



SECONDARY 4 PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS Paper 2

4049/02

2 September 2025 (Tuesday)

2 hours 15 minutes

CANDIDATE
NAME

Solutions

CLASS

4	-		
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INDEX NUMBER

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Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your full name, class and index number in the spaces above.

Write in dark blue or black pen in the space provided for each question.

You may use a HB pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

The number of marks is given in brackets [] at the end of each question or part question.

If working is needed for any question, it must be shown in the space below the question.

Omission of essential working will result in loss of marks.

The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142.

For Examiner's Use		
Q1	8	
Q2	10	
Q3	9	
Q4	9	
Q5	5	
Q6	6	
Q7	11	
Q8	8	
Q9	8	
Q10	6	
Q11	10	
Total	90	

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1) \dots (n-r+1)}{r!}$

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

Answer all the questions.

A4.2 Use of factor theorem, including factorising polynomials and solving cubic equations

- 1 (i) It is given that $f(x) = x^4 - px^3 + 7x^2 + x - q$ has a quadratic factor $x^2 - 2x - 3$.

Show that $p = 5$ and $q = 12$.

[5]

$$x^2 - 2x - 3 = (x - 3)(x + 1)$$

Given $f(-1) = 0$ and $f(3) = 0$

$$(-1)^4 - p(-1)^3 + 7(-1)^2 + (-1) - q = 0$$

$$(3)^4 - p \cdot (3)^3 + 7 \cdot (3)^2 + 3 - q = 0$$

$$q - p = 7 \text{ ---- (1)}$$

$$27p + q = 147 \text{ ---- (2)}$$

$$(2) - (1): \quad 28p = 140$$

$$p = 5 \text{ and } q = 12 \quad (\text{shown})$$

- (ii) Hence, solve the equation $x^4 - px^3 + 7x^2 + x - q = 0$.

[3]

$$f(x) = x^4 - 5x^3 + 7x^2 + x - 12$$

$$= (x^2 - 2x - 3)(x^2 + bx + 4) \text{ [by inspection]}$$

By comparing coefficients of x^2 : $7 = 4 - 2b - 3$

$$b = -3$$

$$(x^2 - 2x - 3)(x^2 - 3x + 4) = 0$$

$$(x - 3)(x + 1)(x^2 - 3x + 4) = 0$$

$$x - 3 = 0 \text{ or } x + 1 = 0 \text{ or } x^2 - 3x + 4 = 0$$

Since discriminant of $x^2 - 3x + 4 = 0$ is

$(-3)^2 - 4(1)(4) = -7 < 0$, there are no real solutions for $x^2 - 3x + 4 = 0$.

Solution: $x = 3$ and $x = -1$

A6.2 Simplifying expressions involving exponential and logarithmic functions**2 (a) Without using a calculator,**find the integer value of $(5^{\lg 2}) (2^{\lg 3}) (5^{\lg 9}) (2^{\lg 6})$.

[3]

$$\begin{aligned}
 (5^{\lg 2}) (2^{\lg 3}) (5^{\lg 9}) (2^{\lg 6}) &= (5^{\lg 2 + \lg 9}) (2^{\lg 3 + \lg 6}) \\
 &= (5^{\lg(2 \times 9)}) (2^{\lg(3 \times 6)}) \\
 &= (5^{\lg 18}) (2^{\lg 18}) \\
 &= (5 \times 2)^{\lg 18} \\
 &= 10^{\lg 18} \\
 &= 18
 \end{aligned}$$

A6.1 Logarithmic functions including laws of logarithms and change of base**(b)** Given that $\log_2(y^2) = 4 - \log_{0.5} x$, express y in terms of x .

[3]

$$\begin{aligned}
 \log_2(y^2) &= 4 \log_2 2 - \frac{\log_2 x}{\log_2 2^{-1}} \\
 \log_2(y^2) &= \log_2 2^4 + \frac{\log_2 x}{1} \\
 \log_2(y^2) &= \log_2 2^4 + \log_2 x \\
 \log_2(y^2) &= \log_2(16x) \\
 y^2 &= 16x \\
 y &= \pm 4\sqrt{x}
 \end{aligned}$$

A6.2 Solving simple equations involving exponential and logarithmic functions**(c)** Solve the equation $2e^x = 3 - 5\sqrt{e^x}$.

[4]

$$\begin{aligned}
 2e^x &= 3 - 5e^{\frac{x}{2}} \\
 \text{Let } y &= e^{\frac{x}{2}}. \\
 2y^2 &= 3 - 5y \\
 2y^2 + 5y - 3 &= 0 \\
 (2y - 1)(y + 3) &= 0 \\
 \text{Since } e^{\frac{x}{2}} > 0 \text{ for all } x, &y = 0.5 \\
 e^{\frac{x}{2}} &= 0.5 \\
 \frac{x}{2} &= \ln 0.5 \\
 x &= 2 \ln 0.5 = -1.39 \text{ (3 s.f.)}
 \end{aligned}$$

A6.3 Using exponential and logarithmic functions as models

- 3 A metal cube is heated to a temperature of 205°C before being dropped into a liquid. As the cube cools, its temperature, $T^{\circ}\text{C}$, t minutes after it enters the liquid is given by

$$T = K + 175e^{-mt}, \text{ where } K \text{ and } m \text{ are constants.}$$

- (i) Show that the value of K is 30. [1]

$$\begin{aligned} \text{At } t = 0, T &= 205^{\circ}\text{C}. \\ K + 175e^{-m(0)} &= 205 \\ K &= 205 - 175 \\ K &= 30 \quad (\text{shown}) \end{aligned}$$

When $t = 3$, the temperature of the cube reaches 128°C .

- (ii) Find the value of m . [3]

$$\begin{aligned} \text{When } t = 3, T &= 128^{\circ}\text{C}. \\ 30 + 175e^{-m(3)} &= 128 \\ 175e^{-3m} &= 98 \\ e^{-3m} &= \frac{98}{175} \\ -3m &= \ln\left(\frac{98}{175}\right) \\ m &= 0.19327 \\ m &= 0.193 \quad (3 \text{ s.f.}) \end{aligned}$$

- (iii) Find the rate at which the temperature of the cube is decreasing at the instant when $t = 8$. [3]

$$\begin{aligned} T &= 30 + 175e^{-0.19327t} \\ \frac{dT}{dt} &= -33.82225e^{-0.19327t} \\ \text{When } t &= 8, \\ \frac{dT}{dt} &= -33.82225e^{-0.19327(8)} \\ &= -7.2063 \\ &= -7.21 \quad (3 \text{ s.f.}) \end{aligned}$$

Temperature of the cube is decreasing at a rate of 7.21°C per minute.

- (iv) Explain why the temperature of the cube can never fall below 30°C . [2]

$$\begin{aligned} \text{Since } e^{-0.19327t} &> 0 \text{ for } t \geq 0, \\ 175e^{-0.19327t} &> 0, \\ 30 + 175e^{-0.19327t} &> 30 \\ \text{Therefore, } T &> 30 \end{aligned}$$

Hence the temperature of the cube can never fall below 30°C .

A5.3 Use of the general term]

- 4 (a) In the expansion of $(2 + x^2)\left(\frac{1}{2x} - ax\right)^5$, there is no term in x . Given that $a \neq 0$, find the value of the constant a . [5]

$$\begin{aligned}
 \text{General term for } \left(\frac{1}{2x} - ax\right)^5 &= \binom{5}{r} \left(\frac{1}{2x}\right)^{5-r} (-ax)^r \\
 &= \binom{5}{r} \left(\frac{1}{2}\right)^{5-r} (-a)^r x^{2r-5} \\
 2r - 5 &= 1 \text{ and } 2r - 5 = -1 \\
 r &= 3 \text{ and } r = 2 \\
 \text{Term with } x^{-1} &= \binom{5}{2} \left(\frac{1}{2}\right)^{5-2} (-a)^2 x^{4-5} \\
 &= \frac{5}{4} a^2 \left(\frac{1}{x}\right) \\
 \text{Term with } x &= \binom{5}{3} \left(\frac{1}{2}\right)^{5-3} (-a)^3 x^{6-5} \\
 &= -\frac{5}{2} a^3 x \\
 (2 + x^2) \left(\frac{1}{2x} - ax\right)^5 \\
 &= (2 + x^2) \left(-\frac{5}{2} a^3 x + \frac{5}{4} a^2 \left(\frac{1}{x}\right) + \dots\right) \\
 &= \left(-5a^3 + \frac{5}{4} a^2\right) x + \dots \\
 -5a^3 + \frac{5}{4} a^2 &= 0 \\
 \frac{5}{4} a^2 (-4a + 1) &= 0 \\
 a = 0 \text{ (rejected) or } a &= \frac{1}{4}
 \end{aligned}$$

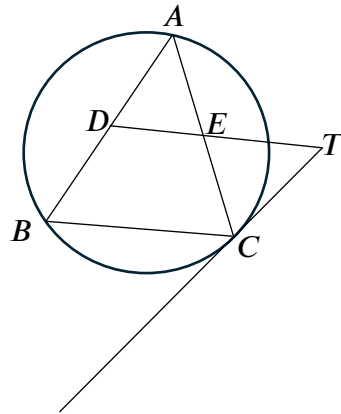
A5.1 Use of Binomial Theorem for positive integer n

- (b) Write down and simplify the first three terms in the expansion of $\left(2 + \frac{x^2}{2}\right)^5$ in ascending powers of x . Hence find the estimated value of $(2.005)^5$, showing all your workings clearly. [4]

$$\begin{aligned}
 \left(2 + \frac{x^2}{2}\right)^5 &= 2^5 + \binom{5}{1} 2^4 \left(\frac{x^2}{2}\right) + \binom{5}{2} 2^3 \left(\frac{x^2}{2}\right)^2 + \dots \\
 &= 32 + 40x^2 + 20x^4 + \dots \quad \text{---- (1)} \\
 \text{Solving } \left(2 + \frac{x^2}{2}\right)^5 &= 2.005^5 \text{ gives } x = 0.1 \\
 \text{Substitute } x = 0.1 \text{ into (1):} \\
 2.005^5 &= 32 + 40(0.1)^2 + 20(0.1)^4 \\
 &= 32 + 40(0.01) + 20(0.0001) \\
 &= 32.402
 \end{aligned}$$

G3.1 Use of tangent-chord theorem

5



The diagram shows a circle passing through the vertices of a triangle ABC . Points D and E are the midpoints of AB and AC respectively. The tangent to the circle at C meets DE extended at the point T . Prove that points A, D, C and T lie on a circle. [5]

By midpoint theorem, DE is parallel to BC .

Thus, $\angle ADE = \angle ABC$. (corresponding angles)

By tangent-chord theorem,
 $\angle ABC = \angle ECT$

Since $\angle ADE = \angle ECT$,
they are angles in same segment,
hence, points A, D, C and T lie on a circle. (shown)

G1.6 Use of the expressions of $a \cos \theta + b \sin \theta$ in the form $R \cos(\theta \pm \alpha)$ or $R \sin(\theta \pm \alpha)$

6 The expression $3 \cos \theta + 5 \sin \theta$ is defined for $0 \leq \theta \leq \frac{\pi}{2}$ radians.

- (i) Express $3 \cos \theta + 5 \sin \theta$ in the form $R \cos(\theta - \alpha)$, where $R > 0$ and α is an acute angle in radian. [2]

$$\begin{aligned} 3 \cos \theta + 5 \sin \theta &= R \cos(\theta - \alpha) \\ 3 \cos \theta + 5 \sin \theta &= \sqrt{3^2 + 5^2} \cos(\theta - \alpha), \text{ where } \alpha = \tan^{-1} \left(\frac{5}{3} \right) \\ &= \sqrt{34} \cos(\theta - 1.0303) \text{ (working value)} \\ &= 5.83 \cos(\theta - 1.03) \text{ (3 s.f.)} \end{aligned}$$

Hence,

- (ii) solve the equation $3 \cos \theta + 5 \sin \theta = 4$, [2]

$$\begin{aligned} 3 \cos \theta + 5 \sin \theta &= 4 \\ \sqrt{34} \cos(\theta - 1.0303) &= 4 \\ \cos(\theta - 1.0303) &= \frac{4}{\sqrt{34}} \\ \text{Basic angle} &= \cos^{-1} \left(\frac{4}{\sqrt{34}} \right) = 0.81482 \\ \theta - 1.0303 &= 0.81482, -0.81482 \\ \theta &= 1.84512, 0.21548 \\ \text{As } \theta \text{ is acute, } \theta &= 0.215 \text{ radians (3 s.f.)} \\ &\text{(accept 0.216 rad if exact form is used)} \end{aligned}$$

- (iii) find the minimum value of $\frac{1}{(3 \cos \theta + 5 \sin \theta)^2 + 1}$. [2]

$$\begin{aligned} -\sqrt{34} &\leq 3 \cos \theta + 5 \sin \theta \leq \sqrt{34} \\ 0 &\leq (3 \cos \theta + 5 \sin \theta)^2 \leq 34 \\ 1 &\leq (3 \cos \theta + 5 \sin \theta)^2 + 1 \leq 35 \\ \frac{1}{35} &\leq \frac{1}{(3 \cos \theta + 5 \sin \theta)^2 + 1} \leq 1 \\ \text{Minimum value} &= \frac{1}{35} \end{aligned}$$

G2.5 Transformation of given relationships, including $y = ax^n$ and $y = kb^x$, to linear form to determine the unknown constants from a straight line graph

- 7 (a) The mass, y mg, of a radioactive substance decreases with time, x hours, after the start of the experiment. The research analyst claims that the data can be modelled by an equation of the form $x^2y = a + bx^2$, where a and b are constants. Values of y for different values of x have been collected. Explain how a straight line graph can be drawn to represent the formula, and state how the values of a and b could be obtained from the line. [4]

Rearrange the given formula into a straight-line graph equation as follows, $x^2y = bx^2 + a$ where x^2y is plotted against x^2 . When line is drawn, the value of b is the gradient and the value of a is the **vertical intercept** of the graph.

Alternatively,

$$y = a\left(\frac{1}{x^2}\right) + b \text{ where } y \text{ is plotted against } \frac{1}{x^2}.$$

- (b) The table shows experimental values of two variables R and t .

t	10	20	30	40	50
R	2.32	2.79	3.31	4.74	4.90

It is known that R and t are related by the equation $R = R_0(3^{-kt})$, where

R_0 and k are constants. An error was made in recording one of the values of R .

- (i) Plot $\ln R$ against t and draw a straight line graph to

illustrate the information. [2]

t	10	20	30	40	50
$\ln R$	0.842	1.03	1.20	1.56	1.59

$$R = R_0(3^{-kt})$$

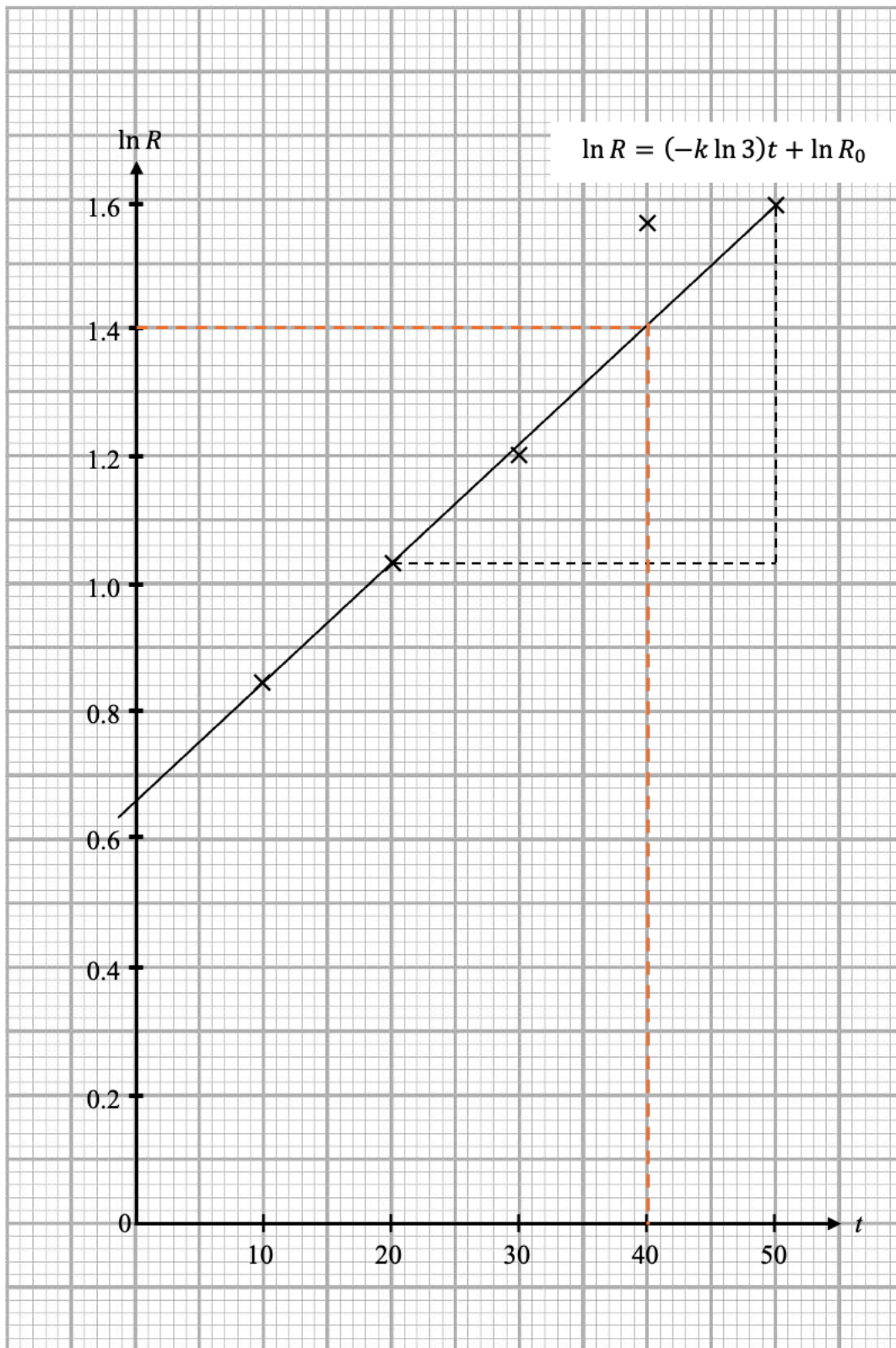
$$\ln R = \ln R_0(3^{-kt})$$

$$\ln R = \ln R_0 + \ln 3^{-kt}$$

$$\ln R = \ln R_0 - kt \ln 3$$

$$\ln R = (-k \ln 3)t + \ln R_0$$

$\therefore$ Plot $\ln R$ against t



- 7 (b) (ii) From your straight line graph, identify the incorrect reading and suggest a corrected value for R . [2]

$R = 4.74$ is the incorrect reading

From the graph, $\ln R = 1.4$

$$R = e^{1.4} = 4.06 \text{ (3 s.f.)}$$

- (iii) Use your graph to estimate the values of R_0 and k . [3]

$$R = R_0(3^{-kt})$$

$$\ln R = (-k \ln 3)t + \ln R_0$$

$$\text{Vertical intercept} = \ln R_0$$

$$= 0.66 \text{ (from the graph)}$$

$$\text{Thus, } R_0 = e^{0.66} = 1.93 \text{ (3 s.f.)}$$

$$\text{Gradient} = -k \ln 3$$

$$= \frac{1.59 - 1.03}{50 - 20}$$

$$= 0.018666$$

$$\text{Thus, } k = \frac{0.018666}{-\ln 3}$$

$$= -0.0170 \text{ (3 s.f.)}$$

C1.15 Evaluation of definite integrals

8 (a) Given that $\int_1^6 f(x) \, dx = 14$ and $\int_1^3 f(x) \, dx = 8$, find

(i) $\int_3^6 f(x) \, dx$, [1]

$$\begin{aligned} \int_3^6 f(x) \, dx &= 14 - 8 \\ &= 6 \end{aligned}$$

(ii) $\int_1^6 (4x - 3f(x)) \, dx$. [2]

$$\begin{aligned} \int_1^6 (4x - 3f(x)) \, dx &= \int_1^6 4x \, dx - 3 \int_1^6 f(x) \, dx \\ &= [2x^2]_1^6 - 3(14) \\ &= 72 - 2 - 42 \\ &= 28 \end{aligned}$$

C1.4 Derivatives of x^n , for any rational n , $\sin x$, $\cos x$, $\tan x$, e^x , and $\ln x$ together with constant multiples, sums and differences

- 8 (b) It is given that $f(x) = \ln(\sin 2x)$. Show that $f''(x) + [f'(x)]^2 + 4 = 0$. [5]

$$f(x) = \ln(\sin 2x)$$

$$f'(x) = \frac{2 \cos 2x}{\sin 2x}$$

$$f''(x) = \frac{\sin 2x (-4 \sin 2x) - 2 \cos 2x (2 \cos 2x)}{(\sin 2x)^2}$$

$$= \frac{-4 \sin^2 2x - 4 \cos^2 2x}{(\sin 2x)^2}$$

$$= \frac{-4(\sin^2 2x + \cos^2 2x)}{(\sin 2x)^2}$$

$$= -\frac{4}{(\sin 2x)^2}$$

$$f''(x) + [f'(x)]^2 + 4 = -\frac{4}{(\sin 2x)^2} + \left[\frac{2 \cos 2x}{\sin 2x} \right]^2 + 4$$

$$= -\frac{4}{(\sin 2x)^2} + \frac{4 \cos^2 2x}{\sin^2 2x} + 4$$

$$= \frac{-4 + 4 \cos^2 2x}{\sin^2 2x} + 4$$

$$= \frac{-4(1 - \cos^2 2x)}{\sin^2 2x} + 4$$

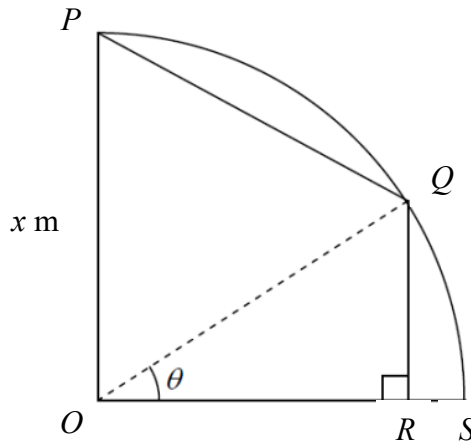
$$= \frac{-4 \sin^2 2x}{\sin^2 2x} + 4$$

$$= -4 + 4$$

$$= 0 \quad (\text{shown})$$

C1.10 Apply differentiation to maxima and minima problems

- 9 The diagram shows a garden plot designed in the shape of a trapezium $OPQR$ inscribed in a quadrant $OPQS$ with a fixed radius x metres and angle $QOR = \theta$ radians.



- (i) Show that the area, $A \text{ m}^2$, of trapezium $OPQR$ is given by $A = \frac{1}{4}x^2(\sin 2\theta + 2\cos\theta)$.

[3]

$$\begin{aligned}
 A &= \left(\frac{1}{2}\right)(x \cos \theta)(x \sin \theta + x) \\
 &= \frac{1}{2}x^2 \sin \theta \cos \theta + \frac{1}{2}x^2 \cos \theta \\
 &= \frac{1}{2}x^2 \left(\frac{\sin 2\theta}{2}\right) + \frac{1}{2}x^2 \cos \theta \\
 &= \frac{1}{4}x^2 \sin 2\theta + \frac{1}{2}x^2 \cos \theta \\
 &= \frac{1}{4}x^2(\sin 2\theta + 2\cos \theta) \quad (\text{shown})
 \end{aligned}$$

- (ii) Given that θ can vary, find

(a) the value of θ for which A has a stationary value.

[3]

$$\begin{aligned}
 \frac{dA}{d\theta} &= \frac{1}{4}x^2(2\cos 2\theta - 2\sin \theta) \\
 \text{For stationary value of } A, \quad \frac{dA}{d\theta} &= 0 \\
 \frac{1}{4}x^2(2\cos 2\theta - 2\sin \theta) &= 0 \\
 \frac{1}{2}x^2(\cos 2\theta - \sin \theta) &= 0 \\
 \frac{1}{2}x^2(1 - 2\sin^2 \theta - \sin \theta) &= 0 \\
 1 - 2\sin^2 \theta - \sin \theta &= 0 \\
 2\sin^2 \theta + \sin \theta - 1 &= 0 \\
 (2\sin \theta - 1)(\sin \theta + 1) &= 0 \\
 \sin \theta = \frac{1}{2} \quad \text{or} \quad \sin \theta &= -1 \\
 \theta = \frac{\pi}{6} \text{ rad, or } \theta = \frac{3\pi}{2} \text{ rad. (rej, because } \theta < \frac{\pi}{2} \text{)}
 \end{aligned}$$

- 9 (ii) (b) the stationary value of A at $x = 10$ and determine whether it is a maximum or a minimum. [2]

$$\frac{d^2A}{d\theta^2} = \frac{1}{4}x^2(-4\sin 2\theta - 2\cos \theta)$$

Subt. $\theta = \frac{\pi}{6}$ and $x = 10$ into $\frac{d^2A}{d\theta^2}$,

$$\begin{aligned}\frac{d^2A}{d\theta^2} &= \frac{1}{4}(10)^2 \left(-4\sin 2\left(\frac{\pi}{6}\right) - 2\cos \frac{\pi}{6}\right) \\ &= \frac{1}{4}(100)(-5.1961) \\ &= -129.90 < 0\end{aligned}$$

Since $\frac{d^2A}{d\theta^2} < 0$, A is a maximum when $\theta = \frac{\pi}{6}$.

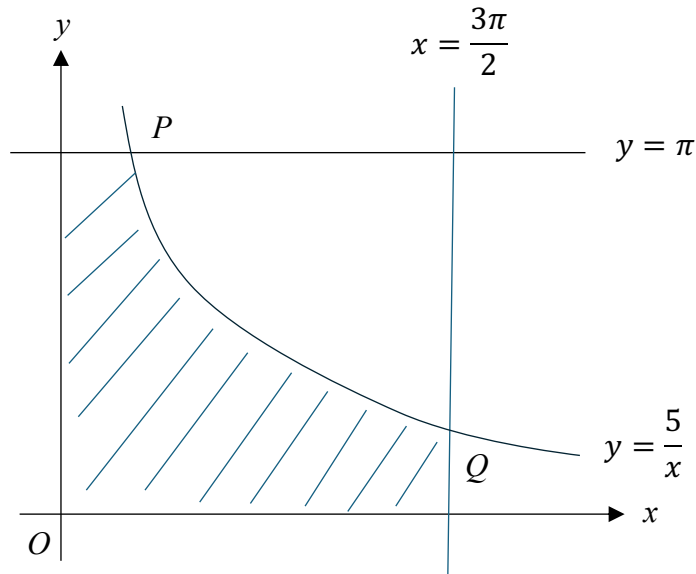
Subt. $\theta = \frac{\pi}{6}$ into A ,

Maximum area of A

$$\begin{aligned}&= \frac{1}{4}(10)^2 \left(\sin 2\left(\frac{\pi}{6}\right) + 2\cos \frac{\pi}{6}\right) \\ &= 64.951 \\ &= 65.0 \text{ cm}^2 \text{ (3 s.f.)}\end{aligned}$$

C1.16 Finding the area of a region bounded by a curve and line(s).

- 10** The diagram shows part of the curve $y = \frac{5}{x}$, $y = \pi$ and $x = \frac{3\pi}{2}$. The curve $y = \frac{5}{x}$ cuts the line $y = \pi$ at P and cuts the line $x = \frac{3\pi}{2}$ at Q .



- (i)** Find the coordinates of P and of Q in terms of π .

[2]

When $y = \pi$, $\frac{5}{x} = \pi$,

Therefore, $x = \frac{5}{\pi}$

Point P is $\left(\frac{5}{\pi}, \pi\right)$.

When $x = \frac{3\pi}{2}$, $y = 5 \div \frac{3\pi}{2} = \frac{10}{3\pi}$

Point Q is $\left(\frac{3\pi}{2}, \frac{10}{3\pi}\right)$.

- (ii)** Find the area of the shaded region bounded by $y = \frac{5}{x}$, $y = \pi$, $x = \frac{3\pi}{2}$, the y -axis and the x -axis.

[3]

$$\begin{aligned}
 \text{Shaded area} &= \pi \left(\frac{5}{\pi}\right) + \int_{\frac{5}{\pi}}^{\frac{3\pi}{2}} \frac{5}{x} \, dx \\
 &= 5 + 5[\ln x]_{\frac{5}{\pi}}^{\frac{3\pi}{2}} \\
 &= 5 + 5 \left[\ln \left(\frac{3\pi}{2}\right) - \ln \left(\frac{5}{\pi}\right) \right] \\
 &= 10.4 \text{ units}^2 \text{ (3 s.f.)}
 \end{aligned}$$

- (iii) Explain why the area of the shaded region is between 5 units^2 and $\frac{3\pi^2}{2} \text{ units}^2$. [1]

$$\text{Area of big rectangle} = \pi \left(\frac{3\pi}{2} \right)$$

$$= \frac{3\pi^2}{2} \text{ units}^2$$

$$\text{Area of small rectangle} = \left(\frac{3\pi}{2} \right) \left(\frac{10}{3\pi} \right)$$

$$= 5 \text{ units}^2$$

Therefore, area of shaded region is between 5 units^2 and $\frac{3\pi^2}{2} \text{ units}^2$.

C1.18 Application of differentiation and integration to problems involving displacement, velocity and acceleration of a particle moving in a straight line

- 11** A particle, travelling in a straight line, has velocity, v cm/s, given by $v = 3t^2 - pt + 4$, where p is a constant and t seconds is the time after passing through a fixed point O . The particle comes to instantaneous rest when $t = 2$.

(i) Show that $p = 8$.

[1]

$$\begin{aligned} v &= 3t^2 - pt + 4 \\ \text{When } v &= 0 \text{ and } t = 2, \\ 3(2)^2 - p(2) + 4 &= 0 \\ 2p &= 16 \\ p &= 8 \quad (\text{shown}) \end{aligned}$$

(ii) Find the range of values of t during which the particle moves in the positive direction.

[3]

$$\begin{aligned} 3t^2 - 8t + 4 &> 0 \\ (3t - 2)(t - 2) &> 0 \\ \text{The range of values of } t &\text{ is } t < \frac{2}{3} \text{ or } t > 2. \\ \text{Since } t \geq 0, \text{ the range of values of } t &\text{ is } 0 \leq t < \frac{2}{3} \text{ or } t > 2. \end{aligned}$$

(iii) Find the minimum velocity of the particle.

[3]

$$\begin{aligned} \frac{dv}{dt} &= 6t - 8 \\ \text{For minimum/maximum velocity, } \frac{dv}{dt} &= 0 \\ 6t - 8 &= 0 \\ t &= \frac{4}{3} \\ \frac{d^2v}{dt^2} &= 6 > 0 \\ \text{At } t = \frac{4}{3}, \text{ velocity is a minimum value.} \\ \text{At } t = \frac{4}{3}, \text{ minimum velocity} &= 3\left(\frac{4}{3}\right)^2 - 8\left(\frac{4}{3}\right) + 4 \\ &= -\frac{4}{3} \text{ cm/s} \\ \text{Accepts } -1.33 \text{ cm/s (3 s.f.)} \end{aligned}$$

(iv) Calculate the total distance travelled by the particle for the first 3 seconds.

[3]

$$s = \int v \, dt$$

$$= t^3 - 4t^2 + 4t + c, \text{ where } c \text{ is an arbitrary constant}$$

When $s = 0$ and $t = 0$, $c = 0$

Therefore, $s = t^3 - 4t^2 + 4t$

Particle is at rest when $v = 0$.

$$3t^2 - 8t + 4 = 0$$

$$(3t - 2)(t - 2) = 0$$

$$t = \frac{2}{3} \text{ or } t = 2$$

$$\text{When } t = \frac{2}{3}, \quad s = \frac{32}{27} \text{ cm}$$

$$\text{When } t = 2, \quad s = 0 \text{ cm}$$

$$\text{When } t = 3, \quad s = 3 \text{ cm}$$

Total distance travelled for the first 3 seconds

$$= \frac{32}{27} + \frac{32}{27} + 3$$

$$= 5\frac{10}{27} \text{ cm}$$

Accepts 5.37 cm (3 s.f.)