General Certificate of Education Ordinary Level 2006
Additional Mathematics 4018/01
Syllabus 4018 Paper 1
SUGGESTED Solutions & Marking Scheme Mark Allocation
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1		**NOT** within the 4049 syllabus			
	(i)	$x \notin A$	B1		
	(ii)	$\mathrm{n}(B') = 16$	B1		
	(iii)	$C \cap D = \varnothing$	B1		
					[3]
2	(i)	$a = 2$	B1		
	(ii)	$\frac{360°}{b} = 120° \Rightarrow b = \frac{360°}{120°} = 3$	B1		
	(iii)	$c = -1$	B1		
					[3]
3	(i)	$y = 8(3x-4)^{-2}$			
		$\frac{\mathrm{d}y}{\mathrm{d}x} = (-2) \cdot 8(3x-4)^{-3} \cdot 3$	M1		
		$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{48}{(3x-4)^3}$	A1		
		When $x = 2$, gradient is -6.	A1		
	(ii)	**NOT** within the 4049 syllabus	E2		
					[5]

4		**NOT** within the 4049 syllabus			
	(i)	Unit vector of $3\mathbf{i}-4\mathbf{j}=\frac{1}{\sqrt{3^2+(-4)^2}}\begin{pmatrix}3\\-4\end{pmatrix}=\frac{1}{5}\begin{pmatrix}3\\-4\end{pmatrix}$ Unit vector of $4\mathbf{i}+3\mathbf{j}=\frac{1}{\sqrt{3^2+4^2}}\begin{pmatrix}4\\3\end{pmatrix}=\frac{1}{5}\begin{pmatrix}4\\3\end{pmatrix}$	M1	Attempt to find either unit vector s.o.i.	
		$\overrightarrow{OP}=\frac{10}{5}\begin{pmatrix}3\\-4\end{pmatrix}=2\begin{pmatrix}3\\-4\end{pmatrix}=\begin{pmatrix}6\\-8\end{pmatrix}$ $\Rightarrow \overrightarrow{OP}=6\mathbf{i}-8\mathbf{j}$	A1		
		$\overrightarrow{OQ}=\frac{15}{5}\begin{pmatrix}4\\3\end{pmatrix}=3\begin{pmatrix}4\\3\end{pmatrix}=\begin{pmatrix}12\\9\end{pmatrix}$ $\Rightarrow \overrightarrow{OQ}=12\mathbf{i}+9\mathbf{j}$	A1		
	(ii)	$\overrightarrow{PQ}=\begin{pmatrix}12\\9\end{pmatrix}-\begin{pmatrix}6\\-8\end{pmatrix}=\begin{pmatrix}6\\17\end{pmatrix}$	M1	Find $\overrightarrow{QP}$ o.e.	
		$\left\lvert\overrightarrow{PQ}\right\rvert=\left\lvert\begin{pmatrix}6\\17\end{pmatrix}\right\rvert=\sqrt{6^2+17^2}$	M1	Apply distance formula o.e.	
		$\left\lvert\overrightarrow{PQ}\right\rvert=\sqrt{325}=5\sqrt{13}$ $\Rightarrow \lambda=5$	A1		
					[6]

5		**NOT** within the 4049 syllabus			
	(i)	Let **A** and **B** represent the number of aircrafts for each type and the number of Economy, Business and First seats in each type of aircraft respectively.			
		$\mathbf{A}=\begin{pmatrix}5 & 8 & 4 & 10\end{pmatrix}$	B1		
		$\mathbf{B}=\begin{pmatrix}300 & 60 & 40\\ 150 & 50 & 20\\ 120 & 40 & 0\\ 100 & 0 & 0\end{pmatrix}$	B1		
	(ii)	Let **C** represent the total number of seats in each class.			
		$\mathbf{C}=\begin{pmatrix}5 & 8 & 4 & 10\end{pmatrix}\begin{pmatrix}300 & 60 & 40\\ 150 & 50 & 20\\ 120 & 40 & 0\\ 100 & 0 & 0\end{pmatrix}=\begin{pmatrix}4180 & 860 & 360\end{pmatrix}$	B2	B1 for any two correct B1 for all correct	
	(iii)	Let **D** represent the percentage of empty seats for Economy, Business and First seats on that particular day.			
		$\mathbf{D}=\begin{pmatrix}0.05\\ 0.1\\ 0.2\end{pmatrix}$	B1		
	(iv)	$\mathbf{CD}=\begin{pmatrix}4180 & 860 & 360\end{pmatrix}\begin{pmatrix}0.05\\ 0.1\\ 0.2\end{pmatrix}=\begin{pmatrix}367\end{pmatrix}$			
		367 seats	B1		
					[6]

6		$\left(2-\frac{x}{2}\right)^6 = 2^6 + \binom{6}{1}(2)^{6-1}\left(-\frac{x}{2}\right) + \binom{6}{2}(2)^{6-2}\left(-\frac{x}{2}\right)^2 + \binom{6}{3}(2)^{6-3}\left(-\frac{x}{2}\right)^3 + \ldots$ $= 64 - 96x + 60x^2 + \ldots$	B3	B1 for each simplified term	
		Coefficient of $x^2 = 60(k) + (-96)(1)$	M1		
		$60k - 96 = 84$ $60k = 180$	M1		
		$k = 3$	A1		
					[6]
7		<u>**NOT**</u> within the 4049 syllabus			
	(i)	Minimum value of f $= -11$	M1	s.o.i.	
		$\mathrm{f}(3) = 9\left(3-\frac{1}{3}\right)^2 - 11 = 53$ $\mathrm{f}(-3) = 9\left(-3-\frac{1}{3}\right)^2 - 11 = 89$	M1		
		$\mathrm{R_f} = [-11, 89]$	A1		
	(iia)	$\left(\frac{1}{3}, -11\right)$	B1		
		Minimum point	B1		
	(iib)	$\left(\frac{1}{3}, 11\right)$	B1		
		Maximum point	B1		
					[7]

8	(a)	$\lg(x+12) = 1 + \lg(2-x)$ $\lg(x+12) - \lg(2-x) = 1$ $\lg\left(\dfrac{x+12}{2-x}\right) = 1$	M1	Quotient low s.o.i.	
		$\dfrac{x+12}{2-x} = 10$	M1	Remove lg	
		$x+12 = 20-10x$ $11x = 8$ $x = \dfrac{8}{11}$	A1		
	(b)	$\log_2 p = a$ $p = 2^a$	M1		
		$\log_8 q = b$ $\dfrac{\log_2 q}{\log_2 2^3} = b$	M1	Change of base low s.o.i	
		$\dfrac{\log_2 q}{3} = b$ $\log_2 q = 3b$ $q = 2^{3b}$	M1		
		$\dfrac{p}{q} = c = a - 3b$	A1		
					[7]

9	(i)	$\frac{dy}{dx}=\frac{2(x+3)-(2x-4)}{(x+3)^2}$	M1	Attempt to use quotient rule	
		$=\frac{2x+6-2x+4}{(x+3)^2}$ $=\frac{10}{(x+3)^2}$	A1		
		$(x+3)^2\geq 0 \Rightarrow \frac{10}{(x+3)^2}>0 \Rightarrow \frac{dy}{dx}>0$	A1	Show that $\frac{dy}{dx}$ is increasing	
		Since $\frac{dy}{dx}>0$ and $\frac{dy}{dx}\neq 0$, the curve has no turning points.	AG	Explanation given	
	(ii)	When $y=0$, $x=2$. Hence, $P(2, 0)$.	B1	s.o.i.	
		When $x=2$, $\frac{dy}{dx}=\frac{10}{(2+3)^2}=\frac{2}{5}$	M1		
		$y=\frac{2}{5}x+c$ $0=\frac{2}{5}(2)+c \Rightarrow c=-\frac{4}{5}$			
		$Q\left(0, -\frac{4}{5}\right)$	M1		
		Area $=\frac{1}{2}(2)\left(\frac{4}{5}\right)$	M1	Shoelace method o.e.	
		$=0.8\text{ units}^2$	A1		
					[8]

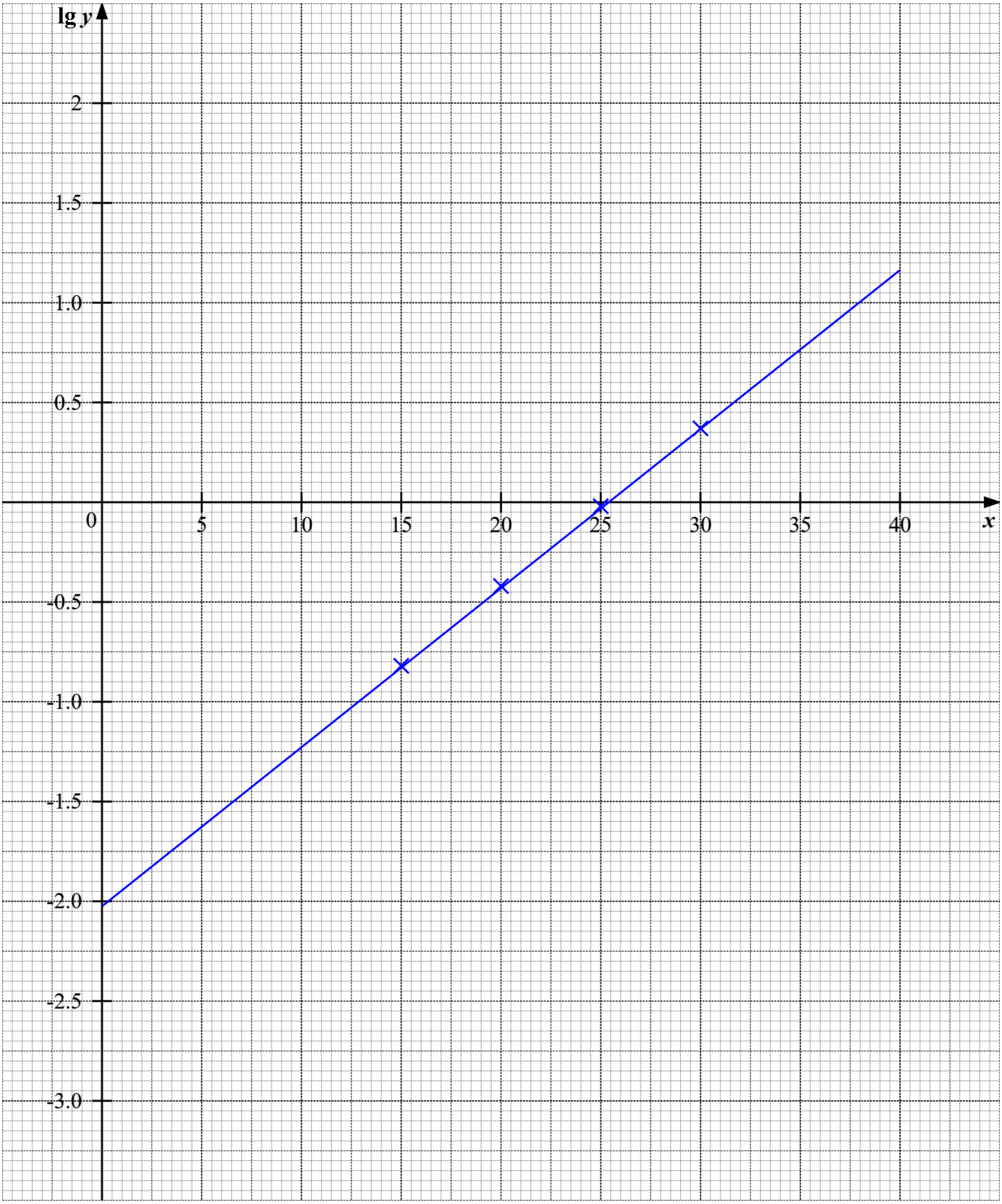
10	(i)	$\mathrm{f}(x)=(x-1)(x-k)\left(x-k^2\right)$	M1	o.e.	
		$\mathrm{f}(2)=7$ $(2-1)(2-k)\left(2-k^2\right)=7$	M1	Remainder theorem s.o.i.	
		$4-2k-2k^2+k^3=7$ $k^3-2k^2-2k-3=0$	A1 AG		
	(ii)	Let $\mathrm{f}(k)=k^3-2k^2-2k-3$			
		$\mathrm{f}(3)=3^3-2(3)^2-2(3)-3=0$ Since $\mathrm{f}(3)=0$, by Factor Theorem, $(k-3)$ is a factor of $\mathrm{f}(k)$.	B1	Factor Theorem s.o.i.	
		Attempt to long divide or compare coefficients	M1	s.o.i.	
		$\mathrm{f}(k)=(k-3)\left(k^2+k+1\right)$	M1		
		$f(k)=0$ $(k-3)\left(k^2+k+1\right)=0$ $k-3=0$ or $k^2+k+1=0$			
		For $k^2+k+1=0$, Discriminant $=1^2-4(1)(1)=-3<0$ Since discriminant < 0, $k^2+k+1=0$ has no real roots.	M1		
		Hence, the only real value of k is $k=3$.	A1 AG		
					[8]

11	(a)	$2\cot x = 1+\tan x$ $\dfrac{2}{\tan x} = 1+\tan x$ $2 = \tan x + \tan^2 x$ $\tan^2 x + \tan x - 2 = 0$	M1	Obtain equation	
		$(\tan x - 1)(\tan x + 2) = 0$	M1	Factorise o.e.	
		$\tan x - 1 = 0$ $\tan x = 1$ $\alpha_1 = 45°$ $\tan x + 2 = 0$ $\tan x = -2$ $x = \tan^{-1}(2)$ $= 63.43494882°$	M1		
		$x = \alpha_1,\ 180° + \alpha_1,\ 180° - \alpha_2,\ 360° - \alpha_2$ $= 45°,\ 225°,\ 116.6°,\ 296.6°$ (1 d.p.)	A2	A1 for any pair of correct answers	
	(b)	$6\sin(2y+1) + 5 = 0$ $6\sin(2y+1) = -5$ $\sin(2y+1) = -\dfrac{5}{6}$	M1	Make $\sin(2y+1)$ the subject	
		$\alpha = \sin^{-1}\left(\dfrac{5}{6}\right) = 0.98511077833$	M1	Basic angle s.o.i.	
		$2y+1 = \pi + \alpha,\ 2\pi - \alpha$	M1	Identify quadrants	
		$y = \dfrac{\pi+\alpha-1}{2},\ \dfrac{2\pi-\alpha-1}{2}$ $= 1.56,\ 2.15$ (3 s.f.)	A2		
					[10]

12		**EITHER**			
	(i)	When $y=0, x=-\ln 2$. $A(-\ln 2, 0)$	B1	o.e.	
		When $x=0$, $y=3$. $B(0,3)$	B1		
	(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x}=2\mathrm{e}^{-2x}$	M1		
		When $x=0$, $\frac{\mathrm{d}y}{\mathrm{d}x}=2$.	M1	Find gradient s.o.i.	
		Gradient of normal $=-\frac{1}{2}$ Equation of normal is $y=-\frac{1}{2}x+3$.	M1	o.e.	
		When $y=0, x=6$. $C(6,0)$	A1		
	(iii)	Area $=\int_{-\ln 2}^{0} 4-\mathrm{e}^{-2x}\,\mathrm{d}x+\frac{1}{2}(6)(3)$	M2		
		$=\left[4x+\frac{\mathrm{e}^{-2x}}{2}\right]_{-\ln 2}^{0}+9$	M1	Correct integration	
		$=\frac{1}{2}-\left[4(-\ln 2)-1\right]+9$	M1	Substitution	
		$=10.3$ units^2 (3 s.f.)	A1 AG		
					[11]

12		**OR**			
	(i)	$\begin{array}{|c|c|c|c|c|} \hline x & 15 & 20 & 25 & 30 \\ \hline \lg y & -0.82 & -0.42 & -0.02 & 0.37 \\ \hline \end{array}$			
		Plotting of all points above	P1		
		Line drawn on suitable axes	L1		
	(ii)	$y = 10^{-A} b^x$ $\lg y = \lg\left(10^{-A} b^x\right)$ $\lg y = \lg 10^{-A} + \lg b^x$ $\lg y = x \lg b - A$	M1	Express in $Y = mX + c$ o.e.	
		$\text{Gradient} = \dfrac{0.85-(-1.9)}{36-1.5} = \dfrac{11}{138}$	M1		
		$\lg b = \dfrac{11}{138}$ $b = 10^{\frac{11}{138}}$ $= 1.201462292$ $= 1.20$ (3 s.f.)	A1	(± 0.02)	
		$A = 2.025$	B1	(± 0.05)	
	(iii)	$y = 10$ $\lg y = 1$	M1		
		$x = 38$	A1	(± 0.5)	
	(iv)	$y^5 = 10^{-x}$ $\lg y^5 = \lg 10^{-x}$ $5 \lg y = -x$ $\lg y = -\dfrac{x}{5}$	M1		
		Drawing of line on graph	B1		
		$x = 7.25$	A1	(± 0.5)	
					[11]

END OF MARKING SCHEME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