ST JOSEPH'S INSTITUTION
PRELIMINARY EXAMINATION 2025
(YEAR 4) ADDITIONAL MATHEMATICS 4049/01

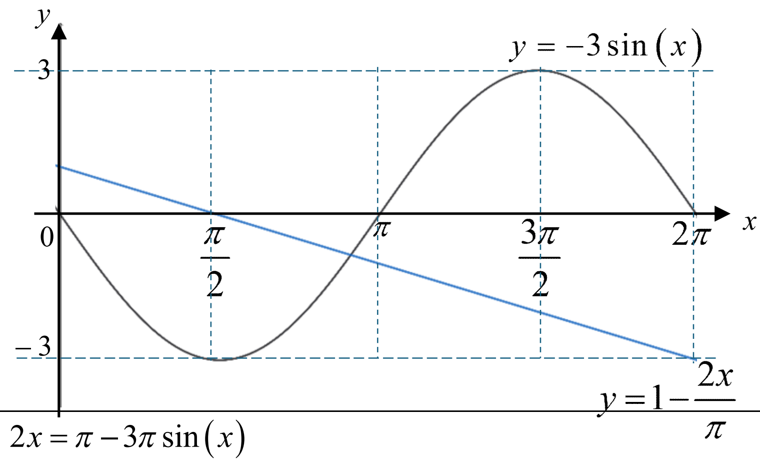
Qn	Answer
1	$x\sqrt{18}-\sqrt{72}=x\sqrt{3}$ $x\sqrt{18}-x\sqrt{3}=\sqrt{72}$ $x\left(\sqrt{18}-\sqrt{3}\right)=\sqrt{72}$ $x=\dfrac{\sqrt{72}}{\left(\sqrt{18}-\sqrt{3}\right)}$ $x=\dfrac{\sqrt{72}}{\left(\sqrt{18}-\sqrt{3}\right)}\times\dfrac{\left(\sqrt{18}+\sqrt{3}\right)}{\left(\sqrt{18}+\sqrt{3}\right)}$ $=\dfrac{\sqrt{1296}+\sqrt{216}}{18-3}$ $=\dfrac{36+6\sqrt{6}}{15}$ $\underline{a=12}$, $\underline{b=6}$
2	$\int\left(x-\dfrac{5}{2x}\right)^2\mathrm{d}x=\int\left(x^2-5+\dfrac{25}{4}x^{-2}\right)\mathrm{d}x$ $=\underline{\dfrac{x^3}{3}-5x-\dfrac{25}{4x}+c}$
3	$x^2+3xy=x+6$ ----- (1) $2y-x=2$ $y=\dfrac{1}{2}x+1$ ----- (2) Sub (2) into (1); $x^2+3x\left(\dfrac{1}{2}x+1\right)=x+6$ $x^2+\dfrac{3}{2}x^2+3x-x-6=0$ $2x^2+3x^2+4x-12=0$ $(5x-6)(x+2)=0$ $\underline{x=\dfrac{6}{5}\text{ or }x=-2}$

4	$-2x^2+6x-9=-2\left(x^2-3x+\frac{9}{2}\right)$ $=-2\left(x^2-3x+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2+\frac{9}{2}\right)$ $=-2\left(\left(x-\frac{3}{2}\right)^2+\frac{9}{4}\right)$ $\underline{=-2\left(x-\frac{3}{2}\right)^2-\left(\frac{9}{2}\right)}$ $-2\left(x-\frac{3}{2}\right)^2-\left(\frac{9}{2}\right)$ Since $\left(x-\frac{3}{2}\right)^2\geq 0$ $\underline{-2\left(x-\frac{3}{2}\right)^2}\leq 0$ **and** $\underline{-2\left(x-\frac{3}{2}\right)^2-\left(\frac{9}{2}\right)\leq-\left(\frac{9}{2}\right)}$ for all real values of x. Therefore $\underline{-9+6x-2x^2}$ **is negative** for all real values of x.
5(a)	Let x = Angle *QPA* *angle APB* = x (*AP* bisects angle *QPB*) *Angle ABP* = x (alt. seg thm) **Since angle *APB* = angle *ABP* = *x*** **Triangle ABP is isosceles**
5(b)(i)	Angle *ACP* = = x ($\angle$ in the same seg) Angle *ACP* = angle *APD* = x Angle *CAP* = angle *PAD* (common $\angle$) Triangle *ACP* and Triangle *APD* (AA Similarity Test)

5(b)(ii)	$\dfrac{AD}{AP}=\dfrac{AP}{AC}$ $AP^2 = AC\times AD \quad (\because AP = AB)$ $AB^2 = AC\times AD$
6	$\dfrac{2x^4+x^3+9x^2+7x+3}{(x+1)(x^2+5)}$ Long division to get the quotient and remainder $\dfrac{2x^4+x^3+9x^2+7x+3}{(x+1)(x^2+5)}$ $=2x-1+\dfrac{2x+8}{(x+1)(x^2+5)}$ $\dfrac{2x+8}{(x+1)(x^2+5)}=\dfrac{A}{x+1}+\dfrac{Bx+C}{x^2+5}$ $2x+8=A(x^2+5)+(Bx+C)(x+1)$ Let $x=-1$ $2(-1)+8 = A((-1)^2+5)$ $6=6A$ $A=1$ Let $x=0$ $8=1(5)+(C)(1)$ $C=3$ Let $x=2$ $2(2)+8=1(2^2+5)+(2B+3)(2+1)$ $2B+3=1$ $B=-1$ $2x-1+\dfrac{2x+8}{(x+1)(x^2+5)}$ $\underline{=2x-1+\dfrac{1}{(x+1)}+\dfrac{3-x}{(x^2+5)}}$

7(a)	$2x^3 + ax^2 + bx - 10 = (x-1)^2(2x+c)$ Comparing constant, $c = -10$ Comparing x^2, $a = -10 - 4 = \underline{\mathbf{-14}}$ Comparing x, $b = 20 + 2 = \underline{\mathbf{22}}$
7(b)	$2\left(\frac{1}{2}\right)^3 - 14\left(\frac{1}{2}\right)^2 + 22\left(\frac{1}{2}\right) - 10 + k = 0$ $\underline{k = \frac{9}{4}}$
8(a)	$750 = \frac{4}{3}\pi x^3 + \pi(x)^2 y$ $\pi(x)^2 y = 750 - \frac{4}{3}\pi x^3$ $y = \dfrac{750 - \frac{4}{3}\pi x^3}{\pi(x)^2}$ $y = \frac{750}{\pi x^2} - \frac{4x}{3}$
8(b)	$S = 4\pi x^2 + 2\pi x\left(\frac{750}{\pi x^2} - \frac{4x}{3}\right)$ $\underline{= 4\pi x^2 + \frac{1500\pi x}{\pi x^2} - \frac{8\pi x^2}{3}}$ $= \frac{4}{3}\pi x^2 + \frac{1500}{x}$

8(c)	$S=\frac{4}{3}\pi x^2+\frac{1500}{x}$ $\frac{dS}{dx}=\frac{8}{3}\pi x-\frac{1500}{x^2}$ $\frac{8}{3}\pi x-\frac{1500}{x^2}=0$ $\frac{8}{3}\pi x=\frac{1500}{x^2}$ $8\pi x^3=4500$ $x=\sqrt[3]{\frac{4500}{8\pi}}$ (=5.6362) Therefore, $S=\frac{4}{3}\pi\left(\sqrt[3]{\frac{4500}{8\pi}}\right)^2+\frac{1500}{\sqrt[3]{\frac{4500}{8\pi}}}$ ≈ 399.201 = **$\underline{399\text{ cm}^2}$**
9(a)	$\underline{a=3}$ Sub $\left(\frac{2\pi}{3},0\right)$ into $y=a+6\cos bx$ $0=3+6\cos b\left(\frac{2\pi}{3}\right)$ $\cos b\left(\frac{2\pi}{3}\right)=-\frac{1}{2}$ $b\left(\frac{2\pi}{3}\right)=\frac{2\pi}{3}\text{(rej)},\frac{4\pi}{3}$ $\underline{b=2}$

9(b)(i)	$y = 1 - \frac{2x}{\pi}$ $\Rightarrow \; y = -3\sin x$ 
9(b)(ii)	$2x = \pi - 3\pi\sin(x)$ $\frac{2x}{\pi} = 1 - 3\sin x$ $-3\sin(x) = 1 - \frac{2x}{\pi}$ No. of solution(s) = <u>**1**</u>
10(a)	$\frac{8-4}{k+2} \times \frac{8-4}{k-6} = -1$ $\frac{4}{k+2} \times \frac{4}{k-6} = -1$ $(k+2)(k-6) = -16$ $k^2 - 4k - 12 + 16 = 0$ $k^2 - 4k + 4 = 0$ $(k-2)^2 = 0$ $\underline{k = 2}$
10(b)	Let the coordinates of E be (E_x, E_y) $\left(\frac{2+E_x}{2}, \frac{8+E_y}{2}\right) = (-2, 4)$ $(E_x, E_y) = (-6, 0)$ $2 \times \frac{1}{2}\begin{vmatrix} 2 & -6 & 2a & 2 \\ 8 & 0 & -a & 8 \end{vmatrix} = 96$

	$\left[6a+16a-\left(-48-2a\right)\right]=96$ $24a=48$ $a=2$ $D\ \underline{(4,-2)}$ **Or** $\dfrac{-a-4}{2a+2}=\dfrac{8-4}{2-6}$ $\dfrac{-a-4}{2a+2}=-1$ $a+4=2a+2$ $a=-2$ $D(4,-2)$
11(a)	$\cot\theta-1=\dfrac{\cos 2\theta}{\sin\theta(\cos\theta+\sin\theta)}$ RHS= $\dfrac{\cos 2\theta}{\sin\theta(\cos\theta+\sin\theta)}$ $=\dfrac{\cos^2\theta-\sin^2\theta}{\sin\theta(\cos\theta+\sin\theta)}$ $=\dfrac{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)}{\sin\theta(\cos\theta+\sin\theta)}$ $=\dfrac{(\cos\theta-\sin\theta)}{\sin\theta}$ $=\cot\theta-1$ $=\text{RHS}$ (proven)
11(b)	$\dfrac{\cos 4\theta}{\sin 2\theta(\cos 2\theta+\sin 2\theta)}=-\dfrac{1}{2}$ $\cot 2\theta-1=-\dfrac{1}{2}$

	$\cot 2\theta = \frac{1}{2}$ $\tan 2\theta = 2$ $-180° \le \theta \le 180°$ $-360° \le 2\theta \le 360°$ Basic angle = 63.4349 $2\theta = -360 + 63.4349$ or $-180 + 63.4349$ or 63.4349 or $180 + 63.4349$ $\theta = \underline{-148.3°, -58.3°, 31.7°, 121.7°}$
12(a) (i)	$90 = 100\left(4^{-k(40)}\right)$ $0.9 = \left(4^{-k(40)}\right)$ $\lg 0.9 = \lg 4^{-k(40)}$ $\lg 0.9 = -40k \lg 4$ $k = 0.00190003866$ $\underline{k = 0.00190\,(3\text{sf})}$
12(a) (ii)	$M = 100\left(4^{-0.00190003866(263)}\right)$ ≈ 50.02 Therefore, % remaining is **<u>50.0%</u>**.
12(b)	$\log_5 x + \log_{25}(x-2) = 1$ $\log_5 x + \frac{\log_5(x-2)}{\log_5 25} = 1$ $\log_5 x + \frac{\log_5(x-2)}{2} = 1$ $\log_5 x^2 + \log_5(x-2) = 2$ $\underline{x^2(x-2) = 25}$ $\underline{x^3 - 2x^2 - 25 = 0}$

13(a)	$\text{Area} = \frac{1}{2}(x)(3x)\sin\frac{2\pi}{3}$ $= \frac{3\sqrt{3}x^2}{4}$ $\frac{\mathrm{d}A}{\mathrm{d}x} = \frac{3\sqrt{3}}{2}x$ $\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{\mathrm{d}A}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t}$ $= \frac{3\sqrt{3}}{2}(4) \times 0.5$ $= \underline{3\sqrt{3}\ \text{cm}^2/\text{s}}$
13(b)	$F = \frac{k}{\sqrt{r}}$ Sub $F = 0.5,\ r = 9,$ $0.5 = \frac{k}{\sqrt{9}}$ $k = 1.5$ $r = \left(\frac{1.5}{F}\right)^2$ $\frac{\mathrm{d}r}{\mathrm{d}F} = -\frac{2(1.5)^2}{F^3}$, sub $F = 0.5$ $\frac{\mathrm{d}r}{\mathrm{d}F} = -\frac{2(1.5)^2}{(0.5)^3}$ = **<u>–36</u>** units per unit increase in F

14(a)	Sub $y=2\sqrt{2+x}$ into $y=2x$ $2\sqrt{2+x}=2x$ $4(2+x)=4x^2$ $x^2-x-2=0$ $(x-2)(x+1)=0$ $x=2$ or $x=-1$ (rej) $\frac{dy}{dx}=2\left(\frac{1}{2}\right)(2+x)^{-\frac{1}{2}}\times(1)$ $=(2+x)^{-\frac{1}{2}}$ Sub $x=2$, $\frac{dy}{dx}=[2+2]^{-\frac{1}{2}}$ $=\frac{1}{2}$ $m_{\perp}=-2$ Let $R=(a, 0)$ $-2=\frac{4-0}{2-a}$ $2-a=\frac{4}{-2}$ $a=4$ $\underline{\therefore R=(4,0)}$
14(b)	y-coord of $P = 4$ Distance $OP = \sqrt{4^2+2^2}=\sqrt{20}$ Distance $PR = \sqrt{(2-4)^2+2^2}=\sqrt{20}$ **Since $OP = PR = \sqrt{20}$, therefore OPR is an isosceles triangle.**