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YISHUN TOWN SECONDARY SCHOOL



PRELIMINARY EXAMINATION 2024 SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC ADDITIONAL MATHEMATICS PAPER 1 (4049/01)

DATE : 21 August 2024

DAY : Wednesday

DURATION : 2 h 15 min

MARKS : 90

READ THESE INSTRUCTIONS FIRST

Do not turn over the cover page until you are told to do so.

Write your name, class and class index number in the spaces at the top of this page.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question. The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question.

The total marks for this paper is 90.

	MARKS	
	OBTAINED	FULL
1		4
2		4
3		5
4		6
5		6
6		6
7		8
8		7
9		8
10		9
11		10
12		8
13		9
TOTAL		90

This question paper consists of **21** printed pages and **1** blank page.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

Answer all questions.

- 1** **(a)** Write down the turning point of the curve $y = -x^2 + 1$, stating clearly whether it is a maximum or minimum point. [2]
- (b)** In a clearly labelled diagram, sketch the graph of $y = -x^2 + 1$, indicating clearly the coordinates of the turning point and axial intercepts. [2]

- 2 A cylinder has radius $(2 - \sqrt{2})$ cm and volume $\pi(7\sqrt{8} - 18)$ cm³. Find, without using a calculator, the height of the cylinder, in cm, in the form $(a + b\sqrt{2})$, where a and b are integers. [4]

- 3 (a) Explain why the principal value of $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$ cannot be $-\frac{\pi}{4}$. [1]

- (b) Given that $\cos \theta = -p$, where $p > 0$, and θ is an obtuse angle. Without using a calculator and leaving your answers in terms of p , find

(i) $\cos\left(\frac{\pi}{2} - \theta\right)$, [2]

(ii) $\cot^2 \theta$. [2]

4 (a) Find the remainder when $4x^3 + 3x^2 + 11x - 3$ is divided by $x + 2$. [2]

(b) When a polynomial $g(x)$ is divided by $x - 1$ and $2x - 1$, the remainders are 1 and 2 respectively. Find the remainder when $g(x)$ is divided by $(x - 1)(2x - 1)$. [4]

5 (a) Prove the identity

$$\cos^2 A \cos^2 B - \sin^2 A \sin^2 B - \sin(A+B) \sin(A-B) = \cos 2A. \quad [4]$$

(b) Hence find the value of A for $-\frac{\pi}{2} \leq A \leq 0$ such that

$$\cos^2 A \cos^2 B - \sin^2 A \sin^2 B - \sin(A+B) \sin(A-B) = -0.241. \quad [2]$$

- 6** **(a)** The curve $\frac{x}{y} + y = 1$ and the line $x - 2y + 6 = 0$ intersect at the points A and B . Find the x -coordinate of A and of B . [3]

(b) Hence solve for the value(s) of x in the simultaneous equations

$$\frac{\lg x}{\lg y} + \lg y = 1,$$

$$\lg \frac{x}{y^2} = -6.$$

[3]

7 The equation of a curve is $y = \frac{\ln x^2}{x^2}$, where $x > 0$.

(a) Show that the gradient of the curve can be written as $\frac{p + q \ln x}{x^r}$, where p , q and r are integers. [2]

(b) Find the range of values of $\ln x$ for the graph to be decreasing. [3]

- (c) The curve passes through the x -axis at point A . Find the equation of the normal to the curve at A . [3]

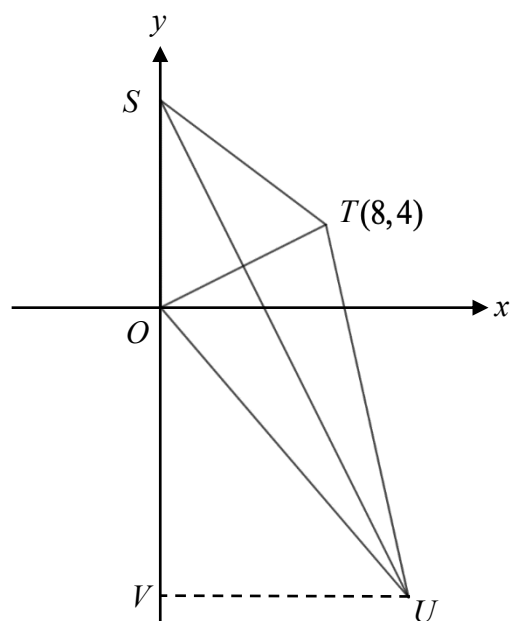
8 (a) Express $\frac{1-3x+10x^2}{x(1-2x)^2}$ in partial fractions. [4]

(b) Hence, find the integral of $\frac{1-3x+10x^2}{x(1-2x)^2}$. [3]

- 9 (a) Show that the solution of the equation $8^x = 3(16^{x+1})$ is $-\frac{\ln 48}{\ln 2}$. [4]

- (b) Express the equation $\log_2 \sqrt{x} + \log_{16}(x-1) = 1$ as a cubic equation in x . [4]

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The diagram shows a kite $OSTU$ in which $OS = TS$ and $OU = TU$. Point S lies on the y -axis, point T is $(8, 4)$ and VU is parallel to the x -axis.

(a) Find the equation of line SU .

[3]

(b) It is given that the coordinates of U are (h, k) .

(i) Express k in terms of h . [1]

(ii) Find the distance between S and U in terms of h . Give your answer in the form of $h\sqrt{a}$, where a is an integer. [3]

(c) In the case where $h = 12$, find the area of the quadrilateral $STUV$. [2]

11 (a) Find the value of k such that $\int_0^{\frac{\pi}{3}} (2 \sin^2 \theta - \sin \theta \cos \theta - 1) \, d\theta = -\frac{2\sqrt{3}+3}{k}$. [5]

(b) The function $f(x)$ and $g(x)$ are given by $f(x) = 3\sin 2x$ and $g(x) = 2\cos x$.

- (i) On the same axes, sketch the graphs of $y = f(x)$ and $y = g(x)$ for $0 \leq x \leq 360^\circ$.

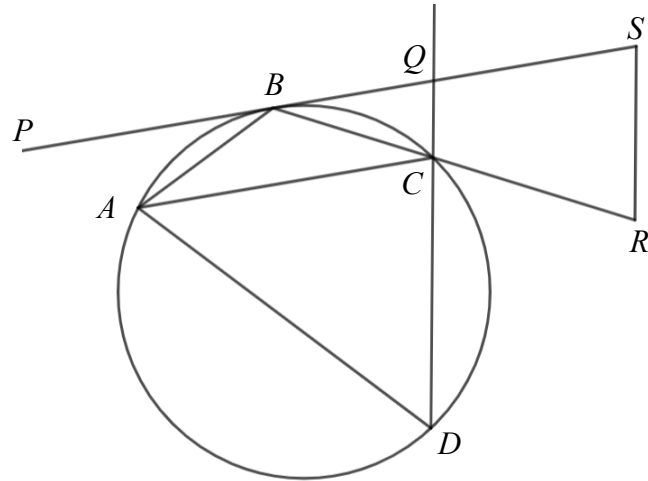
[4]



- (ii) Hence, find the number of solutions of the equation $\sin 2x - \frac{2}{3}\cos x = 0$ for $0 \leq x \leq 360^\circ$.

[1]

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In the diagram, A , B , C and D lie on a circle. PQS is the tangent to the circle at B . DQ and BR are straight lines that meet at C . SR is parallel to QC . Angle ADC is twice the size of angle QBC .

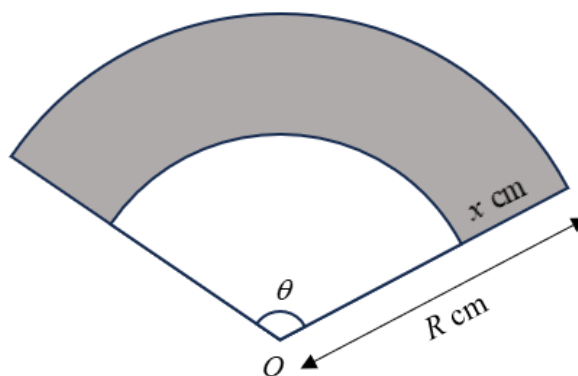
(a) Prove that $AB = BC$.

[4]

(b) Prove that $AB \times SR = BR \times QC$.

[4]

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$$[\text{Area of sector} = \frac{1}{2}r^2\theta]$$

Mr Rich owns a restaurant. His intention is to create a fan-shaped design for his restaurant, which can be modelled by a sector, centre O , with radius R cm. Part of the fan is to be painted gold, leaving an unpainted sector, also centre O , as shown in the diagram. The width of the strip of gold paint is x cm.

- (a) Show that the area, A cm², of the gold-painted region is given by $A = \frac{\theta}{2}(2Rx - x^2)$. [2]

It has been decided that the area of the gold-painted region is to be 880 cm^2 .

- (b) By making θ the subject, find x in terms of R which gives a stationary value of θ and determine its nature. [6]

- (c) Explain why the value of x obtained in part (b) fails to achieve the intended effect of the design of the fan. [1]

End of Paper

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