)x + m = 0$
Since line is tangent to curve,
$(6-n)^2 - 4(10)m = 0$
$(6-n)^2 = 40m$
Since $(6-n)^2 \geq 0, 40m \geq 0, m \geq 0$

Hence m cannot be negative.</td><td>M1: Eliminate x or y

M1: Use of discriminant

M1

A1</td><td>4</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td>(bi)</td><td>$2x^2 - 6x + 1 + k > 0$
$(-6)^2 - 4(2)(1+k) < 0$
$28 - 8k < 0$
$k > 3.5$</td><td>M1: Use of discriminant

A1</td><td>2</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td>(bii)</td><td>$kx^2 + (k+1)x + k < 0$
Discriminant < 0
$(k+1)^2 - 4k^2 < 0$
$-3k^2 + 2k + 1 < 0$
$3k^2 - 2k - 1 > 0$
$(3k+1)(k-1) > 0$
$k < -\frac{1}{3}$ or $k > 1$
Since $k < 0$, $k < -\frac{1}{3}$.</td><td>M1: Use of discriminant

M1: Factorise quadratic

A1</td><td>3</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>5</td><td>(a)</td><td>$c = at + \frac{b}{t}$
$ct = at^2 + b$

Draw ct against t^2.

<table><tr><td>t^2</td><td>0.64</td><td>1</td><td>1.44</td><td>1.96</td><td>3.24</td></tr><tr><td>ct</td><td>16.8</td><td>15.0</td><td>13.0</td><td>10.5</td><td>3.8</td></tr></table>
See graph next page</td><td>B1: Correct statement

M1: Table of values
B1: Plot points
B1: Draw best fit line</td><td>4</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
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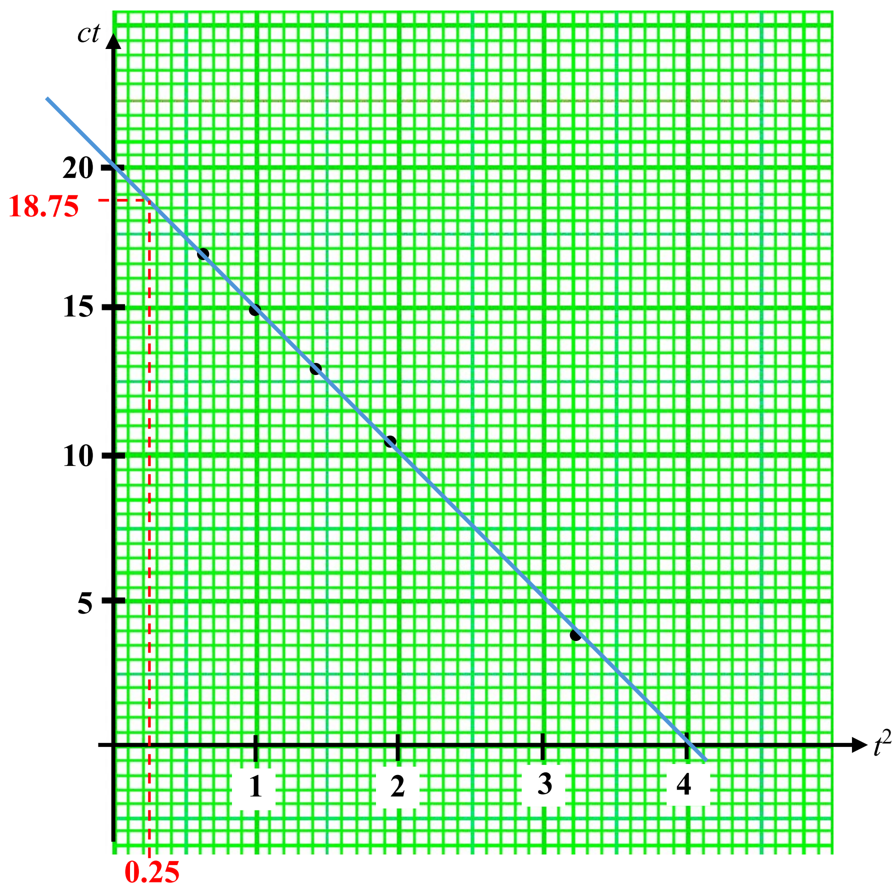
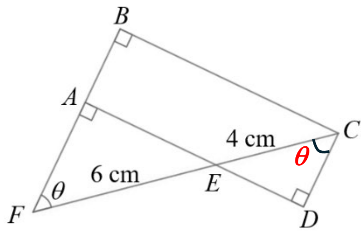
	(b)	$ct = at^2 + b$ $a =$ gradient $= -\frac{20}{4}$ $= -5$ $b =$ vertical intercept $= 20$	M1 A1 (Accept $-5.13 < a < -4.88$) B1 (Accept $19.5 < b < 20.5$)	3
	(c)	When $t = 0.5$, $t^2 = 0.25$. When $t^2 = 0.25$, $ct = 18.75$ $0.5c = 18.75$ $c = 37.5 > 30$ Since the concentration is greater than 30 mg/L, the chemical is not safe for plants when $t = 0.5$ h.	M1 (Accept $18.25 < ct < 19.25$) ($36.5 < c < 38.5$) A1	2

Graph for 5a



Qn No.	Qn Part	Solutions	Marks (Remarks)	Total
6	**(a)**	$y = 2\log_w wx + \log_w(3x-2) - 2$ For y to be defined, $wx > 0$ and $3x - 2 > 0$ $x > 0$ and $x > \frac{2}{3}$ Therefore, $x > \frac{2}{3}$.	B1: Final answer only	1
	(b)	$y = 2\log_w wx + \log_w(3x-2) - 2$ $= \log_w(wx)^2 + \log_w(3x-2) - \log_w w^2$ $= \log_w\left[\frac{w^2x^2(3x-2)}{w^2}\right]$ $= \log_w\left[3x^3 - 2x^2\right]$	M1: Use of power law M1: Realise $2 = \log_w w^2$ M1: Use of product and quotient laws A1: Manipulation to correct format	4
	(c)	$\log_w(3x^3 - 2x^2) = 0$ $3x^3 - 2x^2 = w^0$ $3x^3 - 2x^2 = 1$ $3x^3 - 2x^2 - 1 = 0$ $(x-1)(3x^2 + x + 1) = 0$ $3x^2 + x + 1 = 0$ $x = \frac{-1 \pm \sqrt{1^2 - 4(3)(1)}}{2(3)}$ $= \frac{-1 \pm \sqrt{-11}}{6}$ (no real solutions) or $x = 1$ (only one real root)	M1 M1: Factorisation M1: Using formula or discriminant A1 A1: For value of x	5

Qn No.	Qn Part	Solutions	Marks (Remarks)	Total
7	(a)	$x^3+3x^2-13x-15=9x+9$	M1: Eliminate x or y	
		$x^3+3x^2-22x-24=0$		
		$(x+6)(x-4)(x+1)=0$	M1: Factorise	
		$x=-6,\ x=4$ or $x=-1$		
		$A(-6,-45)$ and $B(-1,0)$	A1, A1	4
	(b)	Shaded region		
		$=\int_{-6}^{-1}\left[(x^3+3x^2-13x-15)-(9x+9)\right]dx$	M1	
		$-\int_{0}^{3}(x^3+3x^2-13x-15)\,dx$	M1	
		$=\int_{-6}^{-1}(x^3+3x^2-22x-24)\,dx$		
		$-\int_{0}^{3}(x^3+3x^2-13x-15)\,dx$		
		$=\left[\frac{x^4}{4}+x^3-11x^2-24x\right]_{-6}^{-1}$	M1: Correct integral	
		$-\left[\frac{x^4}{4}+x^3-\frac{13x^2}{2}-15x\right]_{0}^{3}$	M1: Correct integral	
		$=\left(\frac{1}{4}-1-11+24\right)-(324-216-396+144)$	M1: Substituting limits	
		$-\left[\left(\frac{81}{4}+27-\frac{117}{2}-45\right)-0\right]$		
		$=212.5$ units^2	A1	6

Qn No.	Qn Part	Solutions	Marks (Remarks)	Total
8				
	(a)	$\angle ECD = \angle AFE = \theta$ (alt. $\angle$s) $\sin\theta = \frac{AE}{6} \Rightarrow AE = 6\sin\theta$ $\cos\theta = \frac{CD}{4} \Rightarrow CD = 4\cos\theta$ $\sin\theta = \frac{ED}{4} \Rightarrow ED = 4\sin\theta$ $P = 4 + 6\sin\theta + 4\cos\theta + 4\sin\theta + 6\sin\theta$ $P = 4 + 16\sin\theta + 4\cos\theta$	M1 A1	2
	(b)	$R = \sqrt{16^2 + 4^2} = 4\sqrt{17}$ $\tan\alpha = \frac{4}{16}$ $\alpha = 14.036° \approx 14.0°$ $P = 4 + 4\sqrt{17}\sin(\theta + 14.036°)$	B1 M1 A1	3
	(c)	Max $P = 4 + 4\sqrt{17}$ $\sin(\theta + 14.036°) = 1$ $\theta + 14.036° = 90°$ $\theta = 75.964° \approx 76.0°$	B1 M1 A1	3
	(d)	$4 + 4\sqrt{17}\sin(\theta + 14.036°) = 15$ $\sin(\theta + 14.036°) = 0.66697$ $\theta + 14.036° = 41.834°$ $\theta = 27.8°$	M1 A1	2
	(e)	Since the maximum $P = 4 + 4\sqrt{17} \approx 20.492$, the perimeter of $ABCE$ could be 20 cm.	B1	1

Qn No.	Qn Part	Solutions	Marks (Remarks)	Total
9	(a)	$y=\dfrac{e^{2x}(3x+2)}{5}$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{3}{5}e^{2x}+\dfrac{2}{5}e^{2x}(3x+2)$ $=\dfrac{7}{5}e^{2x}+\dfrac{6}{5}xe^{2x}$ At stationary points, $\dfrac{7}{5}e^{2x}+\dfrac{6}{5}xe^{2x}=0$ $e^{2x}(7+6x)=0$ $e^{2x}=0$ (N.A) or $x=-\dfrac{7}{6}$	M1, M1 M1 A1, A1	5
	(b)	$\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=\dfrac{14}{5}e^{2x}+\dfrac{6}{5}e^{2x}+\dfrac{12}{5}xe^{2x}$ $=\dfrac{12xe^{2x}+20e^{2x}}{5}$ When $x=-\dfrac{7}{6}$, $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=\dfrac{12\left(-\dfrac{7}{6}\right)e^{-\frac{7}{3}}+20e^{-\frac{7}{3}}}{5}$ $=0.116>0$ The stationary point is a minimum point.	M2 M1 A1	4
	(c)	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{5}e^{2x}(7+6x)$ When $x>-1$, $\dfrac{1}{5}e^{2x}>0$, $7+6x>0$ and $\dfrac{1}{5}e^{2x}(7+6x)>0$. Since $\dfrac{\mathrm{d}y}{\mathrm{d}x}>0$, the gradient of the curve is positive when $x>-1$.	B1 B1	2

Qn No.	Qn Part	Solutions	Marks (Remarks)	Total
10	(a)	Gradient of DE $=\dfrac{-3+7}{-4-4}=-\dfrac{1}{2}$ Gradient of normal to $DE=2$ Midpoint of $DE=\left(\dfrac{-4+4}{2},\dfrac{-3-7}{2}\right)$ $=(0,-5)$ Equation of normal: $y=2x-5$ $4(2x-5)=3x-15$ $5x=5$ $x=1$ $y=2-5=-3$ Centre = $(1,-3)$ Radius = $\sqrt{(1+4)^2+(-3+3)^2}=5$ units Equation of circle: $(x-1)^2+(y+3)^2=5^2$ $(x-1)^2+(y+3)^2=25$	M1 M1 M1 M1: Eliminate x or y B1 B1 A1	7
	(b)	Distance between $(-1,1)$ and $(1,-3)$ $=\sqrt{(1+1)^2+(-3-1)^2}$ $=4.47<5$ Since the distance between $(-1,1)$ and $(1,-3)$ is lesser than the radius of the circle, the point $(-1,1)$ lies inside the circle.	M1 A1	2
	(c)	Solving simultaneously $4y=3x-15$ and $y=x-3$: $4(x-3)=3x-15$ $x=-3$ $y=-6$ Since the point of intersection of $y=x-3$ and the normal to the circle is not $(1,-3)$ but $(-3,-6)$, AB is not a possible diameter of the circle.	M1 A1	2

Qn No.	Qn Part	Solutions	Marks (Remarks)	Total
10	**(c)**	**Alternatively,** As $(1,-3)$ is the centre of the circle, sub. $x=1$ into the line $y=x-3$. $y=1-3=-2$ Since $y\neq-3$, the line $y=x-3$ does not pass through the centre $(1,-3)$. Therefore, the line segment PQ is not a possible diameter of the circle.	M1: Sub. centre into the line A1: Conclusion	2