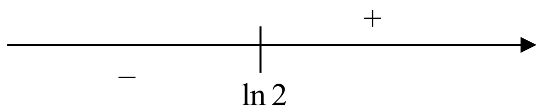
General Certificate of Education Advanced Level 2025
Higher 2 Mathematics 9758/01
Syllabus 9758 Paper 1
SUGGESTED Solutions & Marking Scheme Mark Allocation
Prepared by: Jordan Tew (*@math_stuff_here*)

1	(a)	$u_2 = 3a + 5$	B1	c.a.o.	
		$u_3 = 9a + 20$	B1	FT from u_2	
	(b)	$u_4 = 3(9a+20)+5 = 27a+65$	M1	s.o.i.	
		$a + 3a + 5 + 9a + 20 + 27a + 65 = 110$	M1	Know to sum u_1 to u_4 s.o.i.	
		$40a = 20 \Rightarrow a = \frac{1}{2}$	A1	o.e.	
					[5]
2	(a)	Let the first term be a and common difference be d.			
		$\frac{20}{2}\left(2a+19d\right) = 65$; $\frac{28}{2}\left(2a+27d\right) - 65 = -30$	M1	Attempt to use AP sum formula s.o.i.	
		$10(2a+19d) = 65$ $20a + 190d = 65$; $14(2a+27d) = 35$ $28a + 378d = 35$	M1	Simplify to either equation	
		By GC, $a = 8$ and $d = -\frac{1}{2}$. First term $= 8$ and common difference $= -\frac{1}{2}$.	A1	Conclude, o.e.	
	(b)	$\frac{u_{n+1}}{u_n} = \frac{e^{k(n+2)}}{e^{k(n+1)}} = e^k$	M1	Find $\frac{u_{n+1}}{u_n}$ o.e.	
		As the ratio of subsequent terms is a constant, the sequence is geometric.	A1	Ratio is a constant o.e.	
					[5]

3	(a)	$z=\dfrac{\sqrt{3}+\mathrm{i}}{\sqrt{3}-\mathrm{i}}\times\dfrac{\sqrt{3}+\mathrm{i}}{\sqrt{3}+\mathrm{i}}$	M1	Attempt to multiply by conjugate s.o.i.	
		$z=\dfrac{2+2\sqrt{3}\,\mathrm{i}}{3+1}=\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}\mathrm{i}$	M1	Find z o.e.	
		$\lvert z\rvert=\sqrt{\left(\dfrac{1}{2}\right)^2+\left(\dfrac{\sqrt{3}}{2}\right)^2}=1$	A1		
		$\arg(z)=\tan^{-1}\left(\sqrt{3}\right)=\dfrac{\pi}{3}$	A1	Radians only	
		<u>Alternatively:</u>			
		$\left\lvert\dfrac{\sqrt{3}+\mathrm{i}}{\sqrt{3}-\mathrm{i}}\right\rvert=\dfrac{\lvert\sqrt{3}+\mathrm{i}\rvert}{\lvert\sqrt{3}-\mathrm{i}\rvert}$	M1	Uses $\left\lvert\dfrac{z_1}{z_2}\right\rvert=\dfrac{\lvert z_1\rvert}{\lvert z_2\rvert}$ s.o.i.	
		$\lvert z\rvert=1$	A1		
		$\arg\left(\dfrac{\sqrt{3}+\mathrm{i}}{\sqrt{3}-\mathrm{i}}\right)=\arg\left(\sqrt{3}+\mathrm{i}\right)-\arg\left(\sqrt{3}-\mathrm{i}\right)$	M1	Uses $\arg\left(\dfrac{z}{w}\right)$ $=\arg(z)-\arg(w)$	
		$\arg(z)=\tan^{-1}\left(\sqrt{3}\right)-\left(-\tan^{-1}\left(\sqrt{3}\right)\right)=\dfrac{\pi}{3}$	A1	Radians only	
	(b)	Plotting of A correctly with modulus and argument	B1		
		Plotting of B correctly relative to A	B1		
		Since B is obtained by rotating A by $\dfrac{\pi}{2}$ radians in the anti-clockwise direction, the complex number represented by A is multiplied by i. Hence, the complex number represented by B is iz.	*B1*		
		<u>Alternatively (using exponential form):</u>			
		Since B is obtained by rotating A by $\dfrac{\pi}{2}$ radians in the anti-clockwise direction, the complex number represented by B is $\mathrm{e}^{\mathrm{i}\left(\frac{\pi}{6}+\frac{\pi}{2}\right)}=\mathrm{e}^{\mathrm{i}\frac{\pi}{6}}\times\mathrm{e}^{\mathrm{i}\frac{\pi}{2}}=\mathrm{i}z$.	*B1*		
					[7]

4	(a)	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{3\sqrt{x+1}-(3x+10)\cdot\frac{1}{2}(x+1)^{-\frac{1}{2}}}{x+1}$	M1	Know to use product rule s.o.i.	
		$=\dfrac{3\sqrt{x+1}-\dfrac{3x+10}{2\sqrt{x+1}}}{x+1}=\dfrac{6x+6-3x-10}{2\sqrt{(x+1)^3}}=\dfrac{3x-4}{2\sqrt{(x+1)^3}}$	A1	Simplify	
		x-coordinate of turning point $=\dfrac{4}{3}$	A1	o.e.	
	(b)	$2\sqrt{(x+1)^3}\dfrac{\mathrm{d}y}{\mathrm{d}x}=3x-4$ $\Rightarrow 2\cdot\dfrac{3}{2}(x+1)^{\frac{1}{2}}\dfrac{\mathrm{d}y}{\mathrm{d}x}+2\sqrt{(x+1)^3}\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=3$	M1	Reasonable attempt to differentiate wrt x	
		When $x=\dfrac{4}{3}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$. $2\sqrt{\left(\dfrac{4}{3}+1\right)^3}\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=3\Rightarrow x=0.420848...=0.421$ (3 s.f.)	M1	Value seen	
		Hence, the stationary point at $x=\dfrac{4}{3}$ is a minimum.	A1	M0A0	
					[6]

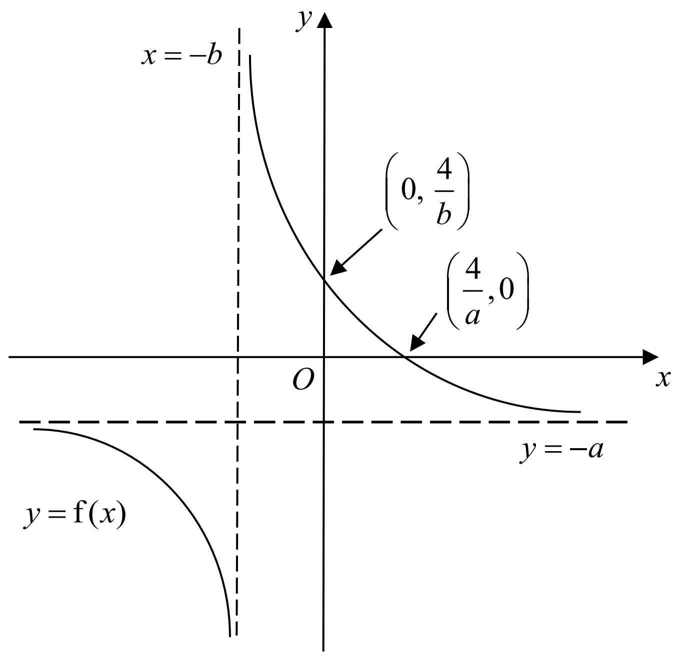
5	(a)	Let $u = \mathrm{e}^x$.			
		$u \geq \dfrac{2}{u} + 1$ $u - \dfrac{2}{u} - 1 \geq 0 \Rightarrow \dfrac{u^2 - u - 2}{u} \geq 0$	M1	Join fraction	
		$\dfrac{(u-2)(u+1)}{u} \geq 0 \Rightarrow \dfrac{\left(\mathrm{e}^x - 2\right)\left(\mathrm{e}^x + 1\right)}{\mathrm{e}^x} \geq 0$ The critical value is $x = \ln 2$.	A1	Find critical value	
		 Hence $x \geq \ln 2$.	A1		
	(b)	$\int_0^1 \left\lvert \mathrm{e}^x - 2\mathrm{e}^{-x} - 1 \right\rvert \mathrm{d}x$ $= \int_0^{\ln 2} -\left(\mathrm{e}^x - 2\mathrm{e}^{-x} - 1\right) \mathrm{d}x + \int_{\ln 2}^1 \left(\mathrm{e}^x - 2\mathrm{e}^{-x} - 1\right) \mathrm{d}x$	M2	M1 each integral with integrand and correct limits	
		$= \left[-\mathrm{e}^x - 2\mathrm{e}^{-x} + x\right]_0^{\ln 2} + \left[\mathrm{e}^x + 2\mathrm{e}^{-x} - x\right]$	M1	Correct integration	
		$= \ln 4 - 4 + \mathrm{e} + \dfrac{2}{\mathrm{e}}$	A1	o.e.	
					[7]

6	(a)	$\frac{\mathrm{d}M}{\mathrm{d}t} = -kM \Rightarrow \frac{1}{M}\frac{\mathrm{d}M}{\mathrm{d}t} = -k$ $\int \frac{1}{M}\,\mathrm{d}M = \int -k\,\mathrm{d}t$	M1	Variable separable s.o.i.	
		$\ln\lvert M\rvert = -kt + c$ $M = \pm e^{-kt+c} = Ae^{-kt},\ A = \pm e^{c}$	M1	Remove modulus after integration	
		When $t = 0$, $M = M_0$. $M = M_0 e^{-kt}$	M1	Finds arbitrary constant	
		When $t = 5730$, $M = \frac{1}{2}M_0$. $\frac{1}{2}M_0 = M_0 e^{-k(5730)} \Rightarrow e^{-5730k} = \frac{1}{2}$ By GC, $k = 0.00012096809 = 0.000121$ (3 s.f.)	A1	o.e.	
	(b)	When $M = 0.2M_0$, $0.2M_0 = M_0 e^{-kt} \Rightarrow e^{-kt} = 0.2$	M1		
		By GC, $t = 13304.64846 = 13300$ (nearest 100)	A1		
					[6]

7	(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}(x\ln x - x) = x\cdot\frac{1}{x} + \ln x - 1$	M1 A1	M1 Attempt to use product rule A1 Differentiate correctly	
		$= \ln x$	AG		
	(b)	Volume generated $= \pi\int_1^{\mathrm{e}} (\ln x)^2\,\mathrm{d}x$	M1	Correct integral	
		$= \pi\left[(\ln x)^2\cdot x\right]_1^{\mathrm{e}} - \pi\int_1^{\mathrm{e}} x\cdot 2\ln x\cdot\frac{1}{x}\,\mathrm{d}x$	M1	Attempt to use by parts integration	
		$= \pi(\mathrm{e}-0) - \pi\left[x\ln x - x\right]_1^{\mathrm{e}}$	M1	Use result from (a)	
		$= (\pi\mathrm{e} - 2\pi)$ units^3	A1	Exact value	
	(c)	Volume generated $= \pi(\mathrm{e})^2(1) - \pi\int_0^1 (\mathrm{e}^y)^2\,\mathrm{d}x$	M2	M1 for cylinder M1 for integral	
		$= 13.18$ units^3 (2 d.p.)	A1		
		<u>Alternatively (using Shell Method in H2 FM):</u>			
		Volume generated $= 2\pi\int_1^{\mathrm{e}} x\ln x\,\mathrm{d}x$	M2	M1 for limits M1 for integrand	
		$= 13.18$ units^3 (2 d.p.)	A1		
					[9]

8	(a)	Let N denote the foot of perpendicular of P to π_1.			
		$\overrightarrow{ON} = \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix} + \alpha \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$, for some $\alpha \in \mathbb{R}$	M1	Find a line that N lies on	
		$\begin{pmatrix} -3+2\alpha \\ 1+\alpha \\ 3\alpha \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = 2$	M1	Substitute back into plane	
		$-6+4\alpha+1+\alpha+9\alpha = 2 \Rightarrow \alpha = \frac{1}{2}$	M1	o.e.	
		$\overrightarrow{ON} = \begin{pmatrix} -2 \\ 1.5 \\ 1.5 \end{pmatrix}$ or $N(-2, 1.5, 1.5)$	A1	o.e.	
	(b)	Let P' denote the reflection of P in π_1.			
		$\overrightarrow{ON} = \dfrac{\overrightarrow{OP} + \overrightarrow{OP'}}{2}$	M1	Attempt to use ratio theorem s.o.i.	
		$\overrightarrow{OP'} = \begin{pmatrix} -4 \\ 3 \\ 3 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$	A1		
	(c)	Normal vector of $\pi_2 = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} \times \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -13 \\ 4 \\ 6 \end{pmatrix}$	B1	c.a.o.	
		Let the acute angle between both planes be θ.			
		$\left\| \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -13 \\ 4 \\ 6 \end{pmatrix} \right\| = \left\| \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \right\| \left\| \begin{pmatrix} -13 \\ 4 \\ 6 \end{pmatrix} \right\| \cos\theta$	M1	Dot product formula s.o.i.	
		$\cos\theta = \frac{4}{\sqrt{3094}} \Rightarrow \theta = 85.9°$ (1 d.p.) or 1.50 rad (3 s.f.)	A1		
					[9]

9	(a)	$y=\sin^{-1} px \Rightarrow \sin y = px$ $\cos y \dfrac{\mathrm{d}y}{\mathrm{d}x} = p$	M1	Differentiate wrt x	
		$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{p}{\cos y} = p\sec y$	A1 AG	Divide by $\cos y$	
	(b)	$\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} = p\sec y \tan y \dfrac{\mathrm{d}y}{\mathrm{d}x}$	M1	Differentiate correctly	
		$\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} = p\sec y \tan y\left(p\sec y\right)$ $= p^2\sec^2 y\tan y$ $= p^2\tan y\left(1+\tan^2 y\right) = p^2\left(\tan y + \tan^3 y\right)$		Use identity and simplify (not needed but used to make working simpler)	
		$\dfrac{\mathrm{d}^3 y}{\mathrm{d}x^3} = p^2\left(\sec^2 y + 3\tan^2 y\sec^2 y\right)$	M1	Any equivalent expression	
		When $x=0$, $y=0$, $\dfrac{\mathrm{d}y}{\mathrm{d}x}=p$, $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}=0$ and $\dfrac{\mathrm{d}^3 y}{\mathrm{d}x^3}=p^3$.	M2	M1 for any two M1 for all correct	
		$y = px + \dfrac{p^3}{6}x^3 + \ldots$	A1	o.e.	
	(c)	$y=\sin^{-1} px \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{p}{\sqrt{1-p^2x^2}}$	M1	Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$	
		Letting $p=3$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{3}{\sqrt{1-9x^2}} \Rightarrow \dfrac{1}{\sqrt{1-9x^2}} = \dfrac{1}{3}\dfrac{\mathrm{d}y}{\mathrm{d}x}$. $\dfrac{1}{\sqrt{1-9x^2}} = \dfrac{1}{3}\dfrac{\mathrm{d}}{\mathrm{d}x}\left(3x+\dfrac{3^3}{6}x^3+\ldots\right) = \dfrac{1}{3}\left(3+\dfrac{27}{2}x^2+\ldots\right)$	M1	Differentiate	
		$\dfrac{1}{\sqrt{1-9x^2}} = 1 + \dfrac{9}{2}x^2 + \ldots$	A1	o.e.	
					[10]

10	(a)		B4	B1 for shape of curve B1 for x and y intercepts B1 for asymptotes B1 for equation of asymptotes	
	(b)	$y=-a+\dfrac{4+ab}{x+b}$	B1		
		(1) Translate a units in the positive y-direction	B1		
		(2) Scale by a factor of $\dfrac{1}{4+ab}$ parallel to the y-axis	B1		
		(1) Translate b units in the positive x-direction	B1		
	(c)	Let $y=\mathrm{f}(x)$ $y=\dfrac{4-ax}{x+b}$ $xy+by=4-ax$	M1	Remove fraction	
		$xy+ax=4-by$ $x=\dfrac{4-by}{y+a} \Rightarrow \mathrm{f}^{-1}(x)=\dfrac{4-bx}{x+a}$	A1		
		$\mathrm{R}_{\mathrm{f}^{-1}}=\mathrm{D}_{\mathrm{f}}=\mathbb{R}\setminus\{-b\}$	B1	o.e.	
	(d)	$\mathrm{f}^{2025}(2)=\dfrac{4-2a}{2+a}$	B1	c.a.o.	
					[12]

11	(a)	$x = 28(\theta\cos\theta + 1)$ $\dfrac{dx}{d\theta} = 28(\cos\theta - \theta\sin\theta)$ $y = 20(2 - \theta\sin\theta)$ $\dfrac{dy}{d\theta} = 20(-\sin\theta - \theta\cos\theta)$	M2	M1 for use of product rule M1 for either derivative seen	
		$\dfrac{dy}{dx} = \dfrac{20(-\sin\theta - \theta\cos\theta)}{28(\cos\theta - \theta\sin\theta)} = \dfrac{5(\sin\theta + \theta\cos\theta)}{7(\theta\sin\theta - \cos\theta)}$	A1	o.e.	
	(bi)	For tangent to be parallel to the y-axis, $7(\theta\sin\theta - \cos\theta) = 0$.	M1	s.o.i.	
		By GC, $\theta = 0.86$ or $\theta = -0.86$ (2 s.f.)	A2	A1 for rounding A1 for values	
	(bii)	When $\theta = 0.8603336$, $x = 43.710697$ When $\theta = -0.8603336$, $x = 12.289303$	M1	Either value seen	
		$CD = 43.710697 - 12.289303 = 31.4$ m (3 s.f.)	A1		
	(ci)	From GC, $\theta = 0$.	B1		
	(cii)	$28 = 28(\theta\cos\theta + 1)$ $\theta\cos\theta + 1 = 1 \Rightarrow \theta\cos\theta = 0$	*M1*	Obtain equation or M0	
		$\theta = \pm\dfrac{\pi}{2}$	*A1*	or B1 for each value of θ	
	(ciii)	$AB = 10\pi$ m or 31.4 m (3 s.f.)	B1	c.a.o.	
	(d)	$0.9\left(\dfrac{1}{2}CD\right) \le AE \le 1.1\left(\dfrac{1}{2}CD\right)$ $14.1396273\text{ m} \le AE \le 17.2817667\text{ m}$	M1	Find a range for *AE* or calculate percentage o.e.	
		$AE = 40 - 26.956307 = 13.0\text{ m}$ (3 $s.f.$) Since *AE* does not lie within the range, the design of this section of the track does not satisfy the condition / is not safe.	A1	Conclude	
					[14]

12	(a)	All the coefficients of the equation are real numbers and thus by Conjugate Root Theorem,	M1	Real numbers and CRT stated o.e.	
		$1-2\text{i}$ is a root of the equation.	A1	M0A0	
	(b)	$(1+2\text{i})^2=1+4\text{i}-4=-3+4\text{i}$ $(1+2\text{i})^3=(1+2\text{i})(-3+4\text{i})=-11-2\text{i}$	M1	Either one seen	
		$-11-2\text{i}+p(-3+4\text{i})+q(1+2\text{i})-10=0$ $-11-2\text{i}-3p+4p\text{i}+q+2q\text{i}-10=0$ $\left(-11-3p+q-10\right)+\left(-2+4p+2q\right)\text{i}=0$	M1	Substitute and attempt to simplify	
		Comparing real parts, $-3p+q-21=0$ $-3p+q=21\ ---(1)$ Comparing imaginary parts, $4p+2q-2=0$ $2p+q=1\ ---(2)$	M1	Either equation seen	
		$(1)-(2)$ $-5p=20\Rightarrow p=-4$ Substitute $p=-4$ into (2) $q=1-2(-4)=9$	A1	Either p or q seen	
		Attempt to long divide or compare coefficients	M1		
		The third root is $z=2$.	A1		
	(c)	Let $z=\frac{1}{w}\Rightarrow w=\frac{1}{z}$. $w=\frac{1}{2}$ or $w=\frac{1}{1+2\text{i}}$ or $w=\frac{1}{1-2\text{i}}$	M1	Knows to replace $z=\frac{1}{w}$ s.o.i.	
		$w=\frac{1}{2}$ or $w=\frac{1\pm 2\text{i}}{5}$	A1	o.e.	
					[10]

END OF MARKING SCHEME

BLANK PAGE