ST JOSEPH'S INSTITUTION
PRELIMINARY EXAMINATION 2025
(YEAR 4)

CANDIDATE NAME			
CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS **4049/01**

Paper 1 **25 August 2025**

Candidates answer on the Question Paper. **2 hours 15 minutes (10:50 – 13:05)**

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen in the space provided.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

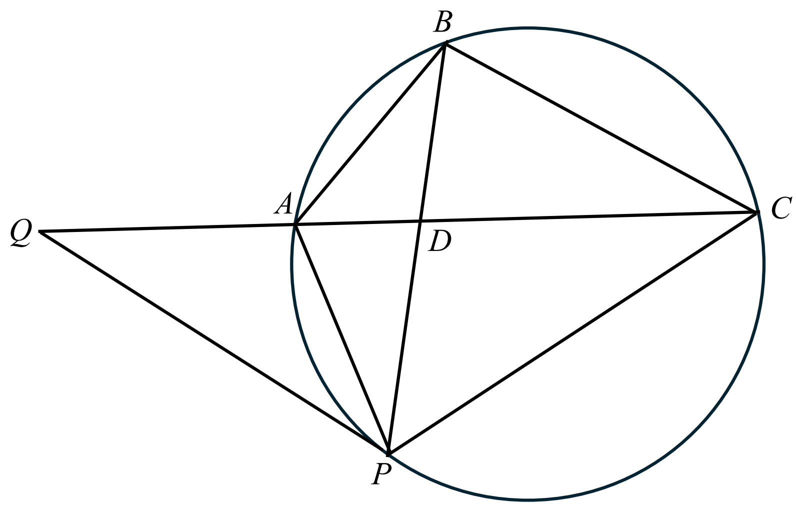
1 Without using a calculator, find the values of the integers a and b for which the solution of the equation $x\sqrt{18} - \sqrt{72} = x\sqrt{3}$ is $\dfrac{a + 2\sqrt{b}}{5}$. [5]

2 Integrate $\left(x-\dfrac{5}{2x}\right)^2$ with respect to x. [3]

3 The curve $x^2 + 3xy = x + 6$ and the line $2y - x = 2$ intersect at two points. Find the x-coordinates of the points of intersection. [3]

4 Express $-9 + 6x - 2x^2$ in the form $a\left(x+b\right)^2 + c$ and hence explain why $-9 + 6x - 2x^2$ is negative for all real values of x. [4]

5



In the diagram, A, B, C and P lie on a circle. The line PQ is the tangent to the circle at P. The tangent meets CA produced at Q. The lines AC and PB intersect at D. AP bisects angle QPB.

(a) Show that triangle ABP is isosceles. [3]

(b) Prove that

(i) triangle ACP is similar to triangle APD, [2]

(ii) $AB^2 = AD \times AC$. [1]

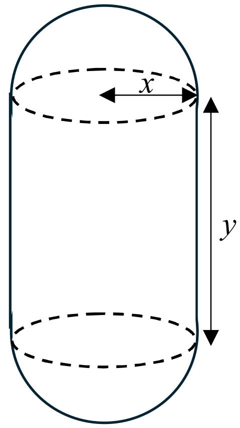
6 Express $\dfrac{2x^4 + x^3 + 9x^2 + 7x + 3}{(x+1)(x^2+5)}$ in partial fractions. [6]

7 It is given that $f(x) = 2x^3 + ax^2 + bx - 10$, where *a* and *b* are constants, has a factor of $(x-1)^2$.

(a) Find the value of *a* and of *b*. [4]

(b) It is given that $g(x) = f(x) + k$, where *k* is a constant. Using the values of *a* and *b* found in **(a)**, find the value of *k* such that $g(x)$ is divisible by $2x - 1$. [2]

8



The diagram shows a solid object made from joining two hemispheres of radius x cm to a cylinder of height y cm. The volume of the object is 750 cm^3.

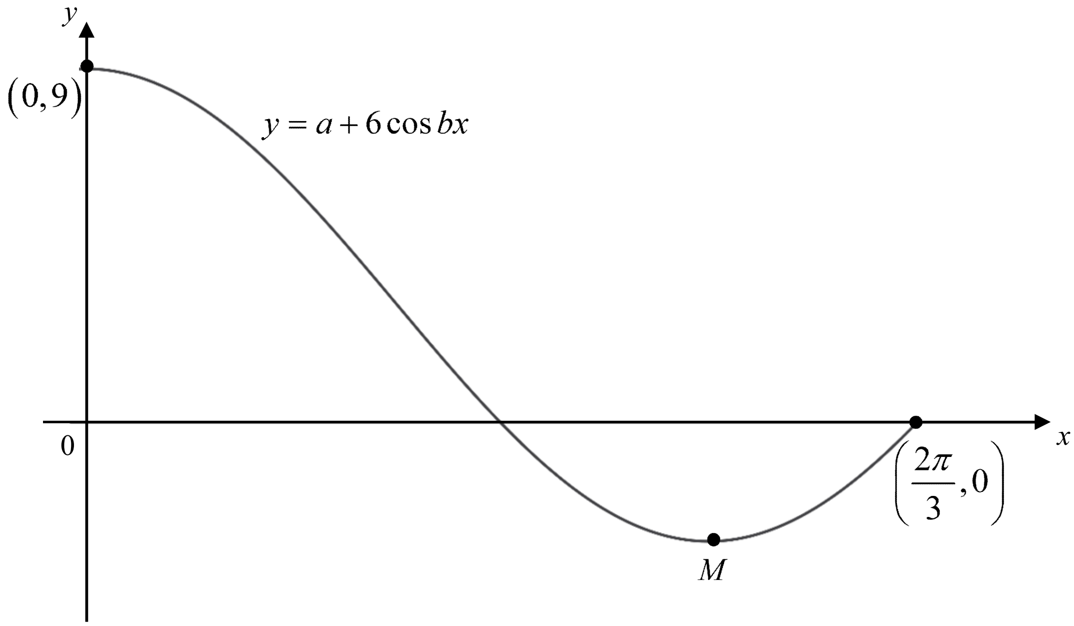
(a) Express y in terms of x. [2]

(b) Show that the surface area, S cm^2, of the object is given by $S = \frac{4}{3}\pi x^2 + \frac{1500}{x}$. [2]

(c) Given that x can vary, find the minimum value of S. [4]

(You are not required to show that S is a minimum.)

9 **(a)**



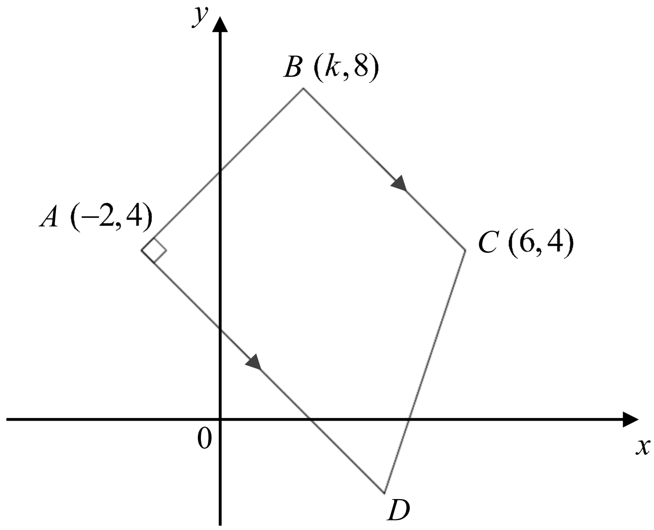
The diagram shows the curve $y = a + 6\cos bx$ for $0 \le x \le \frac{2\pi}{3}$, where a and b are positive constants. The curve meets the x-axis at the point $\left(\frac{2\pi}{3}, 0\right)$. The curve has a maximum point at $(0, 9)$ and a minimum point at M.

Find the value of a and of b. [3]

(b) **(i)** Sketch, on the axes below, the graphs of $y = -3\sin x$ and $y = 1 - \dfrac{2x}{\pi}$ for $0 \le x \le 2\pi$. [3]

(ii) Find the number of solution(s) of the equation $2x = \pi + 3\pi \sin x$ for $0 \le x \le 2\pi$. [2]

10



The diagram shows a trapezium with vertices $A(-2, 4)$, $B(k, 8)$, $C(6, 4)$ and *D*.
The sides *AD* and *BC* are parallel and angle $BAD = 90°$.

(a) Show that $k = 2$. [3]

(b) *BDEF* is a quadrilateral. Point *A* is the midpoint of both *BE* and *DF*. Given that the area of *BDEF* is 96 square units, find the coordinates of *D*. [5]

11 **(a)** Prove the identity $\cot\theta - 1 = \dfrac{\cos 2\theta}{\sin\theta\left(\cos\theta + \sin\theta\right)}$. [3]

(b) Hence solve the equation $\dfrac{\cos 4\theta}{\sin 2\theta\left(\cos 2\theta+\sin 2\theta\right)}=-\dfrac{1}{2}$ for $-180° \le \theta \le 180°$.

[5]

12 **(a)** A certain radioactive substance is known to decay with time such that its mass, M g, after t hours is given by $M = 100\left(4^{-kt}\right)$, where k is a positive constant. The original mass of a block of substance is 100 g. After 40 hours, its mass decreased to 90 g.

(i) Find the value of k. [3]

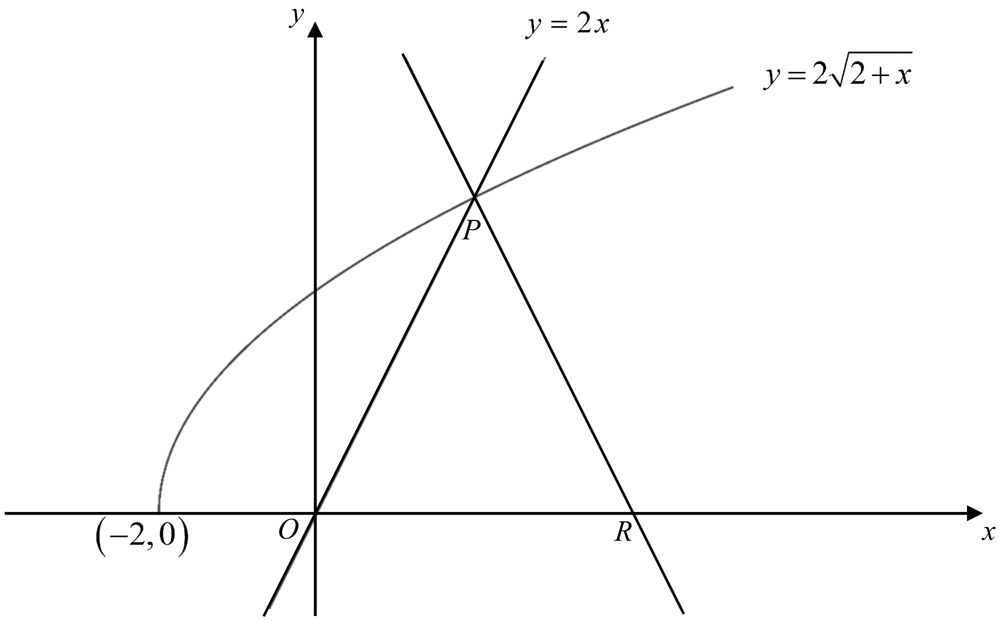
(ii) Calculate the percentage of the original mass of the block remaining when $t = 263$. [2]

(b) Express the equation $\log_5 x + \log_{25}(x-2) = 1$ as a cubic equation in x. [3]

13 **(a)** In the triangle ABC, $AB = x$ cm, $AC = 3x$ cm and angle $BAC = \dfrac{2\pi}{3}$. If x is increasing at the rate of 0.5 cm/s, find the exact rate at which the area is increasing when $x = 4$. [4]

(b) The variables F and r are such that F is inversely proportional to the square root of r. Given that when $F = 0.5$ and $r = 9$, find the rate at which r is changing with respect to F when $F = 0.5$. [4]

14



The diagram shows part of the graph of $y = 2\sqrt{2+x}$. It cuts the x-axis at $(-2, 0)$. The line $y = 2x$ meets the curve at P. The normal to the curve at P meets the x-axis at R.

(a) Find the coordinates of R. [6]

(b) Show that *OPR* is an isosceles triangle. [3]

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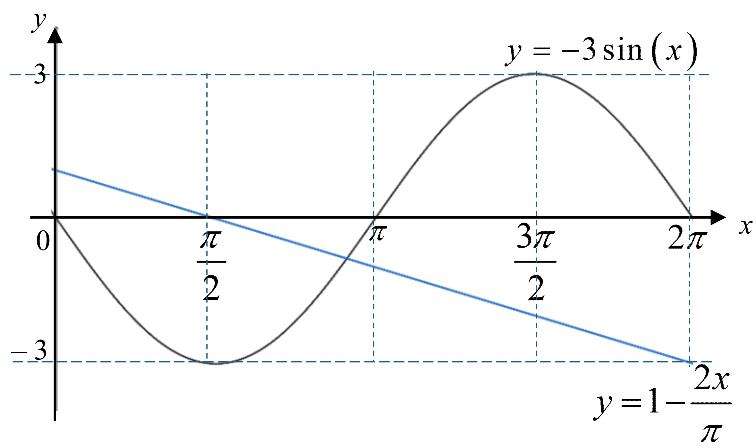
(YEAR 4) ADDITIONAL MATHEMATICS 4049/01
Answer Key

Qn	Answer	
1	$\underline{a=12}, \quad \underline{b=6}$	
2	$\underline{\frac{x^3}{3}-5x-\frac{25}{4x}+c}$	
3	$\underline{x=\frac{6}{5} \text{ or } x=-2}$	
4	$\underline{-2\left(x-\frac{3}{2}\right)^2-\left(\frac{9}{2}\right)}$ $\underline{-2\left(x-\frac{3}{2}\right)^2-\left(\frac{9}{2}\right)\leq-\left(\frac{9}{2}\right)}$ for all real values of x. Therefore $\underline{-9+6x-2x^2}$ **<u>is negative</u>** for all real values of x.	
5(a)	**<u>angle *APB* = angle *ABP*</u>** **<u>Triangle ABP is isosceles</u>**	
5(b)(i)	Triangle *ACP* and Triangle *APD* (AA Similarity Test)	
5(b)(ii)	$AP^2=AC\times AD \quad (\because AP=AB)$ $AB^2=AC\times AD$	
6	$\underline{2x-1+\frac{1}{(x+1)}+\frac{3-x}{(x^2+5)}}$	
7(a)	$c=-10$ $a=\underline{\mathbf{-14}}$ $b=\underline{\mathbf{22}}$	
7(b)	$\underline{k=\frac{9}{4}}$	

8(a)	$y=\dfrac{750}{\pi x^2}-\dfrac{4x}{3}$	
8(b)	$S=4\pi x^2+2\pi x\left(\dfrac{750}{\pi x^2}-\dfrac{4x}{3}\right)$	
8(c)	$x=\sqrt[3]{\dfrac{4500}{8\pi}}$ (=5.6362) $S=399$ **cm²**	
9(a)	$a=3$ $b=2$	
9(b)(i)		
9(b)(ii)	No. of solution(s) = **1**	
10(a)	$k=2$	
10(b)	$D\ (4,-2)$	
11(a)		
11(b)	$\theta=-148.3°,-58.3°,31.7°,121.7°$	
12(a)(i)	$k=0.00190$ (3sf)	
12(a)(ii)	$M=50.02$ Therefore, % remaining is **50.0%**.	
12(b)	$x^3-2x^2-25=0$	
13(a)	$3\sqrt{3}$ cm²/s	
13(b)	$\dfrac{dr}{dF}$ is **–36** units per unit increase in F	

14(a)	$\therefore R = (4,0)$	
14(b)	***OP* = *PR* =** $\sqrt{20}$ **, *OPR* is an isosceles triangle.**	