



**SECONDARY 4
PRELIMINARY EXAMINATION**

**ADDITIONAL MATHEMATICS
Paper 1**

4049/01

29 August 2025 (Friday)

2 hours 15 minutes

CANDIDATE
NAME

MARKING SCHEME

CLASS

4	-		
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INDEX NUMBER

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Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your full name, class and index number in the spaces above.

Write in dark blue or black pen in the space provided for each question.

You may use a HB pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

The number of marks is given in brackets [] at the end of each question or part question.

If working is needed for any question, it must be shown in the space below the question.

Omission of essential working will result in loss of marks.

The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142.

For Examiner's Use		
Q1	3	
Q2	4	
Q3	5	
Q4	5	
Q5	6	
Q6	7	
Q7	8	
Q8	8	
Q9	8	
Q10	8	
Q11	8	
Q12	10	
Q13	10	
Total	90	

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$$

where n is a positive integer and

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

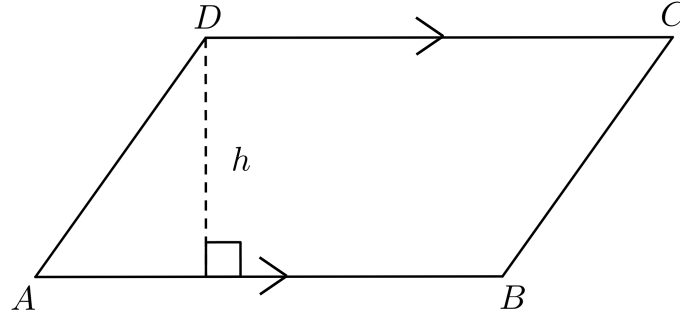
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer all the questions.

- 1 A parallelogram of length AB $(\sqrt{6} - 1)$ cm has an area of $(2\sqrt{6} + 3)$ cm².



Without using a calculator, find the perpendicular height, h , in the form

$(a + b\sqrt{6})$ cm, where a and b are integers.

[3]

Perpendicular height h

$$\begin{aligned}
 &= \frac{2\sqrt{6} + 3}{\sqrt{6} - 1} \\
 &= \frac{2\sqrt{6} + 3}{\sqrt{6} - 1} \times \frac{\sqrt{6} + 1}{\sqrt{6} + 1} \\
 &= \frac{12 + 3 + 5\sqrt{6}}{5} \\
 &= (3 + \sqrt{6}) \text{ cm}
 \end{aligned}$$

2 The function f is given by $f(x) = \sin^2 2x \tan 2x$.

(i) Find $f'(x)$. [2]

$$\begin{aligned} f'(x) &= (2 \sin 2x)(2 \cos 2x) \tan 2x + \sin^2 2x (2 \sec^2 2x) \\ &= 4(\sin 2x \cos 2x) \tan 2x + 2 \sin^2 2x (\sec^2 2x) \\ &= 4 \sin^2 2x + 2 \tan^2 2x \end{aligned}$$

(ii) Explain why f is an increasing function for $0 < x < \frac{\pi}{4}$. [2]

Given $0 < x < \frac{\pi}{4}$
 $\sin^2 2x > 0$ and $\tan^2 2x > 0$,
 hence $4 \sin^2 2x + 2 \tan^2 2x > 0 \Rightarrow f'(x) > 0$.

Since $f'(x) > 0$, f is an increasing function.

- 3 The equation of a curve is $y = (4 + p)x^2 - 2x + p$, where p is a constant.

The curve has a maximum point and intersects the line $y = 2x - 1$.

Find the range of values of p .

[5]

$$\begin{aligned}(4 + p)x^2 - 2x + p &= 2x - 1 \\ (4 + p)x^2 - 4x + p + 1 &= 0\end{aligned}$$

$$a = 4 + p, b = -4, c = p + 1$$

Since the curve intersects the line, $b^2 - 4ac \geq 0$

$$\begin{aligned}16 - 4(4 + p)(p + 1) &\geq 0 \\ 4 - (p^2 + 5p + 4) &\geq 0 \\ -p^2 - 5p &\geq 0 \\ p^2 + 5p &\leq 0 \\ p(p + 5) &\leq 0 \\ -5 &\leq p \leq 0\end{aligned}$$

Since curve has a maximum point, $p + 4 < 0$

$$p < -4$$

Hence

$$-5 \leq p < -4$$

- 4 Show that the curve $y = -x^2 + (m + 4)x - (m^2 + 6)$ is always negative for all real values of m . [5]

$$\begin{aligned}
 \text{Discriminant} &= (m + 4)^2 - 4(-1)(-m^2 - 6) \\
 &= m^2 + 8m + 16 - 4m^2 - 24 \\
 &= -3m^2 + 8m - 8 \\
 &= -3 \left(m^2 - \frac{8}{3}m \right) - 8 \\
 &= -3 \left[\left(m - \frac{4}{3} \right)^2 - \frac{16}{9} \right] - 8 \\
 &= -3 \left(m - \frac{4}{3} \right)^2 + \frac{16}{3} - \frac{24}{3} \\
 &= -3 \left(m - \frac{4}{3} \right)^2 - \frac{8}{3}
 \end{aligned}$$

Since $\left(m - \frac{4}{3} \right)^2 \geq 0$ for all real values of m ,

$$\begin{aligned}
 -3 \left(m - \frac{4}{3} \right)^2 &\leq 0 \\
 -3 \left(m - \frac{4}{3} \right)^2 - \frac{8}{3} &< 0
 \end{aligned}$$

$$\therefore b^2 - 4ac < 0$$

Since $b^2 - 4ac < 0$ and coefficient of x^2 is negative,
 $y = -x^2 + (m + 4)x - (m^2 + 6)$ is always negative for all real values of m . (shown)

ALTERNATIVE

$$\begin{aligned}\frac{dy}{dx} &= -2x + m + 4 \\ &= 0 \\ 2x &= m + 4 \\ x &= \frac{m}{2} + 2\end{aligned}$$

At $x = \frac{m}{2} + 2$

$$\begin{aligned}y &= -\left(\frac{m+4}{2}\right)^2 + (m+4)\left(\frac{m+4}{2}\right) - m^2 - 6 \\ &= \frac{1}{4}(m^2 + 8m + 16) - m^2 - 6 \\ &= -\frac{3}{4}m^2 + 2m - 2 \\ &= -\frac{3}{4}\left(m^2 - \frac{8}{3}m\right) - 2 \\ &= -\frac{3}{4}\left[\left(m - \frac{4}{3}\right)^2 + \frac{16}{9}\right] - 2 \\ &= -\frac{3}{4}\left(m - \frac{4}{3}\right)^2 - 3\frac{1}{3}\end{aligned}$$

Since $\left(m - \frac{4}{3}\right)^2 \geq 0$ for all real values of m ,

$$\begin{aligned}-\frac{3}{4}\left(m - \frac{4}{3}\right)^2 &\leq 0 \\ -\frac{3}{4}\left(m - \frac{4}{3}\right)^2 - 3\frac{1}{3} &< 0 \\ \therefore y - \text{coordinate} &< 0\end{aligned}$$

Since the y -coordinate of the turning point is negative and the graph has a maximum point, $y = -x^2 + (m+4)x - (m^2+6)$ is always negative for all real values of m . (shown)

	ALTERNATIVE
	$ \begin{aligned} y &= -x^2 + (m+4)x - (m^2 + 6) \\ &= -[x^2 - (m+4)x] - (m^2 + 6) \\ &= -\left[\left(x - \frac{m+4}{2}\right)^2 - \left(\frac{m+4}{2}\right)^2\right] - (m^2 + 6) \\ &= -\left(x - \frac{m+4}{2}\right)^2 + \frac{m^2}{4} + 2m + 4 - m^2 - 6 \\ &= -\left(x - \frac{m+4}{2}\right)^2 - \frac{3m^2}{4} + 2m - 2 \end{aligned} $ y-coordinate of maximum point is $-\frac{3m^2}{4} + 2m - 2$ Discriminant of this function $ \begin{aligned} &= 2^2 - 4\left(-\frac{3}{4}\right)(-2) \\ &= 4 - 6 \\ &= -2 < 0 \end{aligned} $ Since discriminant of function is less than zero, and coefficient of m^2 is negative, the y-coordinate of maximum point is always negative. Therefore, $-x^2 + (m+4)x - (m^2 + 6)$ is always negative for all real values of m. (shown)

- 5 The line $4y = 5x - 6$ cuts the curve $x + 4xy - 3y^2 = 7$ at the points P and Q .

Find the length of the line PQ in the form $a\sqrt{b}$, where a and b are integers.

[6]

$$5x = 4y + 6$$

$$x = \frac{4y}{5} + \frac{6}{5} \dots (1)$$

$$x + 4xy - 3y^2 = 7 \dots (2)$$

Substitute (1) into (2) :

$$\left(\frac{4y}{5} + \frac{6}{5}\right)(1 + 4y) - 3y^2 = 7$$

$$(4y + 6)(1 + 4y) - 15y^2 = 35$$

$$16y^2 + 28y + 6 - 15y^2 - 35 = 0$$

$$y^2 + 28y - 29 = 0$$

$$(y - 1)(y + 29) = 0$$

$$y = 1 \text{ or } -29$$

$$x = 2 \text{ or } -22$$

Point P is $(2,1)$ and Point Q is $(-22, -29)$.

Length of line PQ

$$= \sqrt{(2 - (-22))^2 + (1 + 29)^2}$$

$$= \sqrt{1476}$$

$$= 6\sqrt{41} \text{ units}$$

6 Express $\frac{x^3+x^2-10x-22}{(x-3)(x+1)^2}$ in partial fractions.

[7]

$$(x-3)(x+1)^2 = (x-3)(x^2+2x+1) \\ = x^3 - x^2 - 5x - 3$$

$$\begin{array}{r} x^3 - x^2 - 5x - 3 \quad \Bigg| \quad \begin{array}{r} x^3 + x^2 - 10x - 22 \\ (-) \quad x^3 - x^2 - 5x - 3 \\ \hline 2x^2 - 5x - 19 \end{array} \end{array}$$

$$\frac{x^3+x^2-10x-22}{(x-3)(x+1)^2} = 1 + \frac{2x^2-5x-19}{(x-3)(x+1)^2}$$

Let $\frac{2x^2-5x-19}{(x-3)(x+1)^2} = \frac{A}{x-3} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

$$= \frac{A(x+1)^2 + B(x-3)(x+1) + C(x-3)}{(x-3)(x+1)^2}$$

$$2x^2 - 5x - 19 = A(x+1)^2 + B(x-3)(x+1) + C(x-3)$$

Sub $x = -1$

$$2(-1)^2 - 5(-1) - 19 = C(-1-3) \\ -12 = -4C \\ C = 3$$

Sub $x = 3$

$$2(3)^2 - 5(3) - 19 = A(3+1)^2 \\ -16 = 16A \\ A = -1$$

Sub $x = 0$

$$-19 = A + B(-3)(1) + C(-3) \\ -19 = -1 - 3B + 3(-3) \\ 3B = 9 \\ B = 3$$

Hence

$$\frac{x^3+x^2-10x-22}{(x-3)(x+1)^2} = 1 - \frac{1}{x-3} + \frac{3}{x+1} + \frac{3}{(x+1)^2}$$

- 7 (i) Prove that $\frac{1}{\operatorname{cosec} x - \cot x} - \frac{1}{\sin x} = \cot x$. [4]

$$\begin{aligned}
 LHS &= \frac{1}{\operatorname{cosec} x - \cot x} - \frac{1}{\sin x} \\
 &= \frac{1}{\frac{1}{\sin x} - \frac{\cos x}{\sin x}} - \frac{1}{\sin x} \\
 &= \frac{\sin x}{1 - \cos x} - \frac{1}{\sin x} \\
 &= \frac{\sin x}{\sin x(1 + \cos x)} - \frac{1}{\sin x} \\
 &= \frac{1 - \cos^2 x}{\sin x(1 + \cos x)} - \frac{1}{\sin x} \\
 &= \frac{\sin^2 x}{\sin x(1 + \cos x)} - \frac{1}{\sin x} \\
 &= \frac{1 + \cos x}{\sin x} - \frac{1}{\sin x} \\
 &= \cot x \\
 &= RHS \text{ (shown)}
 \end{aligned}$$

- (ii) Hence solve the equation $\frac{1}{\operatorname{cosec} x - \cot x} - \frac{1}{\sin x} = \cot^2 x - 2$ for $0 < x < 2\pi$. [4]

$$\begin{aligned}
 \cot x &= \cot^2 x - 2 \\
 \cot^2 x - \cot x - 2 &= 0 \\
 (\cot x - 2)(\cot x + 1) &= 0 \\
 \cot x &= -1 \quad \text{or} \quad \cot x = 2 \\
 \tan x &= -1 \quad \text{or} \quad \tan x = 0.5 \\
 x &= \frac{3\pi}{4}, \frac{7\pi}{4} \quad \text{or} \quad \text{Basic angle} = 0.46364 \\
 x &= 0.464, 3.61 \text{ radians (3 s.f.)} \\
 x &= 0.464, 2.36, 3.61, 5.50 \text{ radians (3 s.f.)}
 \end{aligned}$$

- 8 (i) Find $\frac{d}{dx} (x^7 \ln 7x)$.

[3]

$$\begin{aligned}\frac{d}{dx} (x^7 \ln 7x) &= 7x^6 \ln 7x + \frac{7x^7}{7x} \\ &= 7x^6 \ln 7x + x^6\end{aligned}$$

- (ii) Hence find $\int x^6 \ln 7x \, dx$.

[4]

$$\begin{aligned}\int \frac{d}{dx} (x^7 \ln 7x) \, dx &= \int 7x^6 \ln 7x + x^6 \, dx \\ x^7 \ln 7x &= \int 7x^6 \ln 7x \, dx + \int x^6 \, dx \\ &= \int 7x^6 \ln 7x \, dx + \frac{x^7}{7} + c_1\end{aligned}$$

, where c_1 is an arbitrary constant

$$\int 7x^6 \ln 7x \, dx = x^7 \ln 7x - \frac{x^7}{7} + c_2$$

, where c_2 is an arbitrary constant

$$\int x^6 \ln 7x \, dx = \frac{x^7 \ln 7x}{7} - \frac{x^7}{49} + c_3$$

, where c_3 is an arbitrary constant

(Accept $\int x^6 \ln 7x \, dx = \frac{x^7 \ln 7x}{7} - \frac{x^7}{49} + c$ as students may only integrate at the final step.)

- (iii) Without additional working, write what the integral will be for $x^{k-1} \ln kx$, where k is an integer.

[1]

$$\int x^{k-1} \ln kx \, dx = \frac{x^k \ln kx}{k} - \frac{x^k}{k^2} + c_4,$$

where c_4 is an arbitrary constant

Similar to part (ii), accept

$$\int x^{k-1} \ln kx \, dx = \frac{x^k \ln kx}{k} - \frac{x^k}{k^2} + c,$$

where c is an arbitrary constant.

- 9 A curve has equation $y = 0.2x^5 + 3x^3 - 10x$.

- (i) Find the x -coordinates of the stationary points, A and B , on the curve. [4]

$$\frac{dy}{dx} = x^4 + 9x^2 - 10$$

For stationary points,

$$\frac{dy}{dx} = 0$$

$$x^4 + 9x^2 - 10 = 0$$

$$(x^2 + 10)(x^2 - 1) = 0$$

$$x = 1 \text{ or } x = -1 \text{ or } x^2 = -10$$

$$(\text{rej. as } x^2 \neq -10)$$

The x – coordinates are $x = 1$ and $x = -1$.

It is given that P is a point on the curve where the **gradient is a minimum**.

- (ii) Show that P and the midpoint of AB are the same point. [4]

$$x = 1 \text{ or } x = -1$$

$$y = -6.8 \text{ or } 6.8$$

$$\text{Midpoint of } AB = \left(\frac{1 - 1}{2}, \frac{-6.8 + 6.8}{2} \right)$$

$$= (0, 0)$$

$$\text{Let } m = x^4 + 9x^2 - 10$$

$$\frac{dm}{dx} = 4x^3 + 18x$$

$$\text{For max./min. gradient, } \frac{dm}{dx} = 0.$$

$$2x(2x^2 + 9) = 0$$

$$x = 0 \text{ or } 2x^2 + 9 = 0 \text{ (rej. as } 2x^2 \neq -9) \\ \Rightarrow y = 0$$

$$\frac{d^2m}{dx^2} = 12x^2 + 18$$

$$\text{Subt. } x = 0, \frac{d^2m}{dx^2} = 18 > 0$$

Therefore, $P = (0, 0)$ is a point on the curve where the **gradient is a minimum**.

Since the midpoint of $AB = (0, 0)$, P and the midpoint of AB are the same. (shown)

- 10 Points $P(-6, -12)$ and $Q(2, -20)$ lie on a circle C_1 with centre A .

The line $7y + 4x + 87 = 0$ passes through the centre of the circle.

- (i) Find the coordinates of A and the radius of the circle, C_1 . [5]

$$\begin{aligned}\text{Midpoint of } PQ &= \left(\frac{-6+2}{2}, \frac{-12+(-20)}{2} \right) \\ &= (-2, -16)\end{aligned}$$

$$\text{Gradient } PQ = \frac{-12+20}{-6-2} = -1$$

Equation of perpendicular bisector of PQ is

$$y + 16 = 1(x + 2)$$

$$y = x - 14 \quad \text{--- (1)}$$

$$7y + 4x + 87 = 0 \quad \text{--- (2)}$$

Substitute (1) into (2) :

$$7x - 98 + 4x + 87 = 0$$

$$x = 1 \Rightarrow y = -13$$

$$A = (1, -13)$$

$$\begin{aligned}\text{Radius} &= \sqrt{(1-2)^2 + (-13-(-20))^2} \\ &= \sqrt{1^2 + 7^2} \\ &= \sqrt{50} \\ &= 5\sqrt{2} \text{ units}\end{aligned}$$

- (ii) Hence find the equation of the circle C_1 . [1]

$$(x - 1)^2 + (y + 13)^2 = 50$$

- (iii) Another circle C_2 is obtained by reflecting circle C_1 in the line $y = x$.

Find the equation of the circle, C_2 . [2]

$$\text{New centre} = (-13, 1)$$

Equation of circle C_2 is

$$(x + 13)^2 + (y - 1)^2 = 50$$

- 11 A spherical scoop of ice cream with radius x cm rests on top of an inverted cone with height 18 cm and radius 7.5 cm.

As the ice cream scoop melts, the radius of the ice cream scoop decreases at a constant rate of 0.00167 cm/s.

- (i) Assuming that the scoop of ice cream retains its shape, find the rate at which the volume of the ice cream scoop is decreasing at the instant when $x = 8$. [3]

$$\begin{aligned}
 V &= \frac{4}{3}\pi x^3 \\
 \frac{dV}{dx} &= 4\pi x^2 \\
 \frac{dV}{dt} &= \frac{dV}{dx} \times \frac{dx}{dt} \\
 \frac{dV}{dt} &= 4\pi x^2(-0.00167) \\
 \text{At } x = 8, \frac{dV}{dt} &= -1.34309 \\
 &= -1.34\text{cm}^3/\text{s} \text{ (3 s.f.)}
 \end{aligned}$$

Volume of the ice cream scoop is decreasing at a rate of 1.34 cm^3 per second.

- (ii) Show that the volume of the melted ice cream in the cone, C , is $C = \frac{25\pi}{432}h^3$,
where h cm is the depth of the melted ice cream in the cone. [1]

Let radius of cone be r ,

Given radius is 7.5 cm and height is 18 cm,

$$\frac{h}{r} = \frac{18}{7.5}$$

$$r = \frac{5}{12}h$$

$$C = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi \left(\frac{5}{12}h\right)^2 h$$

$$= \frac{25\pi}{432}h^3 \text{ (shown)}$$

- (iii) The cone was observed to be 12% filled at some point in time. It is given that the rate of increase of the volume of melted ice cream in the cone is the same rate as the rate of decrease of the volume of the ice cream found in part (i). Assuming no melted ice cream drips outside the cone, find the rate of change of the depth of the melted ice cream in the cone at that instant. [4]

Let h_1 be unknown height.

Using volume of similar figures,

$$\left(\frac{h_1}{18}\right)^3 = \left(\frac{V_1}{V_2}\right)$$

Since cone was filled to 12%, $\frac{V_1}{V_2} = 0.12$

$$h_1 = \sqrt[3]{0.12} \times 18$$

$$= 8.8783$$

$$\frac{dC}{dh} = \frac{25\pi}{144}h^2$$

$$\frac{dh}{dt} = \frac{1}{\frac{dC}{dh}} \times \frac{dC}{dt}$$

Since rate of increase of the volume of melted ice cream in the cone is the same rate as the rate of decrease of volume of the ice cream found in part (i),

At $h = 8.8783$

$$\frac{dh}{dt} = \frac{144}{25\pi(8.8783)^2} \times 1.34309$$

$$= 0.031240$$

$$= 0.0312 \text{ cm/s (3 s. f.)}$$

- 12** An arrow is shot towards a target in an archery competition.

Its height, h m, above the ground at time t seconds after being shot is given by the formula $h = 1.69 + 0.9t - 0.25t^2$.

- (i)** State the height above the ground from which the arrow is shot. [1]

$$h(0) = 1.69 \text{ m}$$

- (ii)** Express h in the form $a + b(t + c)^2$, where a , b and c are constants. [3]

$$\begin{aligned} h &= -0.25t^2 + 0.9t + 1.69 \\ &= -0.25(t^2 - 3.6t) + 1.69 \\ &= -0.25[(t - 1.8)^2 - 3.24] + 1.69 \\ &= -0.25(t - 1.8)^2 + 0.81 + 1.69 \\ &= 2.5 - 0.25(t - 1.8)^2 \end{aligned}$$

- (iii)** Hence state the maximum height attained by the arrow and the time at which this occurs. [2]

greatest height = 2.5 m at 1.8 seconds

- (iv) The arrow missed the target and hit the ground. Explain why the time taken for the arrow to hit the ground is **not** twice the time found in part (iii). [1]

The arrow was shot at 1.69 m above the ground instead of from the ground.

- (v) Find the length of time for which the arrow is at least 2.19 m above the ground. [3]

$$h = 2.5 - 0.25(t - 1.8)^2 \geq 2.19$$

$$0.25(t - 1.8)^2 \leq 0.31$$

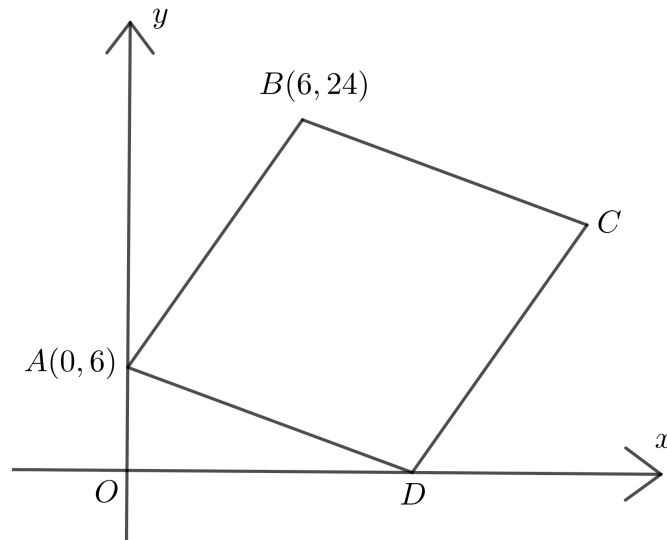
$$(t - 1.8)^2 \leq 1.24$$

$$-\sqrt{1.24} \leq t - 1.8 \leq \sqrt{1.24}$$

$$0.68644 \leq t \leq 2.9135$$

$$\begin{aligned} \text{Length of time} &= 2.9135 - 0.68644 \\ &= 2.22 \text{ s (3 s.f.)} \end{aligned}$$

- 13 The diagram, which is not drawn to scale, shows a rhombus $ABCD$ in which the point A is $(0, 6)$ and point B is $(6, 24)$. The point D lies on the x -axis.



- (i) Find the coordinates of D .

[3]

Let D be $(d, 0)$

$$\begin{aligned}
 \text{length } AB &= \sqrt{(6 - 0)^2 + (24 - 6)^2} \\
 &= \sqrt{360} \text{ units} \\
 \sqrt{(6 - 0)^2 + (0 - d)^2} &= \sqrt{360} \\
 d^2 &= 324 \\
 d &= 18 \text{ or } -18 \\
 &\text{(rejected because } d > 0) \\
 D &= (18, 0)
 \end{aligned}$$

Point D is $(18, 0)$.

- (ii) Find the coordinates of
- C
- .

[4]

$$\begin{aligned}\text{midpoint of } BD &= \left(\frac{6+18}{2}, \frac{24+0}{2} \right) \\ &= (12, 12)\end{aligned}$$

Let C be (a, b) .

$$\text{midpoint of } AC = \left(\frac{0+a}{2}, \frac{6+b}{2} \right)$$

Since it is a rhombus, midpoint BD is the same as midpoint AC .

$$\begin{aligned}\left(\frac{0+a}{2}, \frac{6+b}{2} \right) &= (12, 12) \\ (a, b) &= (24, 18)\end{aligned}$$

Point C is $(24, 18)$.

[NOTE: if students use coordinate adding, they must state the reason why they are able to use it, i.e. parallel sides / equal lengths due to rhombus]

- (iii) Find the ratio of the area of triangle
- AOD
- to the area of rhombus
- $ABCD$
- .

[3]

$$\begin{aligned}\text{Area of triangle } AOD &= \frac{1}{2}(6)(18) \\ &= 54 \text{ units}^2\end{aligned}$$

$$\begin{aligned}\text{Area of rhombus } ABCD \\ &= \frac{1}{2} \sqrt{(-12)^2 + 24^2} \sqrt{24^2 + (12)^2} \\ &= 360 \text{ units}^2\end{aligned}$$

$$\begin{aligned}\text{Ratio} &= 54 : 360 \\ &= 3 : 20\end{aligned}$$

ALTERNATIVE METHOD

$$\begin{aligned}\text{Area of triangle } AOD &= \frac{1}{2}(6)(18) \\ &= 54 \text{ units}^2\end{aligned}$$

$$\begin{aligned}\text{Area of rhombus } ABCD \\ &= \frac{1}{2} \begin{vmatrix} 18 & 24 & 6 & 0 & 18 \\ 0 & 18 & 24 & 6 & 0 \end{vmatrix} \\ &= \frac{1}{2} [(324 + 576 + 36) - (108 + 108)] \\ &= 360 \text{ units}^2\end{aligned}$$

$$\begin{aligned}\text{Ratio} &= 54 : 360 \\ &= 3 : 20\end{aligned}$$