



**ST JOSEPH'S INSTITUTION
PRELIMINARY EXAMINATION 2025
(YEAR 4)**

CANDIDATE
NAME

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CLASS

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INDEX
NUMBER

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ADDITIONAL MATHEMATICS

4049/02

Paper 2

27 August 2025

Candidates answer on the Question Paper.

**2 hours 15 minutes
(09:00 – 11:15)**

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.

Write in dark blue or black pen in the space provided.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer all questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 90.

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

1 (a) Given that $u = 2^x$, express $2^{2x+1} = 5(2^x) + 3$ as an equation in u . [1]

(b) Hence find the value of x for which $2^{2x+1} = 5(2^x) + 3$, giving your answer correct to 2 significant figures. [4]

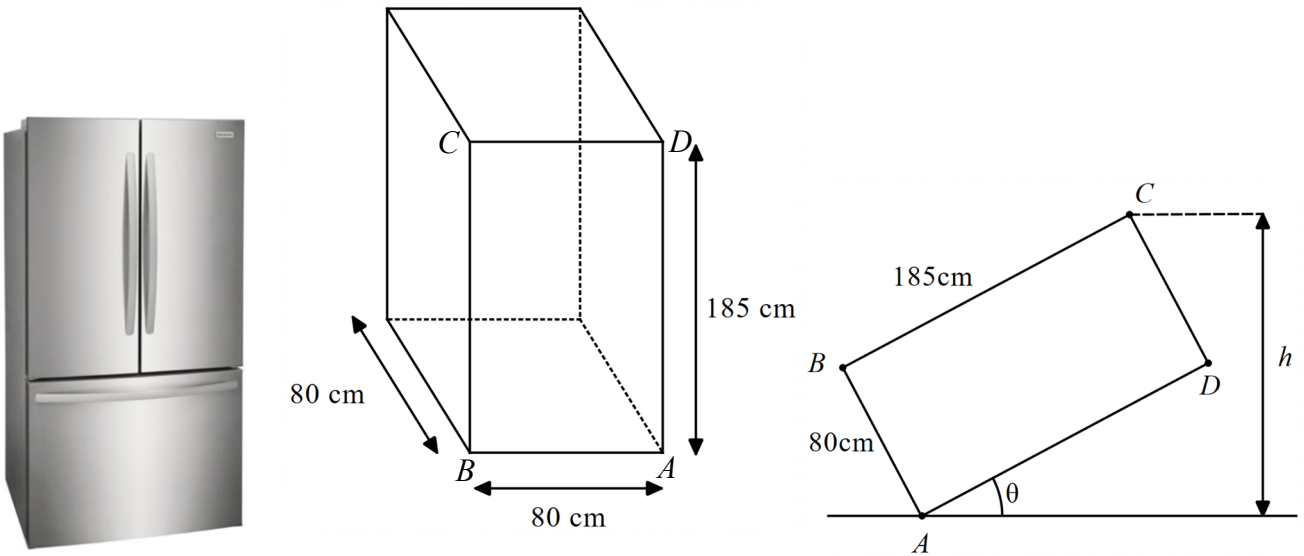
- 2 **(a)** Factorise $(2x+1)^3 - (2x-1)^3$ completely. [2]

- (b)** Hence show that the equation $(2x+1)\left[(2x+1)^3 - (2x-1)^3\right] = 0$ has only one real root. [2]

- 3 (a)** It is given that $y = (x-3)\sqrt{x+1}$. Find $\frac{dy}{dx}$ in the form $\frac{ax-b}{2\sqrt{x+1}}$, where a and b are integers. [4]

- (b)** Hence find the value of $\int_0^3 \frac{9x}{\sqrt{x+1}} dx$. [5]

- 4 A mover needs to deliver a refrigerator to a customer. The refrigerator can be modelled by a cuboid of dimensions 80 cm by 80 cm by 185 cm.



To move the refrigerator into the kitchen through a door, the mover has to tilt the refrigerator such that AD makes an acute angle θ with the horizontal ground.

- (a) Show that the height, h cm, of C above ground is given by $h = 185 \sin \theta + 80 \cos \theta$.
[2]

(b) Express h in the form $R \sin(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [3]

(c) Find the value of θ when $h = 180$. [3]

5 The equation of a curve is $y = 2x^2 + 5$.

- (a)** Find the set of values of x for which $y > 3x^2$ and represent this set on a number line. [4]

The line $y - kx = 3$ is a tangent to the curve at the point P .

(b) Find the possible values of k . [4]

(c) Given that $k > 0$, find the coordinates of P . [3]

- 6** **(a)** Explain why the binomial expansion of $\left(ax + \frac{2}{x}\right)^6$ contains only even powers of x . [2]

- (b)** Write down, and simplify, the first three terms in the expansion of $\left(1 - \frac{x}{a}\right)^3$ in ascending powers of x . [2]

- (c) Given that the coefficient of $\frac{1}{x^4}$ in the expansion of $\left(ax + \frac{2}{x}\right)^6$ is 1728 times the coefficient of x^2 in the expansion of $\left(1 - \frac{x}{a}\right)^3$, find the value of a . [4]

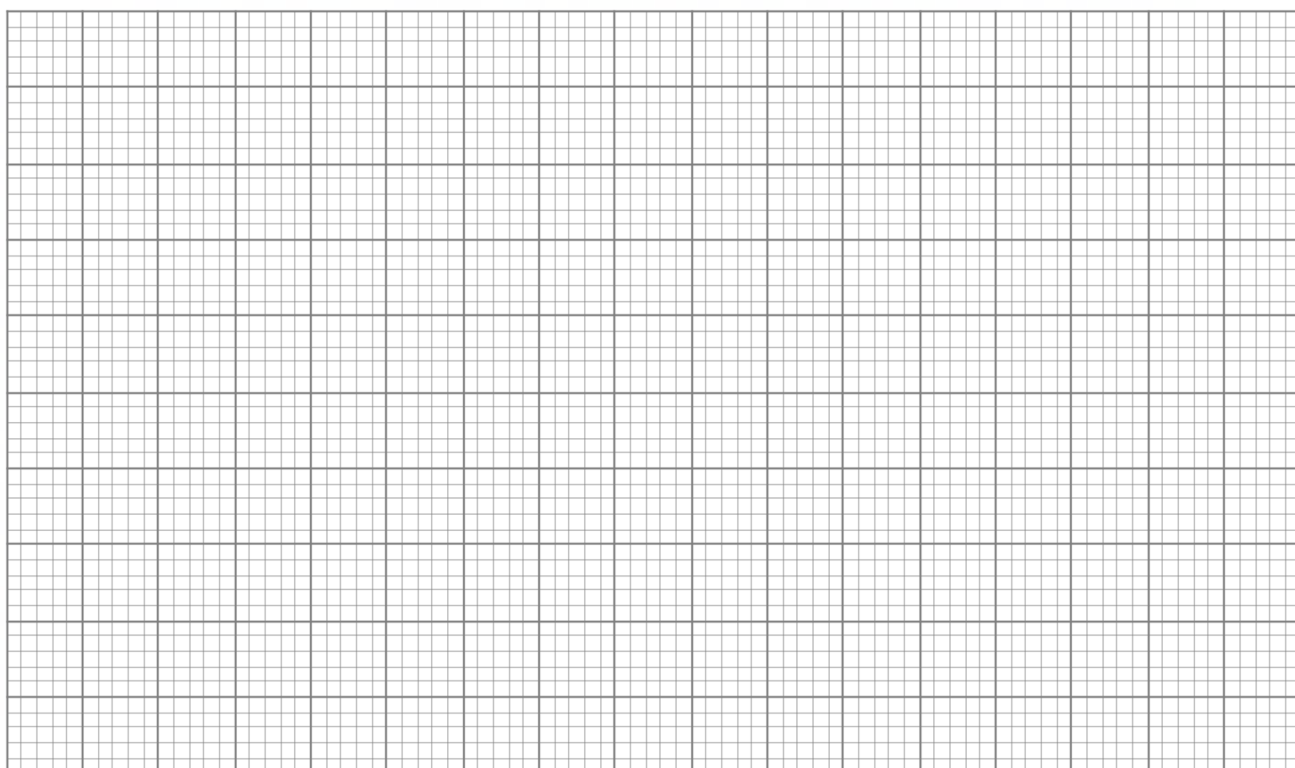
- (d) Using the value of a found in (c), show that there is no term independent of x for $\left(1 - \frac{x}{a}\right)^3 \left(1 + \frac{1}{x}\right)$. [2]

- 7 In an experiment to investigate the relationship between light intensity I lux on a surface and distance d metres from a light source, a student obtained the following results. One of the values of I is incorrect.

d (metres)	2	5	10	20	30
I (lux)	875	141	35	4.22	3.89

It is known that I and d are related by the equation $I = Ad^n$, where A and n are constants.

- (a) Plot $\lg I$ against $\lg d$ and draw a straight line graph. [3]



- (b) Use your graph to
- (i) identify the incorrect reading of I and estimate its correct value, [3]

- (ii) estimate the value of A and of n . [4]

- (c) Using the table of values, explain, with working, how the light intensity I changes when the distance d from the light source is doubled. [1]

- 8** A particle starts from a point O and moves in a straight line so that its velocity, v m/s, is given by $v = 5 - e^{3-2t}$ where t is the time in seconds after leaving O .
- (a)** Find the value of t at which the particle is instantaneously at rest. [3]

- (b) Calculate the distance travelled by the particle in the first second. [4]

- (c) Explain why the velocity of the particle cannot exceed a certain value. [2]

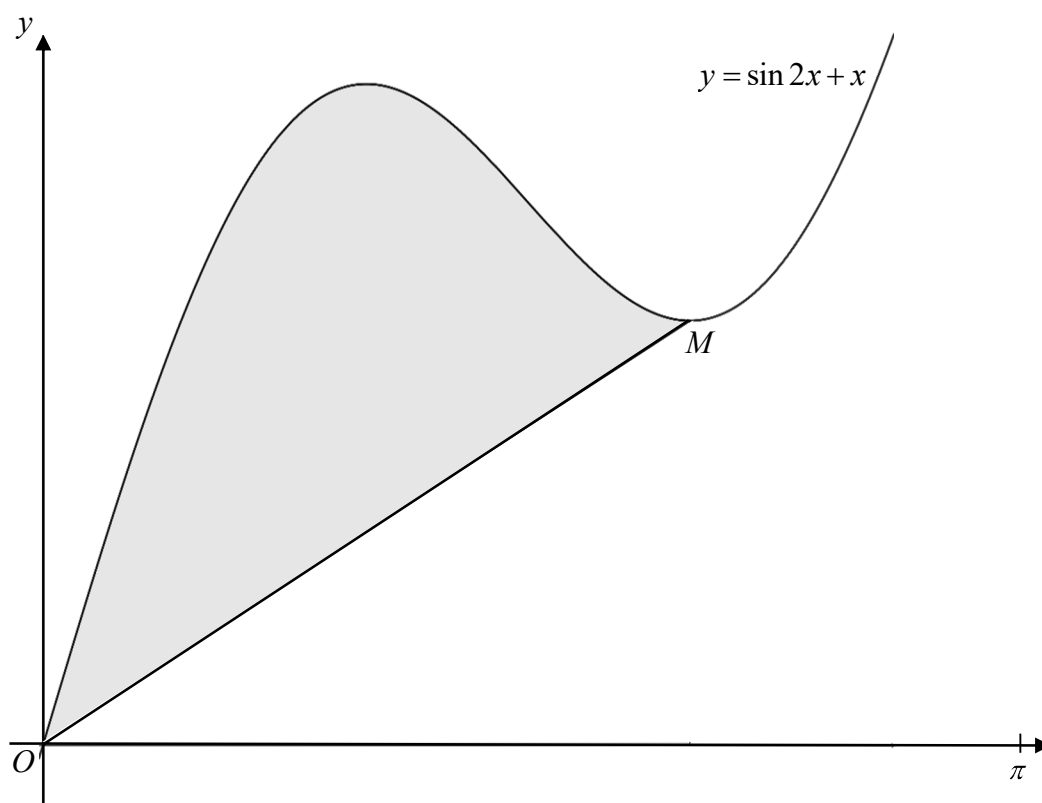
9 A tangent to a circle at the point $(-3,3)$ passes through the origin.

(a) Find the equation of the normal to the circle at the point $(-3,3)$. [3]

- (b) Another normal to the circle passes through the point $(0, -1)$ and is parallel to the line $5y = -2x + 7$. Find the equation of the circle. [6]

- (c) Find the coordinates of the point on the circle which is farthest from the y -axis. Leave your answer in the form $(m + n\sqrt{2}, k)$, where m , n and k are constants. [2]

10



The diagram shows the curve $y = \sin 2x + x$ for $0 \leq x \leq \pi$ radians. The point M is the minimum point of the curve and OM is a straight line.

Show that the area of the shaded region is $\left(\frac{\sqrt{3}\pi}{6} + \frac{3}{4}\right)$ square units. [12]

Continuation of working space for question 10.

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**ST JOSEPH'S INSTITUTION
PRELIMINARY EXAMINATION 2025
(YEAR 4) ADDITIONAL MATHEMATICS 4049/02
Answer Key**

[Turn over

Qn	Solutions	
1(a)	<u>$2u^2 - 5u - 3 = 0$</u>	
1(b)	$x = 1.6$	
2(a)	$2(12x^2 + 1)$	
2(b)	<u>$x = -\frac{1}{2}$</u> is the only real root.	
3 (a)	<u>$\frac{3x-1}{2\sqrt{x+1}}$</u>	
3(b)	<u>24</u>	
4(a)	$h = 185\sin\theta + 80\cos\theta$ (shown)	
4(b)	<u>$h = 25\sqrt{65}\sin(\theta + 23.4^\circ)$</u>	
4(c)	<u>$\theta = 39.9^\circ$</u>	
5(a)	<u>$-\sqrt{5} < x < \sqrt{5}$</u> 	
5(b)	<u>$k = \pm 4$</u>	
5(c)	<u>$P(1, 7)$</u>	
6(a)	x^{6-2r} Powers of x $6 - 2r = 2(3 - r)$ Since the power/exponent can be expressed <u>as a multiple of 2</u> ,	
6(b)	<u>$1 - \frac{3x}{a} + \frac{3x^2}{a^2} + \dots$</u>	
6(c)	<u>$a = 3$</u>	
6(d)	Show is no term independent of x .	
7(a)		

7(b)	incorrect reading of I is 4.22. <u>$I = 8.91$ (3 sf)</u>	
7(c)	<u>$A = 3980$ (3sf)</u> <u>$n = -2.04$</u>	
7(c)	the intensity decrease to 0.248 (one-quarter) of its previous value	
8(a)	<u>$t = 0.695$</u>	
8(b)	Distance in the first second = <u>4.45 m</u>	
8(c)	The particle's velocity <u>cannot exceed 5 m/s.</u>	
9(a)	<u>$y = x + 6$</u>	
9(b)	<u>$(x+5)^2 + (y-1)^2 = 8$</u>	
9(c)	<u>$(-5 - 2\sqrt{2}, 1)$</u>	
10	$\frac{dy}{dx} = 2 \cos 2x + 1$	

	$x = \frac{2\pi}{3}$ $y = -\frac{\sqrt{3}}{2} + \frac{2\pi}{3}$ $\text{Area of shaded region} = \frac{2\pi^2}{9} + \frac{3}{4} - \left(\frac{2\pi^2}{9} - \frac{\sqrt{3}\pi}{6} \right)$ $= \frac{\sqrt{3}\pi}{6} + \frac{3}{4} \quad (\text{shown})$	
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