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YISHUN TOWN SECONDARY SCHOOL



PRELIMINARY EXAMINATION 2024 SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC ADDITIONAL MATHEMATICS PAPER 2 (4049/02)

DATE : 23 August 2024

DAY : Friday

DURATION : 2h 15min

MARKS : 90

READ THESE INSTRUCTIONS FIRST

Do not turn over the cover page until you are told to do so.

Write your name, class and class index number in the spaces at the top of this page.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question. The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total marks for this paper is 90.

	MARKS	
	OBTAINED	FULL
1		6
2		8
3		7
4		10
5		10
6		8
7		9
8		10
9		10
10		12
TOTAL		90

This question paper consists of **21** printed pages and **1** blank page.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

Answer all questions.

- 1 (a)** Given that the curve $y = (4 - k)x^2 + 6kx - 4k$ has a minimum value, find the range of values of k for which the line $y = kx$ meets the curve. [5]

- (b)** State the largest possible integer value of k . [1]

2 Given the following equation $4(9^x) = 15 + 4(3^{x+1}) + 4(3^{-x})$,

- (a)** Using the substitution $u = 3^x$, show that the equation can be written as $4u^3 - 12u^2 - 15u + k = 0$, where k is a constant.

[2]

- (b)** Using the value of k found in **(a)**, show that $2u + 1$ is a factor of $4u^3 - 12u^2 - 15u + k$.

[1]

(c) Hence, find the value(s) of x .

[5]

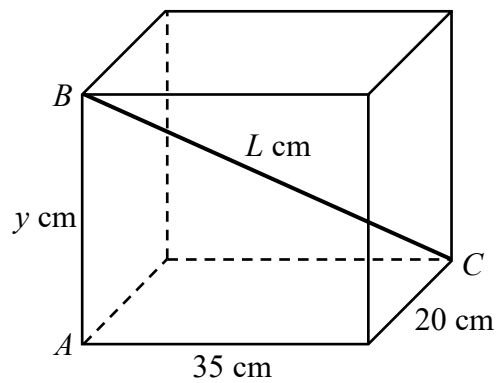
3 It is given that the equation of a curve is $y = e^x \sin x$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [3]

(b) Hence, find $\int e^x \cos x \, dx$. [2]

- (c) Using the results in (a) and (b), find $\int e^x \sin x \, dx$. [2]

- 4 (a) A box with length 35 cm, breadth 20 cm and height y cm is being compressed from the top at a constant rate of 0.005 cm per second. The initial height of the box is 30 cm.



- (i) Find the height of the box after one minute. [2]

- (ii) The length of the diagonal BC is L cm. Show that $L = \sqrt{y^2 + k}$, where k is an integer to be found. [1]

- (iii) Assuming that the shape of the box does not change under the compression, find the rate at which BC is decreasing after one minute. [3]

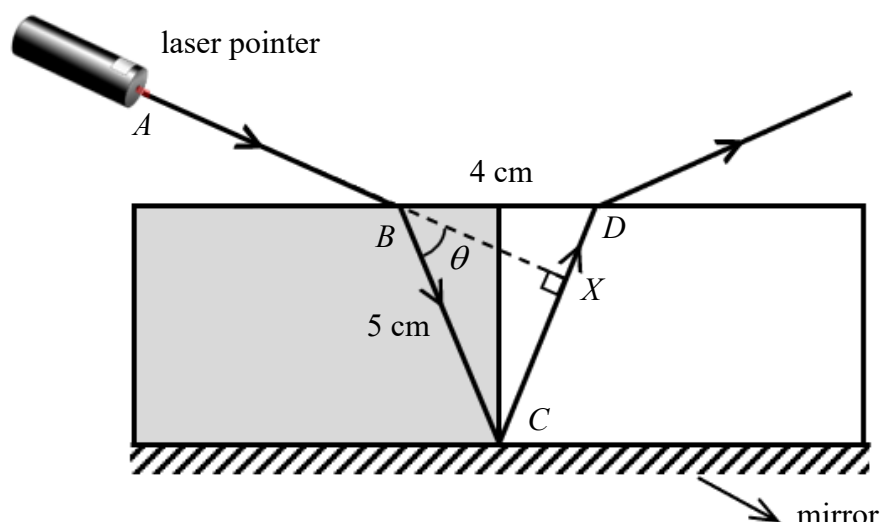
- (b) The density D of the box is related to its volume V by the formula $DV = m$, where m is a constant. Given that when $D = 920$, $V = 0.0245$, find the rate at which the density is changing with respect to V when $V = 0.0196$. [4]

- 5 (a)** Explain why there are only odd powers of x in the expansion $\left(3x^3 - \frac{1}{kx}\right)^9$, where $k > 0$. [3]

- (b)** Given that the coefficient of x^3 in the expansion of $\left(3x^3 - \frac{1}{kx}\right)^9$ is $\frac{567}{16}$, show that the value of k is 2. [4]

- (c) Hence, find the term independent of x in the expansion $(1 - 4x^5)\left(3x^3 - \frac{1}{kx}\right)^9$. [3]

6



Two transparent rectangular blocks made of different materials are placed beside each other on a mirror. A laser pointer is pointed at the blocks and the diagram shows the path of the light ray from the laser pointer. The light ray AB refracts and travels through the path BC . It is then reflected and travels through the path CD , before getting refracted while leaving the prism at D . It is given that angle BCD is equal to angle XBD , and angle XBC is defined as θ , the angle in which the light ray deviates from its original path.

- (a) Show that S cm, the distance travelled by the light ray in the rectangular prism, can be expressed in the form $p + q \sin \theta + r \cos \theta$, where p , q and r are constants to be found. [2]

- (b) Express S in the form $R \sin(\theta + \alpha) + k$, where $R > 0$, $0^\circ \leq \alpha \leq 90^\circ$ and k is an integer. [4]

- (c) Find the value of S and the corresponding value of θ if the light ray is to travel the maximum possible distance within the rectangular prism. [2]

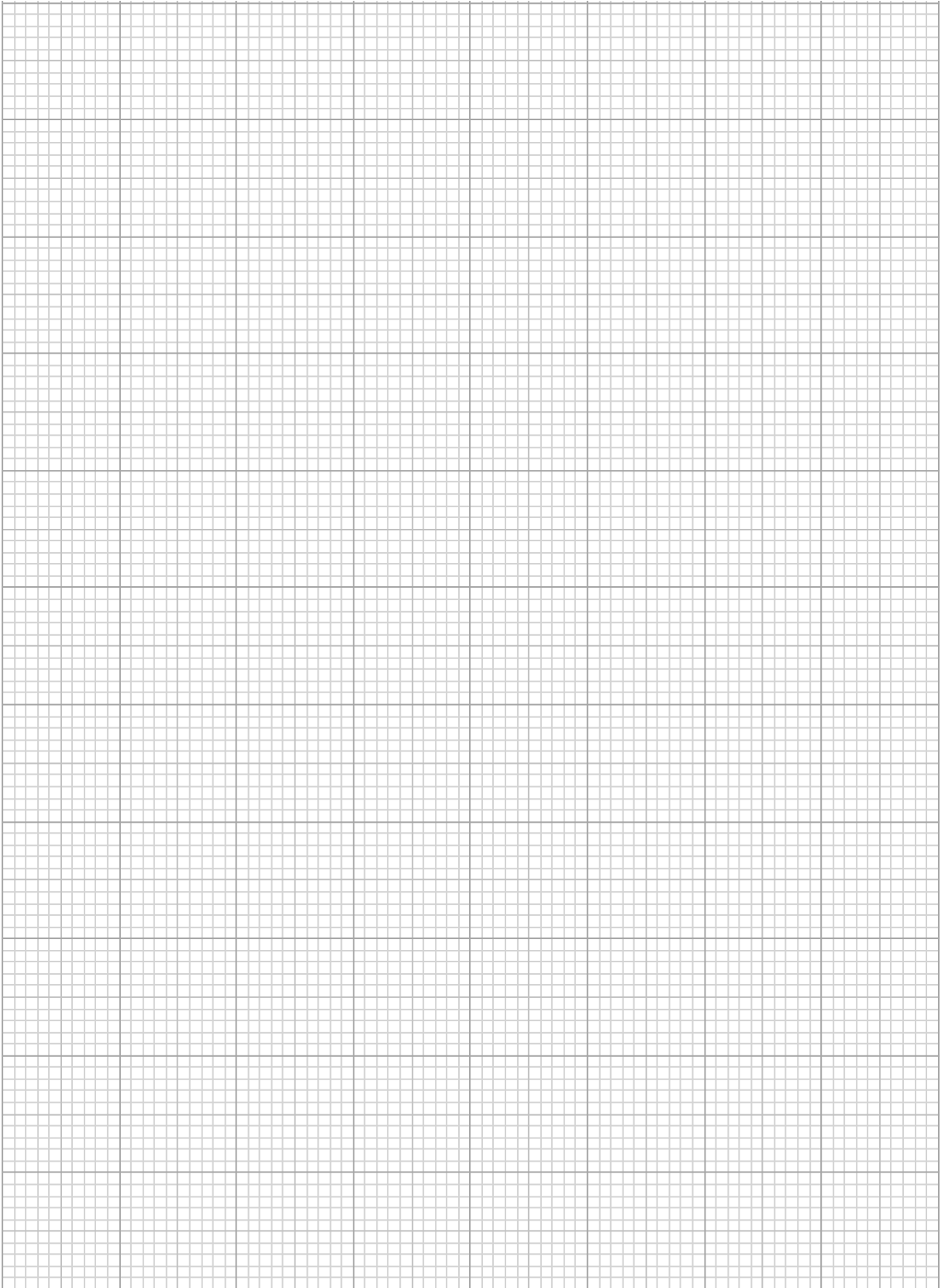
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- 7 (a) Explain how a straight-line graph can be drawn to represent the equation $y = px^q$, where p and q are constants. [2]

- (b) The following table shows experimental values of x and y with the equation $y = px^q$. An error has been made in recording **one** of the values of y .

x	2.82	7.94	14.1	63.1	126
y	50.1	25.1	7.08	6.31	3.98

- (i) On the grid in the next page, draw a straight-line graph based on the experimental values in the table. [3]
- (ii) Hence, calculate the values of p and q . [3]

- (iii) Use the straight line graph drawn in (i) to estimate a value of y to replace the incorrect recording of y . [1]



- 8** A particle travelling in a straight line passes through a fixed point O with its velocity, v m/s, given as $v = e^{2t} - 10e^t + 21$, where t is the time in seconds after it passes O .

(a) Find the acceleration of the particle when it first comes to instantaneous rest. [4]

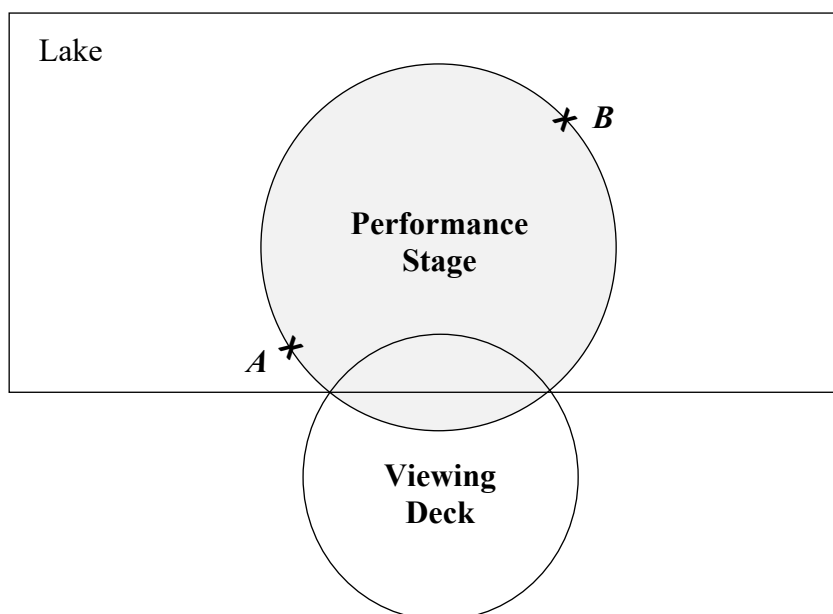
- (b) Find the distance travelled by the particle when the particle reaches minimum velocity.

[6]

- 9 At the Botanic Gardens, a team of landscape architects is embarking on a project to enhance tourism by constructing a performance area on a lake.

The team envisions a design of the performance area that features two overlapping circles. One circle represents the designated performance stage, where artists and performers will showcase their talents against the backdrop of water and greenery. The other circle serves as a viewing deck, offering spectators a vantage point to enjoy the performances.

Below is a simplified sketch of the plan.



The performance stage can be modelled by the equation of a circle given by $x^2 + y^2 - 14x - 20y + 124 = 0$ on the cartesian plane.

- (a) Show that this model suggests that the radius of the performance stage is 5 m.

[2]

- (b) A floor marker is to be placed at the point $(6, 6)$. Explain why any straight line passing through the floor marker will intersect the circumference of the performance stage at two points.

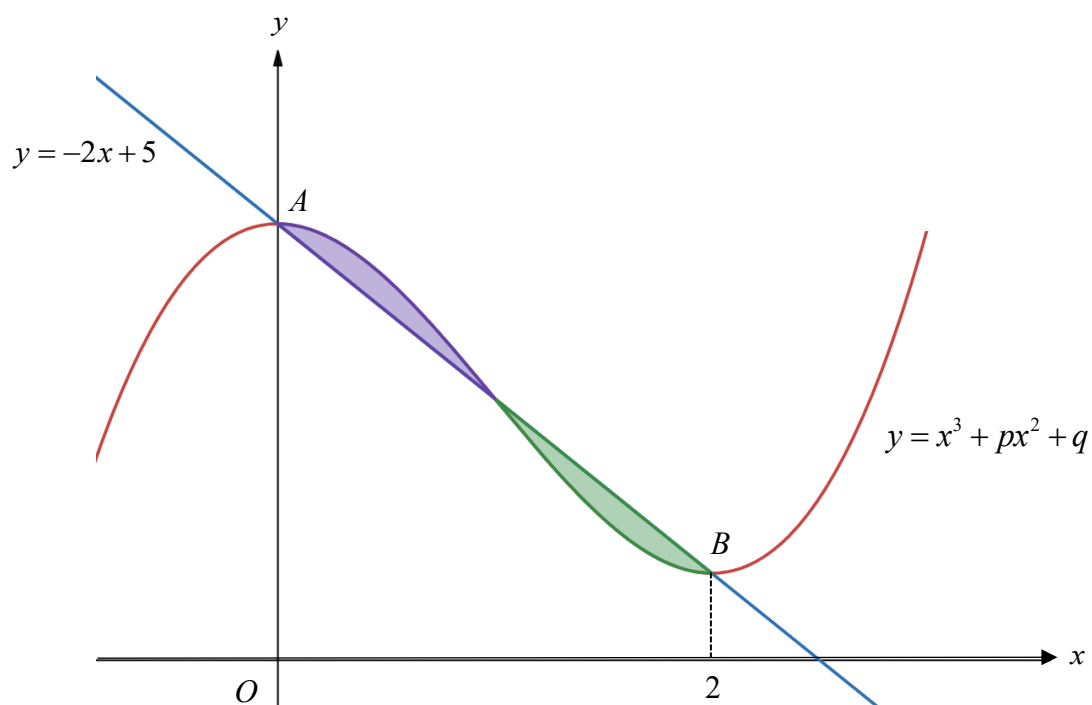
[3]

- (c) Two spotlights at $A(4, 6)$, and $B(10, 14)$, are to be positioned on the circumference of the performance stage such that they form the endpoints of a diameter of the stage. It is observed that placing additional spotlights on the stage equidistant from these 2 spotlights is effective at increasing aesthetic appeal of performances.

Find the equation of line AB and hence determine whether a spotlight placed at $(11, 7)$ is effective at increasing the aesthetic appeal of performances.

[5]

10



The diagram shows the line $y = -2x + 5$ and the curve $y = x^3 + px^2 + q$, where p and q are constants. A and B are stationary points of the curve and they lie on the line. It is given that one of the stationary points of the curve occurs at $x = 2$.

Find the total area of the shaded regions.

[12]

Continuation of working space for question 10.

End of Paper

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Answers Key

Qn	Answer	Qn	Answer
1	(a) $k \leq -\frac{64}{9}$ OR $0 \leq k < 4$ (b) $k = 3$	6	(a) $S = 5 + 5 \sin \theta + 4 \cos \theta$ (b) $S = \sqrt{41} \sin(\theta + 38.7^\circ) + 5$ (c) Max. value of S = 11.4 cm, where $\theta = 51.3^\circ$
2	(a) $4u^3 - 12u^2 - 15u - 4 = 0$ (c) $x = 1.26$	7	(a) $\lg y = q \lg x + \lg p$ / $\ln y = q \ln x + \ln p$. Plot a graph of $\lg y$ / $\ln y$ against $\lg x$ / $\ln x$. (b)(ii) $p = 100 \pm 6$, $q = -0.667 \pm 0.1$ (b)(iii) $y = 17 \pm 1$
3	(a) $\frac{dy}{dx} = e^x \cos x + e^x \sin x$ $\frac{d^2y}{dx^2} = 2e^x \cos x$ (b) $\int e^x \cos x \, dx = \frac{1}{2}(e^x \sin x + e^x \cos x) + c_1$ (c) $\int e^x \sin x \, dx = \frac{1}{2}(e^x \sin x - e^x \cos x) + c_2$	8	(a) $a = -12 \text{ m/s}^2$ (b) Distance = 8.34 m
4	(a)(i) 29.7 cm (a)(ii) $L = \sqrt{y^2 + 1625}$ (a)(iii) Rate of decrease = 0.00297 cm/s (b) $\frac{dD}{dV} = -58700 \text{ g / cm}^6$	9	(b) Length between point and centre of circle = $\sqrt{17} < 5$ (radius of circle). Hence, point is inside circle and any line passing through the point will intersect circumference at 2 points. (c) Equation of line AB: $y = \frac{4}{3}x + \frac{2}{3}$ Yes, it is effective to place spotlight.
5	(c) $-\frac{27}{64}$	10	0.50 units ²