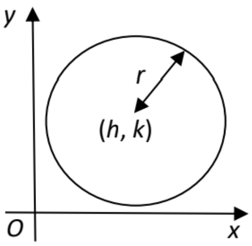
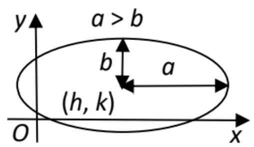
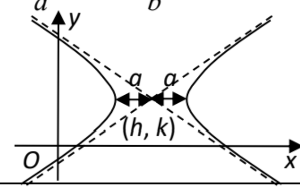
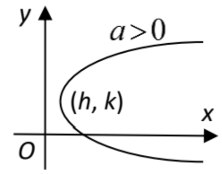
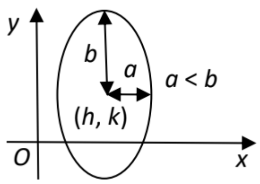
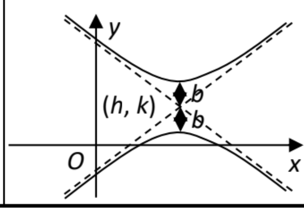
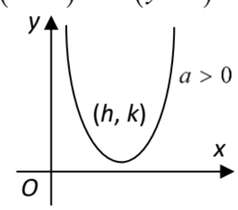
2 Graphs

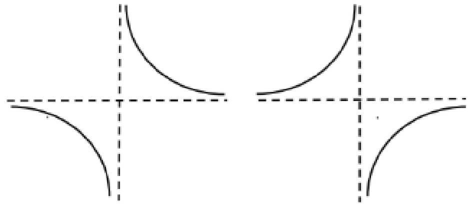
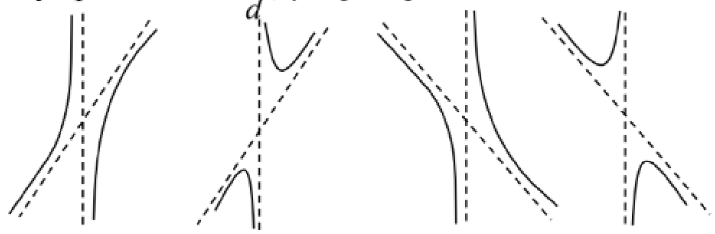
1. Conic Sections
Skill(s) used: Completing the square

Note: To label asymptotes!

Circle	Ellipse	Hyperbola	Parabola
$(x-h)^2+(y-k)^2=r^2$	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$	$(y-k)^2=a(x-h)$
			
		$\frac{(y-k)^2}{b^2}-\frac{(x-h)^2}{a^2}=1$ 	$(x-h)^2=a(y-k)$ 

Hyperbola asymptotes: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=\pm 1 \Rightarrow \frac{(y-k)^2}{b^2}=\frac{(x-h)^2}{a^2}$

Hence, $y-k=\pm\frac{b}{a}(x-h)$ as $x\to\pm\infty$

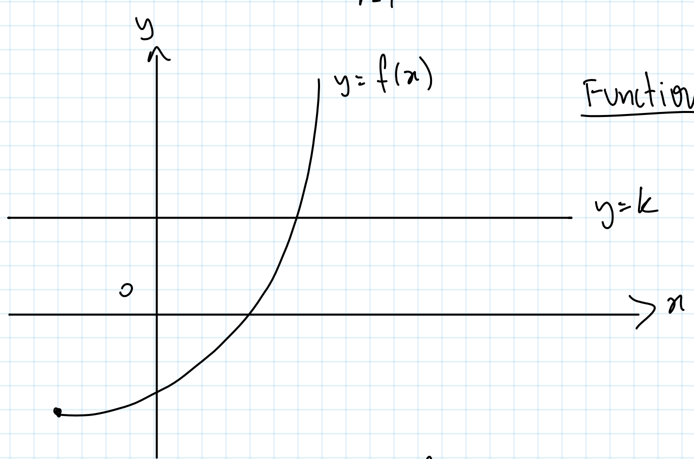
$y=\frac{ax+b}{cx+d}$	$y=\frac{ax^2+bx+c}{dx+e}=(px+q)+\frac{r}{dx+e}$, $a\neq 0$
asymptotes: $x=-\frac{d}{c}$, $y=\frac{a}{c}$	asymptotes: $x=-\frac{e}{d}$, $y=px+q$
	

	AP	GP
u_n	$u_n = a + (n-1)d$ $= S_n - S_{n-1}$	$u_n = ar^{n-1}$
S_n	$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$	$S_n = \frac{a(1-r^n)}{1-r}, \; \lvert r\rvert<1$ $= \frac{a(r^n-1)}{r-1}, \; \lvert r\rvert<1$
S_∞	does not exist	when $\lvert r\rvert<1$, $S_\infty = \frac{a}{1-r}$
Proving	$u_n - u_{n-1} = d$ (constant)	$\frac{u_n}{u_{n-1}} = r$ (constant)

$$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$$

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2 = \left(\sum_{r=1}^{n} r\right)^2$$

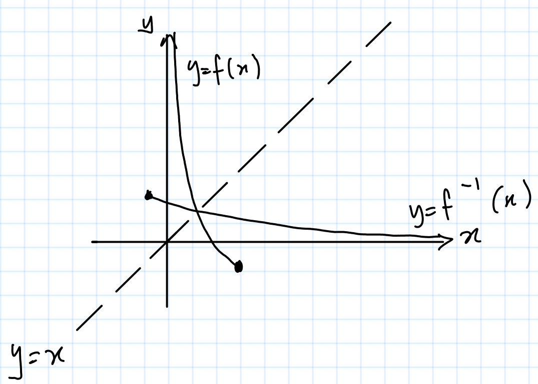


Functions

$y=k$ cuts $y=f(x)$ only ONCE

↳ $f(x)$ is one-to-one

fg exists if $R_g \subseteq D_f$.



$y=f^{-1}(x)$ is a reflection of $y=f(x)$ about the line $y=x$.

Graphs & Transformations

Translation

$y = f(x+2)$ ⟶ translate 2 units in NEGATIVE x-direction

$y = f(x-2)$ ⟶ translate 2 units in POSITIVE x-direction

$y = f(x) + 2$ ⟶ translate 2 units in POSITIVE y-direction

$y = f(x) - 2$ ⟶ translate 2 units in NEGATIVE y-direction

Scaling

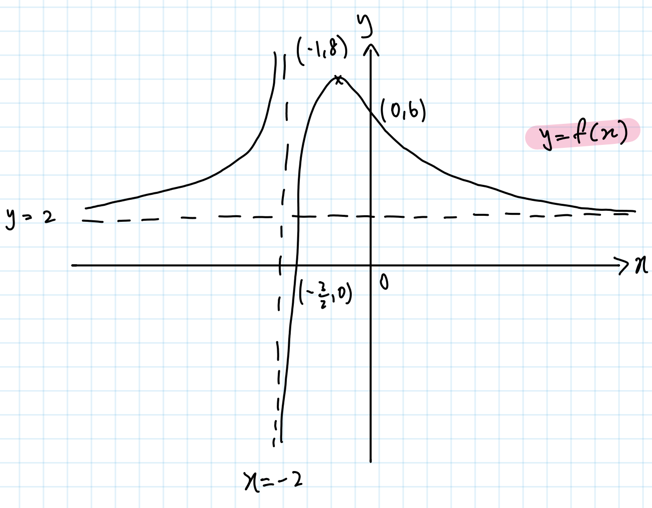
$y = f(2x)$ ⟶ Scale by factor $\frac{1}{2}$ parallel to x-axis

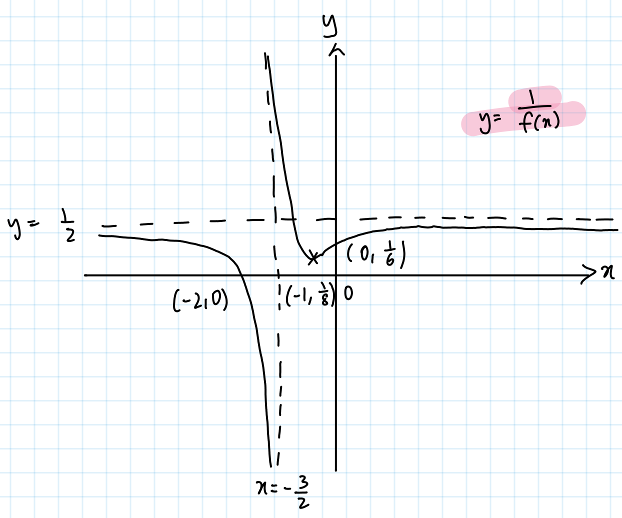
$y = 2f(x)$ ⟶ Scale by factor 2 parallel to y-axis

Reflection

$y = f(-x)$ ⟶ Reflect in y-axis

$y = -f(x)$ ⟶ Reflect in x-axis



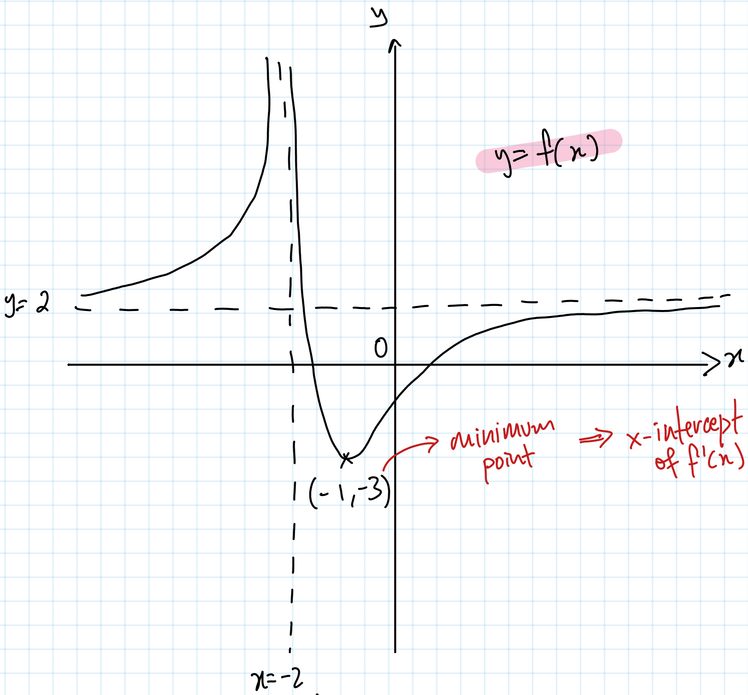


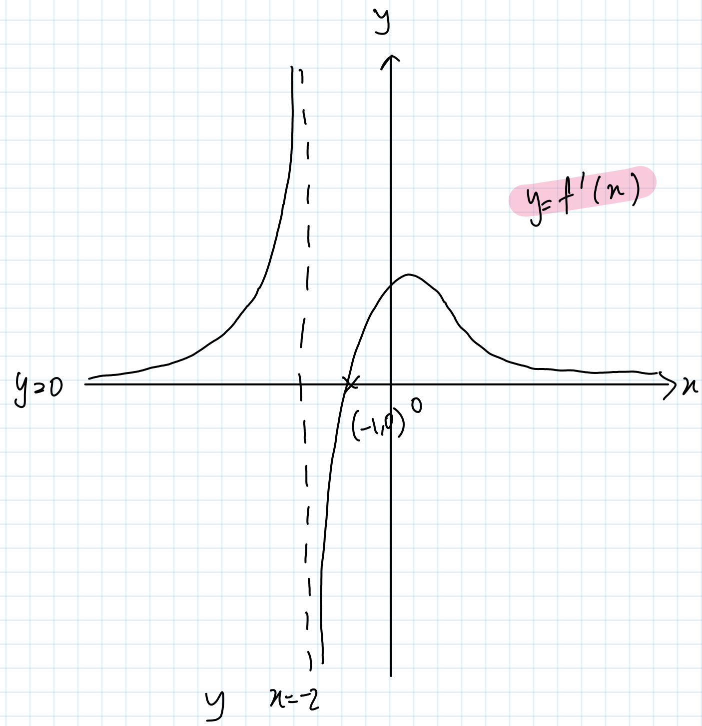
x-intercept ⟷ vertical asymptote

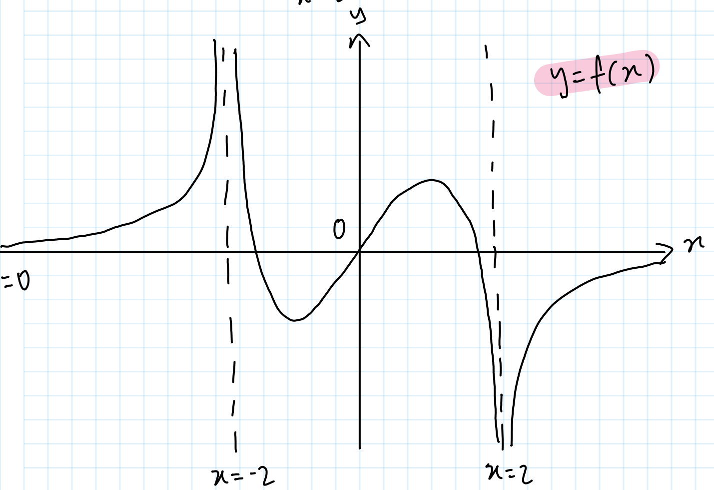
$y = k \longrightarrow y = \frac{1}{k}$

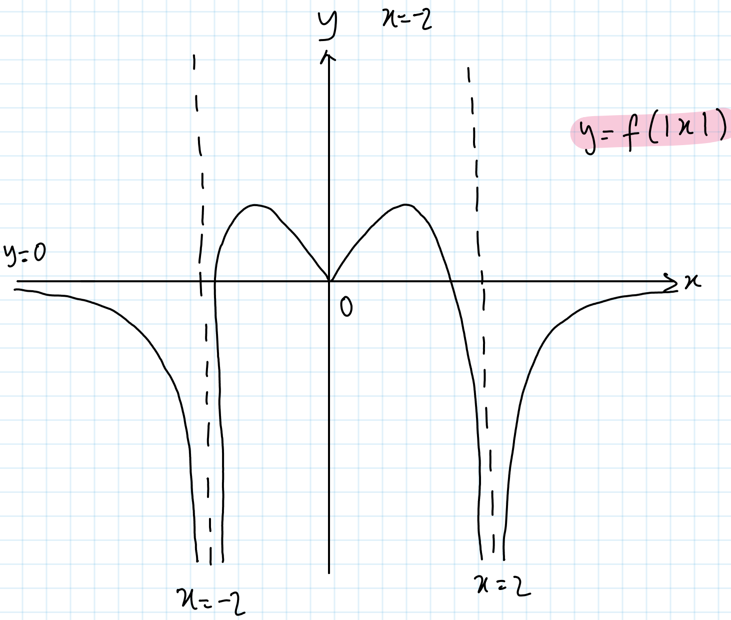
above asymptote ⟷ below asymptote

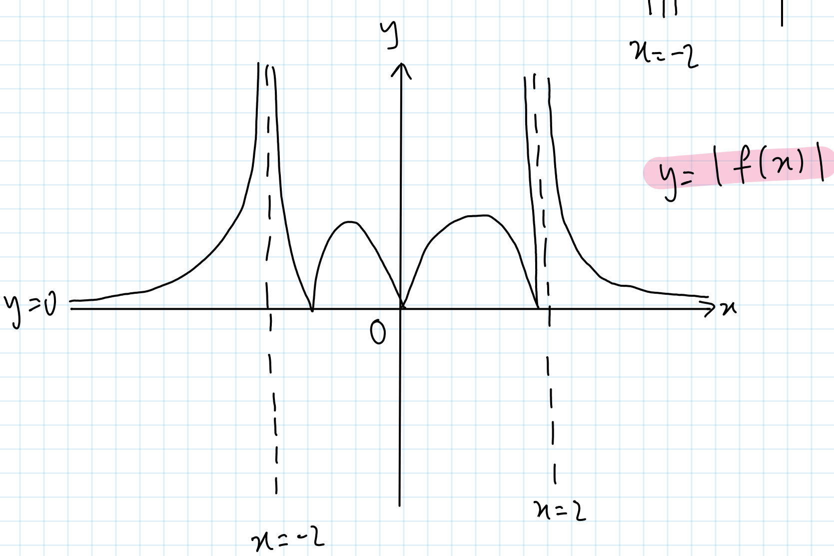
$0^{\pm} \longleftrightarrow \infty^{\pm}$


y
$y = f(x)$
$y = 2$
0
x
$(-1, -3)$
minimum point
⟹ x-intercept of $f'(x)$
$x = -2$


y
$y = f'(x)$
$y = 0$
$(-1, 0)$
0
x
$x = -2$


y
$y = f(x)$
0
x
$y = 0$
$x = -2$
$x = 2$


y
$y = f(|x|)$
$y = 0$
0
x
$x = -2$
$x = 2$


y
$y = |f(x)|$
$y = 0$
0
x
$x = -2$
$x = 2$

Trigonometric Identities & Formulae

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\cot^2\theta + 1 = \operatorname{cosec}^2\theta$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\begin{aligned}\cos 2\theta &= \cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta\end{aligned}$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta$$

$$\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$$

$$\sec(-\theta) = \sec\theta$$

$$\cot(-\theta) = -\cot\theta$$

$$\cos\left(\frac{\pi}{2}-\theta\right) = -\sin\theta$$

$$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta$$

$$\cot\left(\frac{\pi}{2}-\theta\right) = \tan\theta$$

$$\tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta$$

$$\operatorname{cosec}\left(\frac{\pi}{2}-\theta\right) = \sec\theta$$

$$\sec\left(\frac{\pi}{2}-\theta\right) = \operatorname{cosec}\theta$$

$$\sin a \pm \sin b = 2\sin\left(\frac{a\pm b}{2}\right)\cos\left(\frac{a\mp b}{2}\right)$$

$$\cos a + \cos b = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$

$$\cos a - \cos b = 2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

$$\sin a\sin b = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right]$$

$$\cos a\cos b = \frac{1}{2}\left[\cos(a-b) + \cos(a+b)\right]$$

$$\sin a\cos b = \frac{1}{2}\left[\sin(a+b) + \sin(a-b)\right]$$

$$\cos a\sin b = \frac{1}{2}\left[\sin(a+b) - \sin(a-b)\right]$$

θ°	0°	30°	45°	60°	90°	180°	270°
θ rad	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	-1	0	-1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined	0	Not defined
cot	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	Not defined	0
cosec	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	-1	Not defined	-1
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined	-1	Not defined

Differentiation

$$\frac{d}{dx}\left[e^{f(x)}\right] = f'(x)\, e^{f(x)}$$

$$\frac{d}{dx}\left[a^{f(x)}\right] = f'(x)\, a^{f(x)} \ln a$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}\left[\ln f(x)\right] = \frac{f'(x)}{f(x)}$$

$$\frac{d}{dx}(\log_a x) = \frac{1}{x}\log_a e$$

$$\frac{d}{dx}\left[\log_a f(x)\right] = \frac{f'(x)}{f(x)}\log_a e$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\sin^n x) = n(\sin^{n-1} x)\cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\cos^n x) = -n(\cos^{n-1} x)\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\tan^n x) = n(\tan^{n-1} x)\sec^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\sec^n x) = n(\sec^{n-1} x)\sec x \tan x$$
$$= n\sec^n x \tan x$$

$$\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$$

$$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$$

$$\frac{d}{dx}\left[\sin^{-1} f(x)\right] = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$$

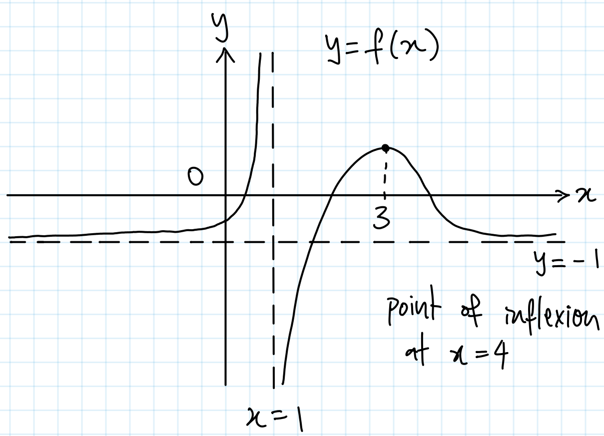
$$\frac{d}{dx}\left[\cos^{-1} f(x)\right] = \frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$$

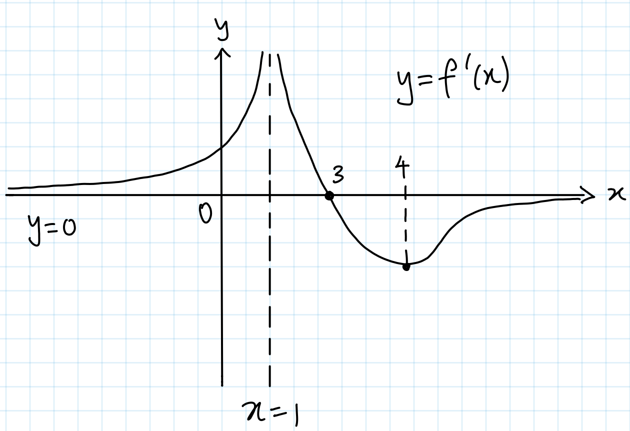
$$\frac{d}{dx}(x^2 y) = x^2\frac{dy}{dx} + y(2x)$$

$$\frac{d}{dx}\left[\tan^{-1} f(x)\right] = \frac{f'(x)}{1+[f(x)]^2}$$

Graph of $y = f'(x)$

	Graph of $y = f(x)$		Graph of $y = f'(x)$
	Mathematical Feature	Geometrical Feature	
1.	Stationary points i.e., $\frac{dy}{dx} = 0$ at $x = a$	Max, Min or stationary point of inflexion at $x = a$	**Intercept at $x = a$**
2.(a)	Vertical asymptote at $x = p$		**Vertical asymptote at $x = p$**
2.(b)	Horizontal asymptote at $y = q$		**Horizontal asymptote at $y = 0$**
3.	Start sketching the graph of $y = f'(x)$ from left to right, using the following checks:		
(i)(a)	$\frac{dy}{dx} > 0$ in interval (a,b)	Curve increasing in interval (a,b)	**Curve above x-axis**
(b)	$\frac{dy}{dx} < 0$ in interval (a,b)	Curve decreasing in interval (a,b)	**Curve below x-axis**
(ii)(a)	$\frac{d^2y}{dx^2} > 0$ in interval (a,b)	Curve is concave upwards in interval (a,b)	**Curve is increasing in interval (a,b)**
(b)	$\frac{d^2y}{dx^2} < 0$ in interval (a,b)	Curve is concave downwards in interval (a,b)	**Curve is decreasing in interval (a,b)**
(iii)	Change in curvature of curve at $x = k$		**There is a turning point (max/min) at $x = k$**





Integration

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$$

$$\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{n+1} + C$$

$$\int f'(x)[f(x)]^n \, dx = \frac{[f(x)]^{n+1}}{n+1} + C$$

$$\int \frac{1}{x} \, dx = \ln|x| + C$$

$$\int \frac{1}{ax+b} \, dx = \frac{1}{a}\ln|ax+b| + C$$

$$\int \frac{f'(x)}{[f(x)]^n} \, dx = \ln|f(x)| + C$$

$$\int e^x \, dx = e^x + C$$

$$\int e^{ax+b} \, dx = \frac{1}{a} e^{ax+b} + C$$

$$\int f'(x) e^{f(x)} \, dx = e^{f(x)} + C$$

$$\int a^x \, dx = \frac{a^x}{\ln a} + C$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \sec x \tan x \, dx = \sec x + C$$

$$\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + C$$

$$\int \tan x \, dx = \ln|\sec x| + C$$

$$\int \cot x \, dx = \ln|\sin x| + C$$

$$\int \operatorname{cosec} x \, dx = -\ln|\operatorname{cosec} x + \cot x| + C$$

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C$$

$$\int \frac{1}{x^2+a^2} \, dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{\sqrt{a^2-x^2}} \, dx = \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{x^2-a^2} \, dx = \frac{1}{2a}\ln\left(\frac{x-a}{x+a}\right) + C$$

$$\int \frac{1}{a^2-x^2} \, dx = \frac{1}{2a}\ln\left(\frac{a+x}{a-x}\right) + C$$

By parts:

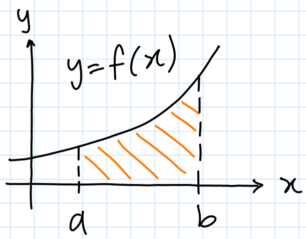
$$\int \left(u\frac{dv}{dx}\right) dx = uv - \int v\frac{du}{dx} \, dx$$

LIATE

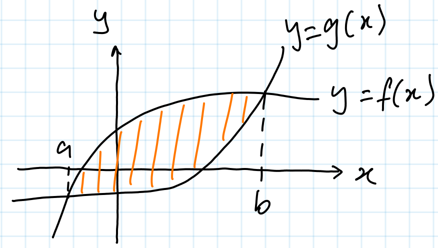
↳ choice of u.

Definite Integrals

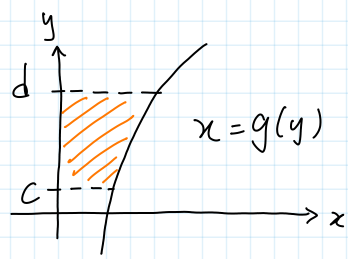
Area



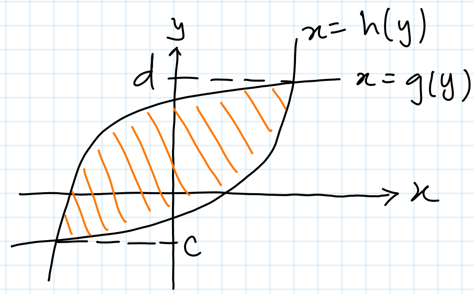
Area $= \int_a^b f(x)\,dx$



Area $= \int_a^b [f(x) - g(x)]\,dx$



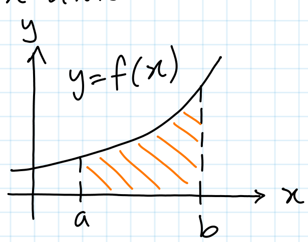
Area $= \int_c^d g(y)\,dy$



Area $= \int_c^d [g(y) - h(y)]\,dy$

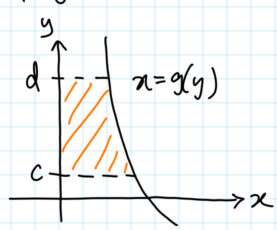
Volume

About x-axis

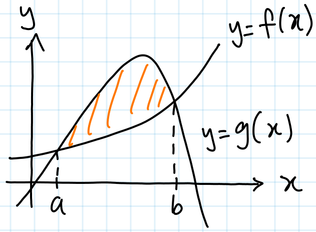


Volume $= \pi \int_a^b y^2\,dx$

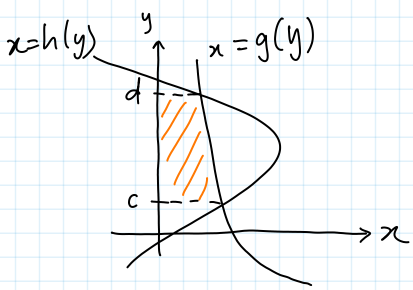
About y-axis



Volume $= \pi \int_c^d x^2\,dy$



Volume $= \pi \int [g(x)]^2 - [f(x)]^2\,dx$



Volume $= \pi \int_c^d [h(y)]^2 - [k(y)]^2\,dy$

Maclaurin Series

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \ldots + \frac{x^n}{n!} f^{(n)} 0 + \ldots$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!} x^r + \ldots \qquad |x|<1$$

$$(1+x)^{-1} = 1 - x + x^2 - x^3 + \ldots + (-1)^r x^r + \ldots \qquad |x|<1$$

$$(1-x)^{-1} = 1 + x + x^2 + x^3 + \ldots + x^r + \ldots \qquad |-x|<1$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!} + \ldots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4} - \frac{x^6}{6!} + \ldots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots \qquad (-1 < x \leq 1)$$

Convergence & Approximation

① Compare 2 series of a $f(x)$ consisting diff. no. of terms

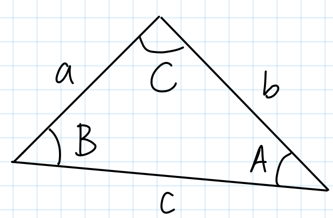
↳ series with a greater no. of terms ⇒ better approximation

② Better approximation if x value is closer to 0.

Small angle approximation

$\sin kx \approx x$ $\qquad$ $\cos kx \approx 1 - \frac{1}{2}(kx)^2$ $\qquad$ $\tan kx \approx kx$

Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$



Cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

Differential Equations

① Direct method

Eg: $\frac{dy}{dx} = \frac{3x}{3x^2+1}$

$y = \frac{3}{2}\int \frac{2x}{x^2+1}dx$

$= \frac{3}{2}\ln(x^2+1) + C$

$\frac{dy}{dx} = f(x)$

② Separation method

Eg: $\frac{du}{dt} = 16 - 9u^2$

$\int \frac{1}{16-9u^2}du = \int 1\, dt$

$\frac{1}{3}\int \frac{1}{4^2-(3u)^2}du = \int 1\, dt$

$\frac{1}{3}\left[\frac{1}{2(4)}\ln\left|\frac{4+3u}{4-3u}\right|\right] = t + C$

$\frac{1}{24}\ln\left|\frac{4+3u}{4-3u}\right| = t + C$

$\frac{dy}{dx} = f(y)$

$\int \frac{1}{f(y)}dy = \int 1\, dx$

$\frac{d^2y}{dx^2} = f(x)$

$\frac{dy}{dx} = \int f(x)\, dx$

$\frac{dy}{dx} = F(x) + C$

$y = \int F(x) + C\ dx$

③ Integrate RHS twice

Eg: $\frac{d^2y}{dx^2} = 16 - 9x^2$

$\frac{dy}{dx} = 16x - 3x^3 + C$

$y = \int 16x - 3x^3 - C\ dx$

$= 8x^2 - \frac{3}{4}x^4 - Cx + d \longrightarrow$ 2 constants: c & d

$\frac{d^2y}{dx^2} = f''(x)$

$\frac{dy}{dx} = f'(x) + C$

$y = f(x) + Cx + d$

④ Substitution (given in question)

Eg: $\frac{dy}{dx} = \frac{1+x+y}{1-x-y}$ } show that $\frac{du}{dx} = \frac{2}{1-u}$

sub $u = x+y$, $\frac{du}{dx} = 1 + \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{du}{dx} - 1$

$\frac{du}{dx} - 1 = \frac{1+u}{1-u}$

$\frac{du}{dx} = \frac{1+u}{1-u} + 1$

$= \frac{1+u+1-u}{1-u}$

$= \frac{2}{1-u}$ [shown!]

$\int(1-u)\,du = \int 2\, dx$

$u - \frac{1}{2}u^2 = 2x + C$

Rate change = Rate ↑ − Rate ↓

Complex Numbers

Cartesian form: $z = x + iy$

$i = \sqrt{-1}$

$\overrightarrow{OP} = x\underset{\sim}{i} + y\underset{\sim}{j}$

$|z| = r = \sqrt{x^2 + y^2}$

$\arg(z) = \theta$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

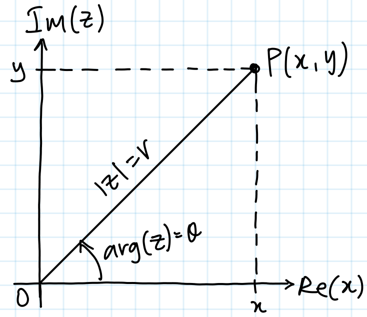
$|z_1 \cdot z_2| = r_1 r_2$
$= |z_1||z_2|$

$\arg(z_1 \cdot z_2) = \theta_1 + \theta_2$
$= \arg(z_1) + \arg(z_2)$

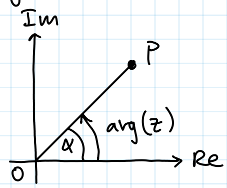
$\left|\frac{z_1}{z_2}\right| = \frac{r_1}{r_2}$
$= \frac{|z_1|}{|z_2|}$

$\arg(z^n) = n \arg(z)$

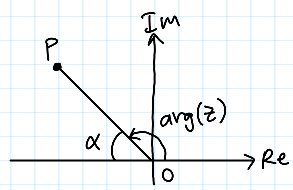
$\arg\left(\frac{z_1}{z_2}\right) = \theta_1 - \theta_2$
$= \arg(z_1) - \arg(z_2)$



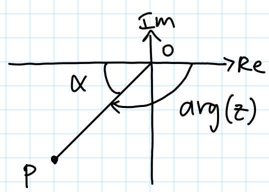
argument in 4 quadrants



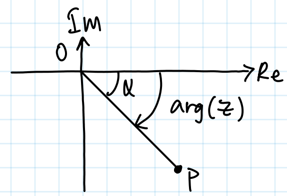
$\arg(z) = \alpha$



$\arg(z) = \pi - \alpha$



$\arg(z) = -(\pi - \alpha)$
$= \alpha - \pi$



$\arg(z) = -\alpha$

Conjugate:

$zz^* = x^2 + y^2$

$z + z^* = 2\,\mathrm{Re}(z)$

$z - z^* = 2i\,\mathrm{Im}(z)$

$(z^*)^* = z$

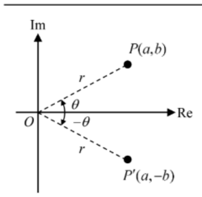
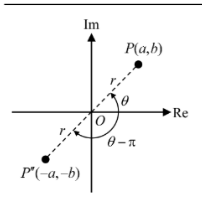
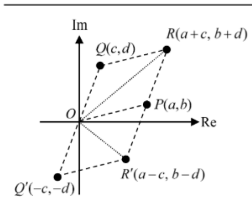
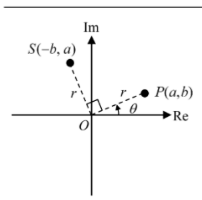
$(kz)^* = kz^*$

$(z_1 \pm z_2)^* = z_1^* \pm z_2^*$

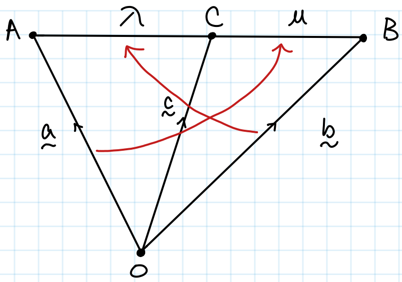
$(z^n)^* = (z^*)$

$(z_1 z_2)^* = z_1^* \cdot z_2^*$

$\left(\frac{z_1}{z_2}\right)^* = \frac{z_1^*}{z_2^*}$

	**Conjugation** • $z^* = a - ib$ represented by point $P'(a, -b)$ • Point P' is a reflection of point P in the real axis • $\lvert z^*\rvert = \lvert z\rvert = r$ • $\arg(z^*) = -\arg(z) = -\theta$
	**Negation** • $-z = -a - ib$ represented by point $P''(-a, -b)$ • Point P'' is a reflection of point P about the origin • $\lvert -z\rvert = \lvert z\rvert = r$ • $\arg(-z) = \arg(z) - \pi = \theta - \pi$
	**Addition and Subtraction** • $z + w = (a + c) + i(b + d)$ represented by point $R(a + c, b + d)$ • $z - w = (a - c) + i(b - d)$ represented by point $R'(a - c, b - d)$
	**Multiplication by i** • $iz = -b + ia$ represented by point $S(-b, a)$ • Point S is obtained by rotating point P anticlockwise about the origin by $\frac{\pi}{2}$ radians • $\lvert iz\rvert = \lvert z\rvert = r$ • $\arg(iz) = \arg(z) + \frac{\pi}{2} = \theta + \frac{\pi}{2}$

Vectors 1



$\vec{PQ} = \underset{\sim}{a}$

$|\vec{PQ}| = |\underset{\sim}{a}|$

$\hat{\underset{\sim}{a}} = \dfrac{\underset{\sim}{a}}{|\underset{\sim}{a}|}$ } unit vector

$\underset{\sim}{a} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$|\underset{\sim}{a}| = \sqrt{x^2+y^2+z^2}$

$\vec{OC} = \dfrac{\mu \vec{OA} + \lambda \vec{OB}}{\mu + \lambda}$

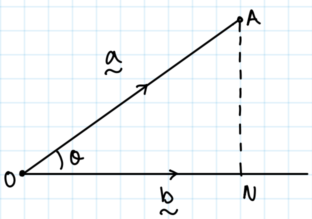
DOT PRODUCT

$\underset{\sim}{a} \cdot \underset{\sim}{b} = |\underset{\sim}{a}||\underset{\sim}{b}| \cos\theta$

properties: $\underset{\sim}{a} \cdot \underset{\sim}{b} = 0$ $(\underset{\sim}{a} \perp \underset{\sim}{b})$

$\underset{\sim}{a} \cdot \underset{\sim}{b} = 1$ $(\underset{\sim}{a} \parallel \underset{\sim}{b})$

$\underset{\sim}{a} \cdot \underset{\sim}{a} = |\underset{\sim}{a}|^2$



Length of projection of $\underset{\sim}{a}$ on $\underset{\sim}{b}$ $(|\vec{ON}|)$

$= |\underset{\sim}{a} \cdot \hat{\underset{\sim}{b}}|$

Projection vector of $\underset{\sim}{a}$ on $\underset{\sim}{b}$ $(\vec{ON})$

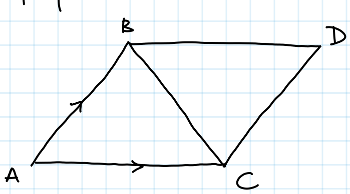
$= (\underset{\sim}{a} \cdot \hat{\underset{\sim}{b}})(\hat{\underset{\sim}{b}}) = \dfrac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{|\underset{\sim}{b}|^2}(\underset{\sim}{b})$

CROSS PRODUCT

$\underset{\sim}{a} \times \underset{\sim}{b} = |\underset{\sim}{a}||\underset{\sim}{b}| \sin\theta \ \hat{\underset{\sim}{n}}$

Properties: $\underset{\sim}{a} \times \underset{\sim}{b} = 0$ $(\underset{\sim}{a} \parallel \underset{\sim}{b})$

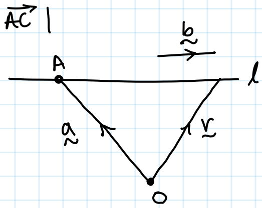
$\underset{\sim}{a} \times \underset{\sim}{a} = 0$



Area of $\triangle ABC = \frac{1}{2}|\vec{AB} \times \vec{AC}|$

Area of ▱$ABDC = |\vec{AB} \times \vec{AC}|$

Equation of a line:



Vector: $l : \underset{\sim}{r} = \underset{\sim}{a} + \lambda \underset{\sim}{b}$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + \lambda \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Parametric: $x = a_1 + \lambda b_1$

$y = a_2 + \lambda b_2$

$z = a_3 + \lambda b_3$

Cartesian: $\lambda = \dfrac{x - a_1}{b_1} = \dfrac{y - a_2}{b_2} = \dfrac{z - a_3}{b_3}$

Relationship between 2 lines

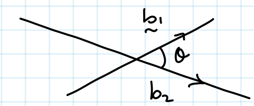
① Parallel: $\underset{\sim}{b}_1 = k \underset{\sim}{b}_2$

② Non-parallel & intersecting: $\underset{\sim}{a}_1 + \lambda \underset{\sim}{b}_1 = \underset{\sim}{a}_2 + \mu \underset{\sim}{b}_2$ with unique λ and μ values.

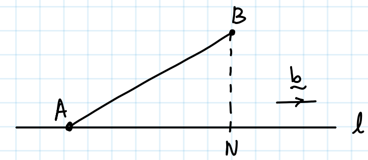
③ Non-parallel & non-intersecting (skew): $\underset{\sim}{a}_1 + \lambda \underset{\sim}{b}_1 = \underset{\sim}{a}_2 + \mu \underset{\sim}{b}_2$ with <u>NO</u> unique λ and μ values.

Angle between 2 lines

$$\theta = \cos^{-1}\left[\frac{|\underset{\sim}{b}_1 \cdot \underset{\sim}{b}_2|}{|\underset{\sim}{b}_1||\underset{\sim}{b}_2|}\right]$$



Foot of ⊥ from point to line:



① Dot product of ⊥ vectors

$\vec{BN} \cdot \underset{\sim}{b} = 0$

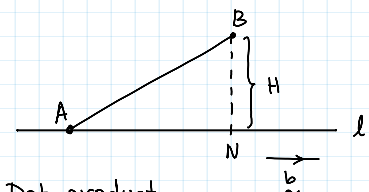
$\vec{BN} = \vec{ON} - \vec{OB}$

② Projection vector

$\vec{AN} = (\vec{AB} \cdot \hat{\underset{\sim}{b}})(\hat{\underset{\sim}{b}})$

$\vec{ON} = \vec{OA} + \vec{AN}$

⊥ distance from a point to a line:



① Dot product

$\vec{BN} \cdot \underset{\sim}{b} = 0$

$H = |\vec{BN}|$

② Cross product

$\sin\theta = \dfrac{|\vec{BN}|}{|\vec{AB}|}$

$|\vec{BN}| = |\vec{AB}| \sin\theta$

$= |\vec{AB}||\hat{\underset{\sim}{b}}| \sin\theta \ |\hat{\underset{\sim}{n}}|$

$= |\vec{AB} \times \hat{\underset{\sim}{b}}|$

③ Length of projection

$\vec{AN} = (\vec{AB} \cdot \hat{\underset{\sim}{b}})(\hat{\underset{\sim}{b}})$

$|\vec{BN}| = \sqrt{|\vec{AB}|^2 - |\vec{AN}|^2}$

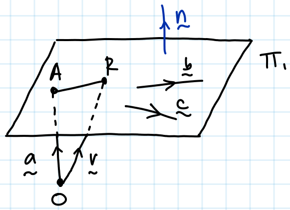
Vectors 2

Equations of a plane

Parametric:

$\Pi_1 = \underset{\sim}{a} + \lambda \underset{\sim}{b} + \mu \underset{\sim}{c}$ direction

$\underset{\sim}{n} = \underset{\sim}{b} \times \underset{\sim}{c}$



Scalar-product:

$\underset{\sim}{r} \cdot \underset{\sim}{n} = \underbrace{\underset{\sim}{a} \cdot \underset{\sim}{n}}_{\text{constant}}$ $\xrightarrow{\div |\underset{\sim}{n}|}$ standard: $\underset{\sim}{r} \cdot \hat{\underset{\sim}{n}} = d$ ↳ $|d|$ ⊥ from origin to plane

Cartesian form: $\underset{\sim}{r} \cdot \underset{\sim}{n} = D$

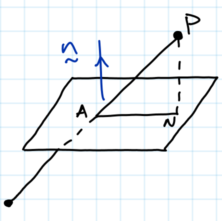
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = D$$

$$ax + by + cz = D$$

Shortest distance from plane to origin:

$$\left| \frac{D}{|\underset{\sim}{n}|} \right|$$

☆ Foot of ⊥ from a point to plane:



$\Pi_1 = \underset{\sim}{r} \cdot \underset{\sim}{n} = D$

① Intersection between line and plane

$l_{\overrightarrow{PN}} : \underset{\sim}{r} = \overrightarrow{OP} + \lambda \underset{\sim}{n}$, $\overrightarrow{ON} = \overrightarrow{OP} + \lambda \underset{\sim}{n}$

$\overrightarrow{PN} \cdot \underset{\sim}{n} = D$ for some λ

Find λ sub into $\overrightarrow{ON}$

$|\overrightarrow{PN}|$ is the ⊥ from P to plane

② Projection vector

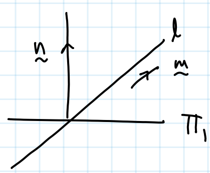
$\overrightarrow{PN}$ is the projection vector of $\overrightarrow{PA}$ on $\underset{\sim}{n}$

$\overrightarrow{PN} = (\overrightarrow{PA} \cdot \hat{\underset{\sim}{n}})(\hat{\underset{\sim}{n}})$

$\overrightarrow{PN} = \overrightarrow{ON} - \overrightarrow{OP}$

$|\overrightarrow{PN}| = |\overrightarrow{PA} \cdot \hat{\underset{\sim}{n}}|$

∡ between a line and a plane



$$\theta = \frac{\pi}{2} - \cos^{-1}\left[\frac{|\underset{\sim}{m} \cdot \underset{\sim}{n}|}{|\underset{\sim}{m}||\underset{\sim}{n}|}\right]$$

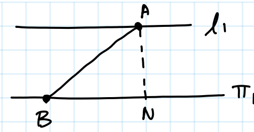
$$= \sin^{-1}\left[\frac{|\underset{\sim}{m} \cdot \underset{\sim}{n}|}{|\underset{\sim}{m}||\underset{\sim}{n}|}\right]$$

Determine if line resides in a plane

$\underset{\sim}{r} = \underset{\sim}{a} + \lambda \underset{\sim}{m}$ resides in plane $\underset{\sim}{r} \cdot \underset{\sim}{n} = D$ if

(i) $\underset{\sim}{m} \cdot \underset{\sim}{n} = 0$ (ii) $\underset{\sim}{a} \cdot \underset{\sim}{n} = D$

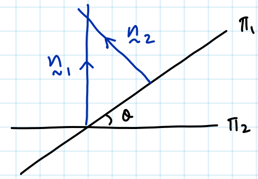
Shortest distance between line & plane



• refer to ☆

∡ between 2 planes

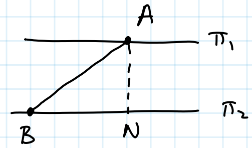
$$\theta = \cos^{-1}\left[\frac{|\underset{\sim}{n}_1 \cdot \underset{\sim}{n}_2|}{|\underset{\sim}{n}_1||\underset{\sim}{n}_2|}\right]$$



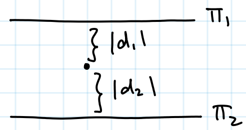
Distance between 2 planes

① • Find a point on Π_1 & Π_2

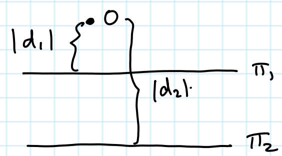
• Refer to ☆



②



(OR)



• $\underset{\sim}{r} \cdot \hat{\underset{\sim}{n}} = d$

• Find $|d|$ for Π_1 and Π_2 then add or subtract accordingly.

• If d is negative ⇒ plane below origin.

$\left|\frac{d_1}{|\underset{\sim}{n}|}\right| + \left|\frac{d_2}{|\underset{\sim}{n}|}\right|$ if d_1 & d_2 of diff. signs

$\left|\frac{d_2}{|\underset{\sim}{n}|}\right| - \left|\frac{d_2}{|\underset{\sim}{n}|}\right|$ if d_1 & d_2 of same signs

Intersection between 2 planes

• 2 planes intersect in a line l.

① Find cartesian for Π_1 & Π_2 then equate.

$\Pi_1 : x - y + z = 4$
$\Pi_2 : x - y - 3z = 8$ } Solve for $x/y/z$ in terms of 1 variable (eg. y)

$x - y + z = 4$
$x = y - z + 4$
$\cancel{y} - z - \cancel{y} - 3z + 4 = 8$
$-4z = 4$
$z = -1$

⇓

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5+y \\ y \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ -1 \end{pmatrix} + y \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\therefore l_1 : \underset{\sim}{r} = \begin{pmatrix} 5 \\ 0 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

② Substitute Π_1 into Π_2

$\Pi_1 : \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix} + \alpha \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + \beta \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$ $\Pi_2 : \underset{\sim}{r} \cdot \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = 8$

$$\begin{pmatrix} 3 + 2\alpha - \beta \\ 3 + \alpha + \beta \\ 4 - \alpha + 2\beta \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} = 8 \Rightarrow \alpha + 2\beta = 5$$

Sub $\alpha + 2\beta$ into Π_1 $\longrightarrow$ $\underset{\sim}{r} = \begin{pmatrix} 13 \\ 8 \\ -1 \end{pmatrix} + \beta \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix}$

$$\therefore l_1 : \underset{\sim}{r} = \begin{pmatrix} 13 \\ 8 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix}$$

DRV

$$E(X) = \Sigma x P(X = x)$$

$$E(X_1 + X_2 + \dots + X_n) = n E(X)$$

$$E(X^2) = \Sigma x^2 P(X = x)$$

$$Var(X) = E(X^2) - [E(X)]^2$$

$$\sigma = \sqrt{Var(X)}$$

$$E(aX \pm b) = a E(X) \pm b$$

$$Var(aX \pm b) = a^2 Var(X)$$

Binomial Distribution

* only $\boxed{2}$ possible outcomes

$E(X) = np$

$X \sim B(n, p)$

$\bar{X} \sim N(np, npq)$

$Var(X) = np(1-p)$
$= npq$

$np > 5$ & $npq > 5$
$\Rightarrow$ can be approximated by a normal distribution.

$$P(X = r) = \binom{n}{r} p^r (1-p)^{n-r}$$

Assumptions: 1) n independent repeated trials

2) Probability of "success" of each trial remains constant (denoted by p).

GC:

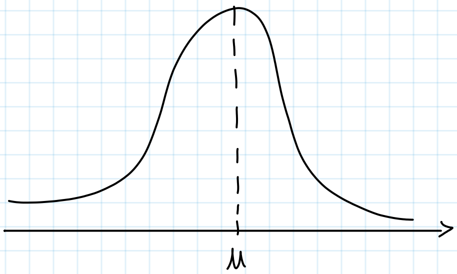
- Probability
  - binompdf for $P(X=r)$
  - binomcdf for $P(X \le r)$

$P(2 < X < 9) = P(3 \le X \le 8)$
$= P(X \le 8) - P(X \le 2)$

Given	In G.C, use	Graph/Table
$X \sim B(n, \frac{1}{20})$, $P(X \le 10) \ge 0.995$	$Y = \text{binomcdf}\left(X, \frac{1}{20}, 10\right)$	Use '**table**' (since n is a positive integer)
$X \sim B(10, p)$, $P(X = 3) = 0.2013$	$Y = \text{binompdf}(10, X, 3)$	Use '**graph**' - set window, Xmin, Xmax, Ymin, Ymax to between 0 and 1 (since p is a probability)
$X \sim B(18, 0.1)$, $P(X < r) > 0.9$	$Y = \text{binomcdf}(18, 0.1, X-1)$	Use '**table**' (since r is a positive integer)

Normal Distribution

$$X \sim N(\mu, \sigma^2)$$



$z = \frac{X - \mu}{\sigma}$ to find μ or σ

invNorm: $P(X<a)$ Left $P(|X-\mu|<c)$ Center $P(X>a)$ Right

$$aX \pm bY \sim N(a\mu_x \pm b\mu_y,\ a^2\sigma_x^2 + b^2\sigma_y^2)$$

$$X_1 + X_2 + \dots + X_n \sim N(n\mu, n\sigma^2)$$

Sampling

$$\sigma^2 = \frac{n-1}{n} s^2$$

Estimation:

Unbiased estimate of population mean

$$\bar{x} = \frac{1}{n}\Sigma x$$
$$= \frac{1}{n}\Sigma(x-c) + c$$

Unbiased estimate of population variance

$$s^2 = \frac{n}{n-1}\sigma^2$$
$$= \frac{1}{n-1}\left[\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right]$$
$$= \frac{1}{n-1}\left[\Sigma(x-c)^2 - \frac{(\Sigma(x-c))^2}{n}\right]$$
$$= \frac{1}{n-1}\left[\Sigma(x-\bar{x})^2\right]$$

Distribution:

	Normal Population	**Non-Normal/Unknown distribution Population**
Known population variance σ^2	$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ *and* $X_1 + X_2 + \dots + X_n \sim N(n\mu, n\sigma^2)$ for any sample size n	$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ approx. *and* $X_1 + X_2 + \dots + X_n \sim N(n\mu, n\sigma^2)$ approx. by **Central Limit Theorem** where $n \geq 30$
Unknown population variance	$\bar{X} \sim N\left(\mu, \frac{s^2}{n}\right)$ approx. where n is large* **In our syllabus, if the population variance is unknown, the sample size n will be large.*	$\bar{X} \sim N\left(\mu, \frac{s^2}{n}\right)$ approx. by **Central Limit Theorem** where $n \geq 30$

Chapter 11: Permutations and Combinations

Counting principles	*Addition principle*: If there are two or more **non–overlapping** categories of ways to perform an operation, and there are m ways in the first category, n ways in the second category and k ways in the last category, there are **($m + n + \ldots + k$)** to perform it.	*Multiplication principle*: If an operation A can be completed in m ways, a second operation B performed in n ways and the last operation in k ways, the successive operations can be completed in $m \times n \times \ldots \times k$ ways.		
Permutations	A *permutation* is an **ordered arrangement of objects** mainly concerned to find the **total number of ways to arrange objects**.	No. of ways to arrange n distinct objects in a straight line…		$n!$
		…where p of them are **identical**.	Note: If there are 2 objects that **must be together**, treat them as a **single unit** and multiply $2!$. If objects must be separated, place them between the **gaps** of the row.	$\frac{n!}{p!}$
		…and q of them are **identical**.		$\frac{n!}{p!q!}$
		No. of ways to arrange r objects out of n **distinct** objects in a straight line		${}^{n}\mathrm{P}_{r}$
Combinations	A *combination* is an **unordered selection** of a **number of objects** from a **given set**.	No. of ways to choose r objects from n distinct objects		${}^{n}\mathrm{C}_{r}$
		If r different balls are distributed to n different urns (such that any urn can contain any number of balls), then number of outcomes…		2^{n}
		Note: When **restrictions** are given, they must be satisfied first before the number of combinations is selected. This can be done by breaking down into different cases, calculating separately and adding/subtracting them. Some useful techniques: **1.** Complementary technique **2.** Grouping technique (order must be taken into account) **3.** Insertion technique (order must be taken to account – use for 3 or more objects) Note: When subdividing into groups of **equal number**, remember to divide by the **number of groups**.		
Permutations in a circle	No. of ways of arranging n distinct objects in a circle if:			
	The positions are **indistinguishable**: $\frac{n!}{n} = (n-1)!$	The positions are **distinguishable**: $n! = n \times (n-1)!$		
	Note that seats become distinguishable if seats are **numbered**, **different colour** and **different shapes/sizes**.			

Chapter 12: Probability

Probability of an event	$\mathrm{P}(A) = \frac{\text{No. of outcomes event occurs}}{\text{No. of possible outcomes}} = \frac{\mathrm{n}(A)}{\mathrm{n}(S)}$				
Complimentary Events	$\mathrm{P}(A) + \mathrm{P}(A') = 1$				
Addition Law	$\mathrm{P}(A \text{ or } B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \text{ and } B)$	$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$			
Mutually exclusive events	2 events are mutually exclusive if **they cannot occur together** (within a situation)				
	$\mathrm{P}(A \cap B) = 0$	$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$	$\mathrm{P}(A \mid B) = 0$ or $\mathrm{P}(B \mid A) = 0$		
Conditional Probability	$\mathrm{P}(B \mid A) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(A)}$	$\mathrm{P}(A \mid B) = \frac{\mathrm{P}(B \cap A)}{\mathrm{P}(B)}$	If A and B are mutually exclusive, $\mathrm{P}(A \mid B)$=0 (Since B occurs, A wont occur, vice versa)		
Independent Events	2 events are independent if the occurrence of one does not affect the other (between diff. situations)				
	$\mathrm{P}(A \mid B) = \mathrm{P}(A)$	$\mathrm{P}(B \mid A) = \mathrm{P}(B)$	$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B)$ *		
Other useful approaches	Few stages with few outcomes: **Tree diagram**	Diff. probabilities for diff. events: **Venn diagram**	Small sample space: **Table of outcomes**	P&C method: **Only WITHOUT** replacement	**Sequences and Series**: For turn-by-turn situations
Tree diagram	How to use a tree diagram: **1.** Multiply the probabilities along the branches to get the end results **2.** On any set of branches that meet up at a point, the probabilities must **add up to 1**. **3.** Check that all end results add up to 1. **4.** To answer any question, find the relevant end results. If more than one satisfy the requirements, **add** these end results together.				
Other formulas	$\mathrm{P}(A \cup B) = 1 - \mathrm{P}(A' \cap B')$	$\mathrm{P}(A \cap B) = 1 - \mathrm{P}(A' \cup B')$	$\mathrm{P}(A) = \mathrm{P}(A \cap B) + \mathrm{P}(A \cap B')$	$\mathrm{P}(A' \mid B) + 1 - \mathrm{P}(A \mid B)$	
Notes	"and" means $\cap$ (i.e. multiply) whereas "or" means $\cup$ (i.e. add)				

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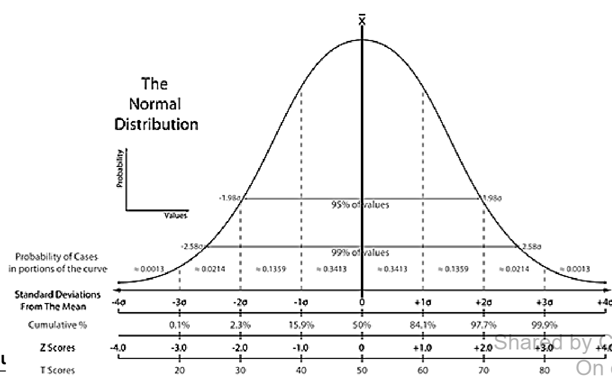
Chapter 16: Hypothesis Testing

Important definitions	H_0 (null hypothesis)	Particular **claim** for a value for the **population mean.**	In hypothesis testing, we try to **reject H_0** as far as possible as it is a more reliable claim compared to **do not reject H_0.**
	H_1 (alternative hypothesis)	Range of values that **excludes** the value specified by null hypothesis.	
	Test statistic	Random variable whose value is calculated from **sample data.**	
	Critical region	Range of values of the test static that leads to the **rejection of H_0** The value of c which determines the critical region is known as the *critical value*.	
	Level of significance, α	Probability of rejecting H_0 given that H_0 is true. (usually around 0.05) • If a result is **significant** at α%, then it is **also significant** at any level **greater** than α%. • If a result is **not significant** at α%, then it is also **not significant** at any level < α%. • The p–value is the **smallest level of significance** at which H_0 can be rejected.	
	p-value	Probability of observing a value of the test statistic as **extreme** or **more extreme** than the one obtained, given that H_0 is true.	p-value $\le \alpha$: Test statistic lies in the critical region, hence we **reject H_0.** p-value $> \alpha$: Test statistic does not lie in the critical region, hence we **do not reject H_0.**
Hypothesis tests	Types of tests	***Left-tail test***: A change in the **decrease direction**. H_1: $\mu < \mu_0$ ***Right-tail test***: A change in the **increase** direction. H_1: $\mu > \mu_0$	***2-tail test***: A difference in **either direction.** H_1: $\mu \neq \mu_0$. Note that α (and p-value) is **divided equally** between the 2 tails of the critical region.
	z-test	• True when population distribution is **Normal** and the population variance is known this holds for **any sample size**, large or small. • Also true when population size is **large** when population distribution is **unknown**. When n is large >50, by **central limit theorem**, … Under H_0, $\bar{X} \sim N\left(\mu_0, \frac{\sigma^2}{n}\right)$ (approximately), test statistic $Z = \frac{\bar{X}-\mu_0}{\sqrt{\sigma^2/n}}$ (approximately).	However, when population *variance* is **unknown**, we use s^2 instead of σ^2. Under H_0, $\bar{X} \sim N\left(\mu_0, \frac{s^2}{n}\right)$ approximately, test statistic $Z = \frac{\bar{X}-\mu_0}{\sqrt{s^2/n}}$ approximately where $s^2 = \frac{1}{n-1}\left[\sum x - \frac{\left(\sum x\right)^2}{n}\right] = \frac{1}{n-1}\sum (x-\bar{x})^2$
	T-test	Use when n is small and population variance is unknown. Given (or assume) that the population distribution is **normal**, Under H_0, $\bar{X} \sim N\left(\mu_0, \frac{\sigma^2}{n}\right)$ approximately, test statistic $T = \frac{\bar{X}-\mu_0}{\sqrt{S^2/n}} \sim t(n-1)$.	
General hypothesis testing procedure	***Critical region method*** (if $\bar{x}, \mu_0$ or n is unknown)	1. Write down H_0 and H_1. 2. Determine an approximate *test statistic* and its distribution. 3. Identify the *level of significance*, α (from qn) 4. Determine the *critical region*. **Multiply** by 2 if there is a 2-tail test. 5. Compute the *test statistic* (using G.C.) 6. Determine if the test static lies *within the critical region*, and state conclusion **in context of the problem**. (since z = ____ (< or >) _____, we (reject / do not reject H_0 at ___% of significance and conclude that there is sufficient evidence that …)	
	p-value method	1. Write down H_0 and H_1. 2. Determine an approximate *test statistic* and its distribution. 3. Identify the *level of significance*, α (from qn) 4. Compute the *test statistic* (using G.C.) 5. Find the p-value (from G.C.). **Multiply** by 2 if there is a 2-tail test. 6. Determine if the test static lies *within the critical region*, and state conclusion **in context of the problem**. (since p = ___ (< or >) α = ___, we (reject / do not reject H_0 at ___% of significance and conclude that there is sufficient evidence that …)	



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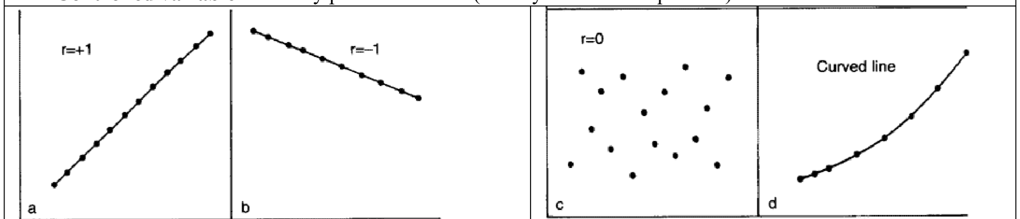
Chapter 17: Correlation and Regression

Scatter diagrams

- A sketch where each axis represents a **variable** and each point represents an **observation.**
- **Controlled variable** is usually plotted at x-axis (usually stated from question).



Positive linear correlation	**Negative** linear correlation	**No** linear correleation	**Non**-linear correleation

Product moment correlation coefficient

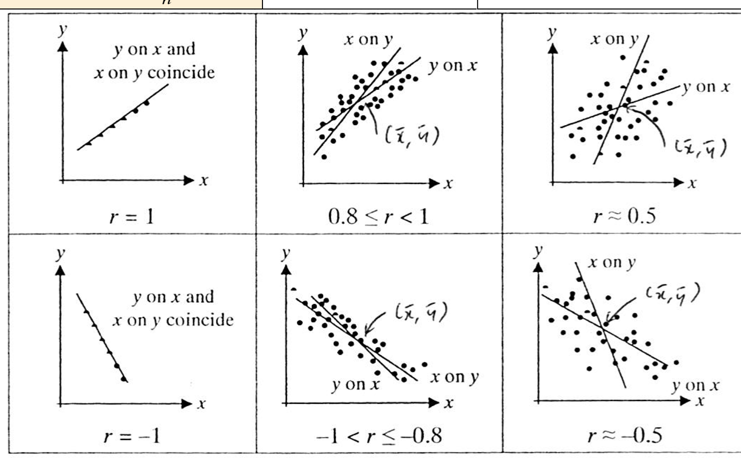
- The *product moment correleation coefficient*, denoted by r, measures the **strength and direction** of a **linear correleation** between the 2 variables.
- $-1 \le r \le 1$
- **Anomalies** are removed from a set of bivariate data to calculate an accurate value of r.
- There may not be direct **cause-effect relationship** between the 2 variables.

$$r = \frac{\sum (x-\bar{x})(y-\bar{y})}{\sqrt{\left\{\sum (x-\bar{x})^2\right\}\left\{\sum (y-\bar{y})^2\right\}}} = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sqrt{\left(\sum x^2 - \frac{\left(\sum x\right)^2}{n}\right)\left(\sum y^2 - \frac{\left(\sum y\right)^2}{n}\right)}}$$

Regression lines

The *linear regression* fits a straight line in the scatter diagram.

Least squares regression line of y on x		Least squares regression line of x on y
$y-\bar{y} = b(x-\bar{x})$ where $b = \frac{\sum (x-\bar{x})(y-\bar{y})}{\sum (x-\bar{x})^2} = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}$	Both regression lines intersect at $(\bar{x}, \bar{y})$	$x-\bar{x} = d(y-\bar{y})$ where $d = \frac{\sum (x-\bar{x})(y-\bar{y})}{\sum (y-\bar{y})^2} = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sum y^2 - \frac{\left(\sum y\right)^2}{n}}$



Choice of regression line

Case	*Estimate y given x*	*Estimate x given y*
x is controlled, y is random	Use y on x	
y is controlled, x is random	Use x on y	
Both x and y is random	Use y on x	Use x on y

Interpolation extrapolation

Interpolating: Estimation using a value **within the given range** of data.	*Extrapolation*: Estimation using a value **outside the given range** of data (unreliable and should be avoided)

Linear law

Non-linear model	*Variable Y*	*Variable X*
$y = a + bx^2$	y	x^2
$y = a + \frac{b}{x}$	y	$\frac{1}{x}$
$y = ax^b$	$\ln y$	$\ln x$

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