General Certificate of Education Advanced Level 2025
Higher 2 Mathematics 9758/02
Syllabus 9758 Paper 2
SUGGESTED Solutions & Marking Scheme Mark Allocation
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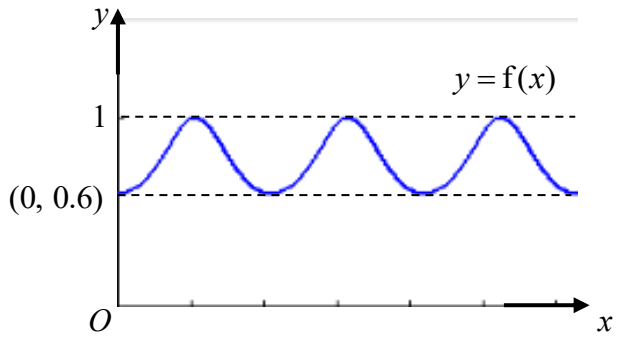
Section A: Pure Mathematics [40 marks]

1	(a)	$8c+11t+5b=114$ $5c+14t+7b=112$ $9c+9t+4b=110$	M2	M1 for any correct equation M1 for all three	
		$c=7,\ t=3,\ b=5$	A1		
	(b)	Let x, y and z denote the number of cars, trains and boats Sam has.			
		$7x+3y+5z=113$	M1	Set up equation	
		Process of Guess and Check, Trial and Error, Algebraic manipulation, inequalities, o.e.	M1	Any valid method seen and reason	
		6 cars, 7 boats, 10 trains	A1		
					[6]

2	(a)	$\overrightarrow{PQ} = \begin{pmatrix} 1 \\ 6 \\ 6 \end{pmatrix} - \begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 6 \end{pmatrix}$	M1	Find $\overrightarrow{PQ}$ s.o.i.	
		$3\overrightarrow{QR} = 4\overrightarrow{PQ}$ $\overrightarrow{QR} = \frac{4}{3}\overrightarrow{PQ} = \frac{4}{3}\begin{pmatrix} 3 \\ 3 \\ 6 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ 8 \end{pmatrix}$	M1	Attempt to find $\overrightarrow{QR}$ o.e.	
		$\overrightarrow{OR} - \overrightarrow{OQ} = \begin{pmatrix} 4 \\ 4 \\ 8 \end{pmatrix} \Rightarrow \overrightarrow{OR} = \begin{pmatrix} 4 \\ 4 \\ 8 \end{pmatrix} + \begin{pmatrix} 1 \\ 6 \\ 6 \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \\ 14 \end{pmatrix}$	A1		
	(b)	$\overrightarrow{SQ} = \begin{pmatrix} 1 \\ 6 \\ 6 \end{pmatrix} - \begin{pmatrix} -1 \\ -2 \\ c \end{pmatrix} = \begin{pmatrix} 2 \\ 8 \\ 6-c \end{pmatrix}$	M1	Find $\overrightarrow{SQ}$ s.o.i.	
		$\overrightarrow{SQ} \cdot \overrightarrow{PQ} = 0 \Rightarrow \begin{pmatrix} 2 \\ 8 \\ 6-c \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 3 \\ 6 \end{pmatrix} = 0$ $6 + 24 + 36 - 6c = 0$ $6c = 66 \Rightarrow c = 11$	A1		
	(c)	$\overrightarrow{PS} = \begin{pmatrix} -1 \\ -2 \\ 11 \end{pmatrix} - \begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -5 \\ 11 \end{pmatrix}$	M1	Find $\overrightarrow{PS}$ s.o.i.	
		$\left\lvert \begin{pmatrix} 1 \\ -5 \\ 11 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 3 \\ 6 \end{pmatrix} \right\rvert = \left\lVert \begin{pmatrix} 1 \\ -5 \\ 11 \end{pmatrix} \right\rVert \left\lVert \begin{pmatrix} 3 \\ 3 \\ 6 \end{pmatrix} \right\rVert \cos\theta$	M1	Reasonable attempt to use dot product s.o.i.	
		$\cos\theta = \frac{54}{\sqrt{7938}} \Rightarrow \theta = 52.7°$ (1 d.p.) or 0.920 rad (3 s.f.)	A1		
					[8]

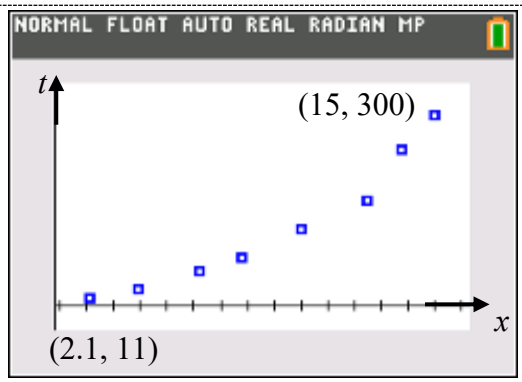
3	(a)	$\frac{dy}{dx} = y^2 \sin 3x \Rightarrow \frac{1}{y^2}\frac{dy}{dx} = \sin 3x$ $\int y^{-2} dy = \int \sin 3x \, dx$	M1	Variable separable s.o.i.	
		$-\frac{1}{y} = -\frac{\cos 3x}{3} + c$	M1	Correct integration	
		When $x = \pi$, $y = 1$. $-1 = \frac{1}{3} + c \Rightarrow c = -\frac{4}{3}$	M1	Find arbitrary constant o.e.	
		$-\frac{1}{y} = -\frac{\cos 3x}{3} - \frac{4}{3} \Rightarrow f(x) = \frac{3}{4 + \cos 3x}$	A1		
	(bi)		B2	B1 for correct maximum, minimum and y-intercept B1 for correct period and shape	
		$c = 1$ or $c = \frac{3}{5}$	B1	o.e.	
	(bii)	$x = \frac{k\pi}{3}$, $k \in \mathbb{Z}$	B2	B1 for equation B1 condition for k	
	(biii)	$f(x + \pi) = \frac{3}{4 - \cos 3x}$	B1	FT from (a)	
	(c)	$\frac{d^2y}{dx^2} = 2y \sin 3x \frac{dy}{dx} + 3y^2 \cos 3x$	M2	M1 for attempt to use product rule M1 for implicit differentiation	
		$\frac{d^2y}{dx^2} = (3\cos 3x) y^2 + 2y \sin 3x \left(y^2 \sin 3x\right)$ $\frac{d^2y}{dx^2} = (3\cos 3x) y^2 + \left(2\sin^2 3x\right) y^3$	A1		
					[13]

4	(a)	Full capacity of bowl $= \frac{1}{3}\pi(15)^2(45-15) = 2250\pi$	M1		
		Time taken to fill the bowl $= \frac{2250\pi}{10\pi} = 225$ seconds	A1		
	(b)	$\frac{\mathrm{d}V}{\mathrm{d}h} = \frac{2}{3}\pi h(45-h) + \frac{1}{3}\pi h^2(-1)$	M1	Product rule s.o.i.	
		$= \frac{1}{3}\pi h\big(2(45-h)-h\big) = \frac{1}{3}\pi h(90-3h)$	M1	Simplify s.o.i.	
		$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}h} \times \frac{\mathrm{d}h}{\mathrm{d}t}$ $10\pi = \frac{1}{3}\pi(12)\big(90-3(12)\big) \times \frac{\mathrm{d}h}{\mathrm{d}t}$	M1	Substitution	
		$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{5}{108}$ cm/s	A1	c.a.o.	
	(c)	$\frac{\mathrm{d}V}{\mathrm{d}t} = 10\pi t \Rightarrow V = \int 10\pi t \, \mathrm{d}t$ $V = 5\pi t^2 + c$	M1	Integrate correctly	
		When $t = 0$, $V = 0$ $0 = 0 + c \Rightarrow c = 0$	M1	Find arbitrary constant	
		When $V = 972\pi$, $972\pi = 5\pi t^2 \Rightarrow t = 13.94274005 = 13.9$ seconds (3 s.f.)	A1		
	(d)	When $V = 972\pi$, $\frac{1}{3}\pi h^2(45-h) = 972\pi \Rightarrow 45h^2 - h^3 = 2916$	M1	Form equation	
		$h = 9$ or $h = 43.456$ (reject) or $h = -7.4558$ (reject)	M1	Must reject for M1	
		$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}h} \times \frac{\mathrm{d}h}{\mathrm{d}t}$ $10\pi\sqrt{\frac{972}{5}} = \frac{1}{3}\pi(9)\big(90-3(9)\big) \times \frac{\mathrm{d}h}{\mathrm{d}t}$	M1	Substitution	
		$\frac{\mathrm{d}h}{\mathrm{d}t} = 0.7377111136 = 0.738$ cm/s (3 s.f.)	A1	M0A0	
					[13]

Section B: Probability and Statistics [60 marks]

5	(a)	P(RB or RY or BY or BR or YR or YB) $=\left(\frac{3}{15}\times\frac{5}{14}+\frac{3}{15}\times\frac{7}{14}+\frac{5}{15}\times\frac{7}{14}\right)\times 2!$	M2	M1 Any correct case considered M1 All cases	
		$=\frac{71}{105}$	A1		
	(b)	P(*n*th counter is red) $=\frac{\binom{12}{n-1}}{\binom{15}{n-1}}\times\frac{3}{16-n}=\frac{36}{445}$	M1	Any equivalent equation or method seen	
		By GC, $n=6$.	A1		
					[5]
6	(a)	$\mathrm{P}(X<7)=0.05$	B1	o.e.	
	(b)	$\mathrm{P}(X<7)=0.05$ $\mathrm{P}\left(Z<\frac{7-\mu}{1.8}\right)=0.05$	M1	Attempt to standardise s.o.i.	
		$\frac{7-\mu}{1.8}=-1.644853626$ $\Rightarrow \mu=9.960736527=9.96$ (3 s.f.)	A1		
	(c)	$\mathrm{P}(X>7)\mathrm{P}(Y>7)=0.38$ $\mathrm{P}(Y>7)=\frac{0.38}{1-0.05}=0.4$	M1	0.4 s.o.i.	
		$\mathrm{P}\left(Z>\frac{7-\lambda}{2.8}\right)=0.4$	M1	Attempt to standardise s.o.i.	
		$\frac{7-\lambda}{2.8}=0.2533471011$ $\Rightarrow \lambda=6.29062811692=6.29$ (3 s.f.)	A1		
	(d)	$2\mathrm{P}(Y>7)=0.800000107$	M1		
		By GC, $a=3.934087593=3.93$ (3 s.f.)	A1		
					[8]

7	(a)	Number of ways $= \frac{8!}{2!2!}$	M1		
		$= 10080$	A1		
	(b)	Number of ways to arrange the 5 odd numbers $= \frac{5!}{2!2!}$	M1		
		Choosing an even number and slotting in between the odd numbers $= \binom{3}{1} \times \binom{4}{1}$	M1		
		Total number of ways $= \frac{5!}{2!2!} \times \binom{3}{1} \times \binom{4}{1} \times 2! = 720$	A1		
	(c)	Number of ways $= \frac{3!6!}{2!2!}$	M1	3!6! s.o.i.	
		$= 1080$	A1		
	(d)	Required probability $= \frac{1}{3}$	B1	FT from (c) if working is shown	
					[8]

8	(a)	The value of the correlation coefficient is not close to 1, showing that the relationship between Long jump and High jump have a weak positive linear correlation.	M1	Make reference to value of r clearly	
		This is also seen in the scatter diagram where there is little to no linear relationship seen in the data. Hence, it suggests that there is no linear relationship.	A1 AG		
	(bi)		B1	Must mark end-points or B0	
		As x increases, t increases at an increasing rate, suggesting that x and t have a non-linear relationship but perhaps a quadratic one.	B1	Non-linear o.e.	
	(bii) (A)	For $t = ax + b$, $r = 0.958$ (3 s.f.) For $t = cx^2 + d$, $r = 0.990$ (3 s.f.)	M1	Either one seen	
		As the <u>magnitude</u> of the value of r for the model $t = cx^2 + d$ is closer to 1, and the scatter diagram shows a non-linear correlation between x and t, $t = cx^2 + d$ is the better model to fit the given data.	A1	A0 if 'magnitude' was not stated o.e.	
	(bii) (B)	$t = 1.25x^2 + 0.876$	B2	B1 for each term	
	(biii) (A)	When $x = 11$, $t = 152.088469 = 152$ seconds (3 s.f.)	B1	FT from (b)(ii)(B)	
	(biii) (B)	The value of r is very close to 1, showing that there is a strong positive linear correlation between t and x^2.	B1		
		The value of $x = 11$ lies within the data range of $[2.1, 15]$.	B1	Must quote the interval $[2.1, 15]$	
					[11]

9	(a)	The <u>event</u> where a randomly chosen refrigerator is faulty is independent of the <u>event</u> where another refrigerator is faulty.	B1	Independence	
		For all the refrigerators, the probability of choosing a faulty refrigerator is always constant.	B1	Constant probability	
	(b)	$\mathrm{P}(X>1)=1-\mathrm{P}(X\leq 1)$	M1		
		$=0.53956797=0.540$ (3 s.f.)	A1		
	(c)	Let W denote the number of days in a 5-day working week where Alan finds more than one faulty refrigerator. $W\sim \mathrm{B}(5,\ 0.53956797)$	M1	Must define clearly	
		$\mathrm{P}(W<2)=\mathrm{P}(W\leq 1)=0.1419422302=0.142$ (3 s.f.)	A1		
	(d)	Let A denote the number of faulty refrigerators Alan finds in a 5-day working week. $A\sim \mathrm{B}(450,\ 0.02)$	M1	Must define clearly	
		$\mathrm{P}(A<10)=\mathrm{P}(A\leq 9)=0.5874127271=0.587$ (3 s.f.)	A1		
	(e)	Required Probability $=\mathrm{P}(X=0)\mathrm{P}(Y=2)+\mathrm{P}(X=1)\mathrm{P}(Y=1)$ $+\mathrm{P}(X=2)\mathrm{P}(Y=0)$	M2	M1 for any correct case considered M1 for all cases	
		$=0.1766878644=0.177$	A1		
					[11]

10	(a)	Both machines operate and pack the packets of flour on their own without any relation to the other. Hence, it is reasonable to assume that both distributions are independent.	B1	o.e.	
	(b)	Let A kg and C kg denote the masses of the packets of flour produced by Machines P and Q respectively. $A \sim \mathrm{N}\left(2.2,\ 0.1^2\right)$ and $C \sim \mathrm{N}\left(2.1,\ 0.05^2\right)$	M1	Must define clearly	
		$\mathrm{E}(A-C) = 2.2-2.1 = 0.1$ $\mathrm{Var}(A-C) = 0.1^2+0.05^2 = 0.0125$	M1	Either one seen	
		$\mathrm{P}(A>C) = \mathrm{P}(A-C>0) = 0.814453351 = 0.814\ (3\text{ s.f.})$	A1		
	(c)	Let $T = A_1+A_2+A_3+C_1+C_2+\ldots+C_5$			
		$\mathrm{E}(T) = 3(2.2)+5(2.1) = 17.1$ $\mathrm{Var}(T) = 3\left(0.1^2\right)+5\left(0.05^2\right) = 0.0425$	M1 M1		
		$\mathrm{P}(T>17) = 0.6861870959 = 0.686\ (3\text{ s.f.})$	A1		
	(d)	Let μ kg denote the <u>population</u> mean mass of the packets produced by Machine P after the adjustment.	B1	Must state 'population mean'	
		$\mathrm{H}_0: \mu = 2.2,\ \mathrm{H}_1: \mu \neq 2.2$	B1		
	(e)	$\bar{x} = \dfrac{4.5}{30}+2 = 2.15$	B1		
		$s^2 = \dfrac{1}{29}\left(1.11-\dfrac{4.5^2}{30}\right) = 0.015$	B1		
	(f)	Under H_0, $\overline{X} \sim \mathrm{N}\left(2.2,\ \dfrac{0.015}{30}\right)$ approximately by Central Limit Theorem as $n = 30$ is large.	M1	Use of CLT	
		By GC, p-value $= 0.0253472347 < 0.05$	M1	Correct p-value	
		Hence we reject H_0.	A1	Stated clearly	
		We can conclude at 5% level of significance that there is sufficient evidence to say that the mean mass of the packets produced by Machine P after adjustment differs from 2.2 kg.	A1	o.e.	

	(g)	After the adjustment, the distribution of the masses of the packets produced by Machine P is unknown. With a sufficiently large sample size of at least 30, the production manager can utilize the Central Limit Theorem to approximate the distribution of the masses of the packets to be normal. With a sample which is not large enough, the Central Limit Theorem cannot be used, lowering the confidence in the conclusion from the hypothesis test.	B1		
	(h)	If the sample was not chosen randomly, it will affect the independence of the packets being chosen as well as the probability of each packet being chosen not to be constant. Hence, this will affect the unbiased estimates of population mean and variance of the masses, lowering the confidence in the conclusion of the hypothesis test.	B1		
					[17]

END OF MARKING SCHEME