ST JOSEPH'S INSTITUTION
PRELIMINARY EXAMINATION 2025
(YEAR 4) ADDITIONAL MATHEMATICS 4049/02

<table>
<tr><th>Qn</th><th>Solutions</th></tr>
<tr><td>1(a)</td><td>$2^{2x+1}=5(2^x)+3$
Let $u=2^x$
$\underline{2u^2-5u-3=0}$</td></tr>
<tr><td>1(b)</td><td>$(2u+1)(u-3)=0$
$u=-\frac{1}{2}$ or $u=3$

$2^x>0$ therefore $2^x\neq-\frac{1}{2}$
$2^x=3$
$\lg 2^x=\lg 3$
$x=\frac{\lg 3}{\lg 2}$
$=\underline{\mathbf{1.6}}$ (2 sf)</td></tr>
<tr><td>2(a)</td><td>$(2x+1)^3-(2x-1)^3=\left[(2x+1)-(2x-1)\right]\left[(2x+1)^2+(2x+1)(2x-1)+(2x-1)^2\right]$
$=(2)\left[4x^2+4x+1+4x^2-1+4x^2-4x+1\right]$
$=2\left(12x^2+1\right)$</td></tr>
<tr><td>2(b)</td><td>$2(2x+1)\left(12x^2+1\right)=0$

$12x^2+1=0$ or $2x+1=0$
Discriminant $=0^2-4(12)(1)$
$=-48<0$
No real roots

$\underline{x=-\frac{1}{2}}$ is the only real root.</td></tr>
</table>

3 (a)	$y=(x-3)\sqrt{x+1}$ $\frac{dy}{dx}=(1)\sqrt{x+1}+(x-3)\left(\frac{1}{2}\right)(x+1)^{-\frac{1}{2}}(1)$ $=\sqrt{x+1}+\frac{x-3}{2\left(\sqrt{x+1}\right)}$ $=\frac{2(x+1)+x-3}{2\sqrt{x+1}}$ $=\frac{2x+2+x-3}{2\sqrt{x+1}}$ $\underline{=\frac{3x-1}{2\sqrt{x+1}}}$
3(b)	$\int_0^3 \frac{3x-1}{2\sqrt{x+1}}\ dx=\left[(x-3)\sqrt{x+1}\right]_0^3$ $6\int_0^3\left(\frac{3x-1}{2\sqrt{x+1}}+\frac{1}{2\sqrt{x+1}}\right)\mathrm{d}x=6\left\{\left[(x-3)\sqrt{x+1}\right]_0^3+\int_0^3\frac{1}{2\sqrt{x+1}}\mathrm{d}x\right\}$ $\int_0^3\left(\frac{9x}{\sqrt{x+1}}\right)\mathrm{d}x=6\left\{\left[(x-3)\sqrt{x+1}\right]_0^3+\int_0^3\frac{1}{2\sqrt{x+1}}\mathrm{d}x\right\}$ $=6\left\{\left[(3-3)\sqrt{3+1}-(0-3)\sqrt{0+1}\right]+\left[\frac{\frac{1}{2}(x+1)^{1/2}}{\frac{1}{2}}\right]_0^3\right\}$ $=6\left\{(3)+\left[(3+1)^{1/2}-(0+1)^{1/2}\right]\right\}$ $=6\left[(3)+(2-1)\right]$ $=\underline{\mathbf{24}}$

4(a)	*Let E be a point below C (and to the left of D)* In ΔCDE, $\cos\theta = \dfrac{CE}{80}$ $CE = 80\cos\theta$ *Let G be a point below D (and to the right of A)* In ΔADG, $\sin\theta = \dfrac{DG}{185}$ $DG = 185\sin\theta$ $h = CE + EF$ $h = 185\sin\theta + 80\cos\theta$ (shown)
4(b)	$h = 185\sin\theta + 80\cos\theta$ $185\sin\theta + 80\cos\theta = R\sin(\theta+\alpha)$ $R\sin\alpha = 80$ $R\cos\alpha = 185$ $R = \sqrt{80^2 + 185^2}$ $= \sqrt{40625}$ $= 25\sqrt{65}$ $\alpha = \tan^{-1}\left(\dfrac{80}{185}\right)$ $= 23.385°$ $\therefore h = 25\sqrt{65}\sin(\theta + 23.4°)$
4(c)	To clear the kitchen door, max h is 180cm, When h = 180cm $25\sqrt{65}\sin(\theta + 23.385°) = 180$ $\sin(\theta + 23.385°) = \dfrac{180}{25\sqrt{65}}$ $\sin(\theta + 23.385°) = \dfrac{36}{5\sqrt{65}}$ Basic angle =63.259° $\theta + 23.385° = 63.259°$ $\theta = 39.9°$

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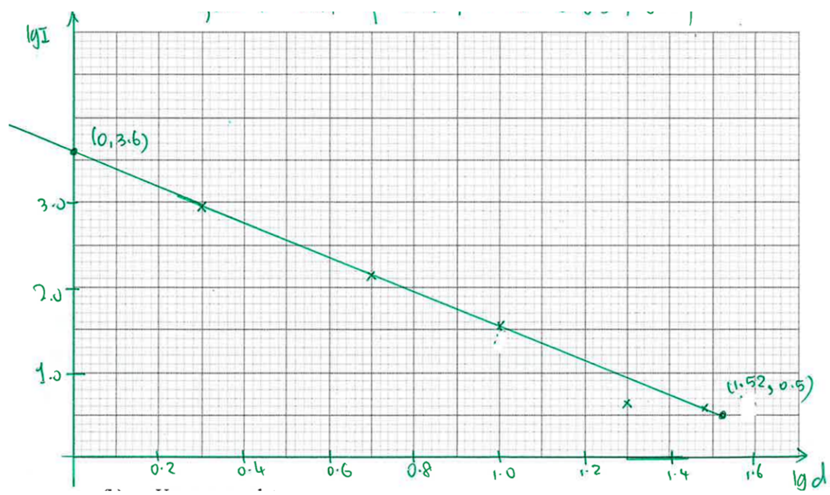
5(a)	$y=2x^2+5$ $2x^2+5>3x^2$ $x^2-5<0$ $(x+\sqrt{5})(x-\sqrt{5})<0$ $\underline{-\sqrt{5}<x<\sqrt{5}}$ 
5(b)	$y=2x^2+5$ ---- (1) $y-kx=3$ $y=kx+3$ ---- (2) $2x^2+5=kx+3$ $2x^2-kx+2=0$ $b^2-4ac=0$ $(-k)^2-4(2)(2)=0$ $k^2=16$ $\underline{k=\pm 4}$
5(c)	$2x^2-4x+2=0$ $x^2-2x+1=0$ $(x-1)^2=0$ $x=1$ $y=4(1)+3$ $\quad =7$ $\underline{P(1,7)}$
6(a)	$\left(ax+\frac{2}{x}\right)^6$ Term $\binom{6}{r}(ax)^{6-r}\left(\frac{2}{x}\right)^r$ $\frac{x^{6-r}}{x^r}=x^{6-2r}$ Powers of x $6-2r=2(3-r)$ Since the power/exponent can be expressed **<u>as a multiple of 2</u>**, therefore all the expansion only contains even powers of x.

6(b)	$\left(1-\frac{x}{a}\right)^3 = \underline{1-\frac{3x}{a}+\frac{3x^2}{a^2}+\ldots}$
6(c)	$\left(ax+\frac{2}{x}\right)^6$ Term in $\frac{1}{x^4}$ $x^{6-2r} = x^{-4}$ $6-2r=-4$ $r=5$ $\frac{1}{x^4}$ term $=\binom{6}{5}(ax)\left(\frac{2}{x}\right)^5 = 192a\left(\frac{1}{x^4}\right)$ $\left(1-\frac{x}{a}\right)^3$ Term in $x^2 = \frac{3}{a^2}(x^2)$ from (b) $192a = \frac{3}{a^2}\times 1728$ $a^3 = \frac{3\times 1728}{192}$ $a^3 = 27$ $\underline{\underline{a=3}}$
6(d)	Term independent of x $\left(1-\frac{3x}{a}+\frac{3x^2}{a^2}+\ldots\right)\left(1+\frac{1}{x}\right)$ $=(1)(1)+\left(-\frac{3x}{3}\right)\left(\frac{1}{x}\right)+\ldots$ There is no term independent of x.

7(a)

d	2	5	10	20	30
I	875	141	35	4.22	3.89
$\lg d$	0.30	0.70	1.00	1.30	1.48
$\lg I$	2.94	2.15	1.54	0.63	0.59



7(b) The incorrect reading of I is 4.22.

From the graph, the correct $\lg I$ reading is 0.95.

$0.95 = \lg I$

$I = 10^{0.95}$

$\underline{I = 8.91 \ \ (3 \text{ sf})}$

7(c) $I = Ad^n$

$\lg I = n \lg d + \lg A$

From the graph

$\lg A = 3.6$

$A = 10^{3.6}$

≈ 3981.071

	$\underline{=3980\ (3\text{sf})}$ $\text{Gradient} = \dfrac{0.5-3.6}{1.52-0}$ ≈ -2.0394 $= -2.04\ (3\text{sf})$ $\underline{n = -2.04}$
7(c)	From the table, $\dfrac{I_1}{I_2} = \dfrac{35}{141}$ = 0.248 (to 3 sf) When the distance doubled, the intensity decrease to 0.248 (one-quarter) of its previous value
8(a)	$v = 5 - e^{3-2t}$ $5 - e^{3-2t} = 0$ $e^{3-2t} = 5$ $\ln e^{3-2t} = \ln 5$ $3 - 2t = \ln 5$ $\underline{t = 0.695}$

8(b)	Distance in the first second $= \left\|\int_{0.69528}^{1}\left(5-e^{3-2t}\right)dt\right\| + \left\|\int_{0}^{0.69528}\left(5-e^{3-2t}\right)dt\right\|$ $= \left\|\left[5t-\left(\frac{e^{3-2t}}{-2}\right)\right]_{0.69528}^{1}\right\| + \left\|\left[5t-\left(\frac{e^{3-2t}}{-2}\right)\right]_{0}^{0.69528}\right\|$ $= \left\|\left[5(1)+\left(\frac{e^{3-2(1)}}{2}\right)\right]-\left[5(0.69528)+\left(\frac{e^{3-2(0.69528)}}{2}\right)\right]\right\|$ $+\left\|\left[5(0.69528)+\left(\frac{e^{3-2(0.69528)}}{2}\right)\right]-\left[5(0)+\left(\frac{e^{3-2(0)}}{2}\right)\right]\right\|$ $= 0.3827356 + \|-4.06636\|$ $= 4.449$ $=$ **<u>4.45 m</u>**
8(c)	$v = 5 - e^{3-2t}$ as t approaches ∞, e^{3-2t} approaches 0 $v =$ **<u>5 m/s</u>** **The particle's velocity <u>cannot exceed 5 m/s</u>,** and this is the greatest velocity attained.
9(a)	$m_t = \frac{3-0}{-3-0}$ $= -1$ Gradient of normal = 1 Equation of normal is $y-3=1(x+3)$ $\underline{y=x+6}$
9(b)	Equation of normal is $y=-\frac{2}{5}x-1$ --- (1) $y=x+6$ --- (2)

	Sub (1) into (2) $-\frac{2}{5}x-1=x+6$ $\frac{7}{5}x=-7$ $x=-5$ Centre = $(-5,1)$ $r^2=(-3+5)^2+(3-1)^2$ $=8$ Therefore equation of circle is $\underline{(x+5)^2+(y-1)^2=8}$
9(c)	$\underline{\left(-5-2\sqrt{2},1\right)}$
10	$y=\sin 2x+x$ $\frac{dy}{dx}=2\cos 2x+1$ $2\cos 2x+1=0$ $\cos 2x=-\frac{1}{2}$ Basic angle = $\frac{\pi}{3}$ $2x=\pi-\frac{\pi}{3},\pi+\frac{\pi}{3}$ Min point $2x=\frac{4\pi}{3}$ $x=\frac{2\pi}{3}$ $y=\sin\left(\frac{4\pi}{3}\right)+\frac{2\pi}{3}$ $=-\frac{\sqrt{3}}{2}+\frac{2\pi}{3}$

Area of region below shaded area

$$=\frac{1}{2}\left(-\frac{\sqrt{3}}{2}+\frac{2\pi}{3}\right)\left(\frac{2\pi}{3}\right)$$

$$=\frac{2\pi^2}{9}-\frac{\sqrt{3}\pi}{6}$$

Area under graph

$$=\int_0^{\frac{2\pi}{3}}\left(\sin(2x)+x\right)\,\mathrm{d}x$$

$$=\left[-\frac{1}{2}\cos 2x+\frac{x^2}{2}\right]_0^{\frac{2\pi}{3}}$$

$$=\left[-\frac{1}{2}\cos 2\left(\frac{2\pi}{3}\right)+\frac{\left(\frac{2\pi}{3}\right)^2}{2}\right]-[-\frac{1}{2}\cos 2(0)]$$

$$=\left[\frac{1}{4}+\frac{2\pi^2}{9}\right]+\frac{1}{2}$$

$$=\frac{2\pi^2}{9}+\frac{3}{4}$$

Area of shaded region $=\frac{2\pi^2}{9}+\frac{3}{4}-\left(\frac{2\pi^2}{9}-\frac{\sqrt{3}\pi}{6}\right)$

$$=\frac{\sqrt{3}\pi}{6}+\frac{3}{4}\quad\text{(shown)}$$