Algebra

A1: Quadratic
Discriminant: $D = b^2 - 4ac$
$D = 0$: 2 real & eql roots
$D > 0$: 2 real & distinct roots
$D < 0$: no real roots
↳ $a > 0$: above x-axis
$a < 0$: below x-axis

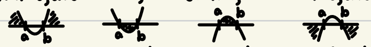
A2: Inequalities
1. $D = 0$: 2 real & eql roots
2. $D > 0$: 2 real & distinct roots
3. $D < 0$: no real roots
4. $D \geq 0$: real roots

	$a>0$	$a<0$
1.	∪	∩
2.	∪	∩
3.	∪	∩

always +ve: $a > 0$ } $D < 0$
always -ve: $a < 0$ }
nvr +ve: $a \leq 0$ } $D \leq 0$
nvr -ve: $a \geq 0$ }

For all values: use $D < 0$

$f(x) > 0, a > 0$; $f(x) < 0, a > 0$; $f(x) > 0, a > 0$; $f(x) < 0, a < 0$



$x < a$ & $x > b$; $a < x < b$; $a < x < b$; $x < a$ & $x > b$

A3: Surds
Laws: $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
$\sqrt{a} \times \sqrt{a} = a$
$a\sqrt{b} \times c\sqrt{d} = ac\sqrt{bd}$
Arithmetic for surds is same as algebra
Rationalisation: $\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{\sqrt{ab}}{b}$
Conjugate of $a \pm b\sqrt{c}$: $a \mp b\sqrt{c}$
↳ use when denominator is in the form $a \pm b\sqrt{c}$
↳ multiply numerator & denominator by conjugate
For eqns with surd on 1 side: square both sides

A4: Polynomials
Long division: $g(x)\overline{)f(x)}$ with $Q(x)$ ← quotient, $f(x)$ ← dividend, $g(x)$ divisor, $R(x)$ ← remainder

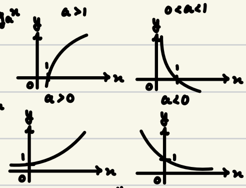
Remainder theorem: $f(x) \div (x-a) \Rightarrow f(a) = R(x)$
Factor theorem: $f(x) \div (x-a)$, $f(a) = 0$, $(x-a)$ is a factor
Cubic identities: $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$
Factorisation: 1. find factor to get $(x-a)(ax^2+bx+c)$ ← quadratic
2. factorise to $(x-a)(x-b)(x-c)$
Partial fractions: $\frac{f(x)}{g(x)}$ ← if improper, use long division

factor	partial fraction
$(ax+b)$	$\frac{A}{ax+b}$
$(ax+b)^2$	$\frac{A}{ax+b} + \frac{B}{(ax+b)^2}$
(x^2+c)	$\frac{Ax+B}{x^2+c}$

A5: Logarithms
$\log_a y = x \Leftrightarrow y = a^x$
Product law: $\log_a xy = \log_a x + \log_a y$
Quotient law: $\log_a \frac{x}{y} = \log_a x - \log_a y$
Power law: $\log_a x^r = r \log_a x$
Change of base law: $\log_a b = \frac{\log_c b}{\log_c a}$
$\log_a M = \log_a N \Rightarrow M = N$
U-sub: let $f(x)$ be U, then sub back into eqn

$\log_{10} = \lg$
$\log_e = \ln$
$\log_a a = 1$
$\log_a 1 = 0$

log: $y = \log_a x$; $a > 1$; $0 < a < 1$
exp: $y = b^{ax}$; $a > 0$; $a < 0$

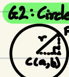


when "t becomes very large" or "extended period of time", $t = \infty$

A6: Binomial
$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r$
$\binom{n}{r} = \frac{n!}{(n-r)!\,r!}$; $n! = n \times (n-1) \times (n-2) \times \dots \times 3 \times 2 \times 1$
$\binom{n}{2} = \frac{n(n-1)}{2}$
$\binom{n}{3} = \frac{n(n-1)(n-2)}{6}$
General formula: $T_{r+1} = \binom{n}{r} a^{n-r} b^r$
↳ $(r+1)^{th}$ term

Geometry

G1: Coordinate Geometry
line eqn: $y - y_1 = m(x - x_1)$
Perpendicular lines: $l_1 \perp l_2 \Leftrightarrow m_1 m_2 = -1$
Area of polygon: shoelace method — pts are taken in ACW direction
$\frac{1}{2}\begin{vmatrix} x_1 & x_2 & \cdots & x_n & x_1 \\ y_1 & y_2 & \cdots & y_n & y_1 \end{vmatrix}$
$= \frac{1}{2}\left[(x_1y_2 + x_2y_3 + \dots + x_ny_1) - (y_1x_2 + y_2x_3 + \dots + y_nx_1)\right]$

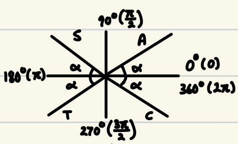
G2: Circles


Standard form: $(x-a)^2 + (y-b)^2 = r^2$

G3: Linear Law
Linear law: convert non-linear graph to $Y = mX + C$
$m = \frac{Y - Y_1}{X - X_1} \Rightarrow Y - Y_1 = m(X - X_1)$

G4: Trigo Function
$1° = \frac{\pi}{180}$ rad
$1 \text{ rad} = \frac{180°}{\pi}$

	$30°\left(\frac{\pi}{6}\right)$	$45°\left(\frac{\pi}{4}\right)$	$60°\left(\frac{\pi}{3}\right)$
$\sin\theta$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\tan\theta$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$



1st Quadrant: $0° < \theta < 90°$, all +ve, $\theta = \alpha$
2nd Quadrant: $90° < \theta < 180°$, only $\sin\theta$ +ve, $\theta = 180° - \alpha$
3rd Quadrant: $180° < \theta < 270°$, only $\tan\theta$ +ve, $\theta = 180° + \alpha$
4th Quadrant: $270° < \theta < 360°$, only $\cos\theta$ -ve, $\theta = 360° - \alpha$

Complementary ∠s: $\sin(90° - \theta) = \cos\theta$
$\cos(90° - \theta) = \sin\theta$
$\tan(90° - \theta) = \frac{1}{\tan\theta}$

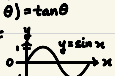
∠s > 360°: $\sin(360n° + \theta) = \sin\theta$
$\cos(360n° + \theta) = \cos\theta$
$\tan(360n° + \theta) = \tan\theta$

-ve ∠s: $\sin(-\theta) = -\sin\theta$
$\cos(-\theta) = \cos\theta$
$\tan(-\theta) = -\tan\theta$

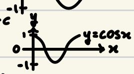
$\csc\theta = \frac{1}{\sin\theta}$
$\sec\theta = \frac{1}{\cos\theta}$
$\cot\theta = \frac{1}{\tan\theta}$

Principal values: $-\frac{\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2}, -1 \leq x \leq 1$
$0 \leq \cos^{-1} x \leq \pi, -1 \leq x \leq 1$
$-\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$

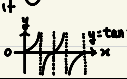
Sine: $y = a\sin(bx) + c$
↳ $b \times$ period $= 360°$



Cosine: $y = a\cos(bx) + c$
↳ $b \times$ period $= 360°$



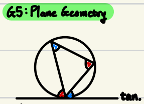
Tangent: $y = a\tan(bx)$
↳ $b \times$ period $= 180°$



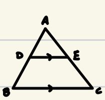
G5: Trigo Identities & Eqns
$\cot\theta = \frac{\cos\theta}{\sin\theta}$
Pythagorean Identities:
$\sin^2\theta + \cos^2\theta = 1$
$1 + \tan^2\theta = \sec^2\theta$
$1 + \cot^2\theta = \csc^2\theta$
Addition formulae:
$\sin(A \pm B) = \sin A\cos B \pm \cos A \sin B$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
R-formula:
$a\sin\theta \pm b\cos\theta = R\sin(\theta \pm \alpha)$
$a\cos\theta \pm b\sin\theta = R\cos(\theta \mp \alpha)$
$R = \sqrt{a^2 + b^2}$
$\alpha = \tan^{-1}\left(\frac{b}{a}\right)$

Double ∠ formulae:
$\sin 2A = 2\sin A\cos A$
$\cos 2A = \cos^2 A - \sin^2 A$
$= 2\cos^2 A - 1$
$= 1 - 2\sin^2 A$
$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$

$y = a\sin bx + c$
$y = a\cos bx + c$
$a = \frac{max - min}{2}$; $\frac{2\pi}{b}$; $c = \frac{max + min}{2}$

G5: Plane Geometry


(alt. segment theorem)



(mid pt. theorem)
$DE = \frac{1}{2}BC$

Calculus

C1: Differentiation
Power rule: $y = x^n, \frac{dy}{dx} = nx^{n-1}$
Constant multiple rule: $\frac{d}{dx}[kf(x)] = k\frac{d}{dx}[f(x)]$
Constant rule: $y = k, \frac{dy}{dx} = 0$
Sum & Difference rule: $\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$
Chain rule: $y = f[g(x)], \frac{dy}{dx} = \frac{df}{dg} \times \frac{dg}{dx}$
Product rule: $\frac{d}{dx}(uv) = v\frac{du}{dx} + u\frac{dv}{dx}$
Quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

C2: Tangent, Normal & ROC
$m_{tan} = f'(x)$
normal ⊥ tan
$m_{normal} = -\frac{1}{f'(x)}$
Increasing function: $\frac{dy}{dx} > 0$
Decreasing function: $\frac{dy}{dx} < 0$
ROC of y at a $= \left.\frac{dy}{dx}\right|_{x=a}$
Connected ROC: $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

C3: Maxima & Minima
1st Derivative test:
1. Find $\frac{dy}{dx} = 0$
2.

x	x^-	x	x^+
$\frac{dy}{dx}$	$f'(x^-)$	0	$f'(x^+)$
gradient		—	
sketch			

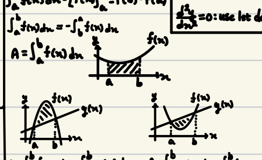
3. ∩ : max pt.
∪ : min pt.
~ or ⌞⌝ : pt. of inflection

2nd Derivative test:
$\frac{d^2y}{dx^2} < 0$: max pt.
$\frac{d^2y}{dx^2} > 0$: min pt.
$\frac{d^2y}{dx^2} = 0$: use 1st derivative test

C5: Differentiation of Trigo, Exp & Log
Trigo:
$\frac{d}{dx}(\sin u) = \cos u \times \frac{du}{dx}$
$\frac{d}{dx}(\cos u) = -\sin u \times \frac{du}{dx}$
$\frac{d}{dx}(\tan u) = \sec^2 u \times \frac{du}{dx}$
Exp & Log:
$\frac{d}{dx}(e^u) = \frac{du}{dx} \times e^u$
$\frac{d}{dx}(\ln u) = \frac{du}{dx} \times \frac{1}{u}$

C6: Integration
General rules:
$\int f(x)\,dx = F(x) + C$, $F'(x) = f(x)$
$\int [f(x)]^n dx = \frac{[f(x)]^{n+1}}{(n+1) \times f'(x)} + C$
$\int kf(x)\,dx = k\int f(x)\,dx$
$\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$
Trigo:
$\int \cos u\,dx = \sin u \div \frac{du}{dx} + C$
$\int \sin u\,dx = -\cos u \div \frac{du}{dx} + C$
$\int \sec^2 u\,dx = \tan u \div \frac{du}{dx} + C$
Exp & Log:
$\int e^u\,dx = e^u \div \frac{du}{dx} + C$
$\int \frac{1}{u}\,dx = \ln u \div \frac{du}{dx} + C$
$\int \frac{f'(x)}{f(x)}\,dx = \ln[f(x)] + C$

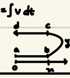
C7: Definite Integration
$\int_a^b f(x)\,dx = [F(x)]_a^b = F(b) - F(a)$
$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
$A = \int_a^b f(x)\,dx$



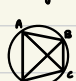
$A = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$; $A = \int_a^b g(x)\,dx - \int_a^b f(x)\,dx$

C8: Kinematics
displacement: $s \Rightarrow s = \int v\,dt$
At a: $s = 0, d = 0$
At b: $s = x, d = x$
At c: $s = x, d = x + y$
At d: $s = 0, d = 2x + y$

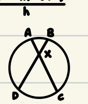
velocity: $v = \frac{ds}{dt} \Rightarrow v = \int a\,dt$
acceleration: $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$



Additional info
Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$
$s = \frac{a+b+c}{2}$
Differentiation by 1st Principles: $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$



(Ptolemy's Theorem)
$AD \times BC + AB \times DC = AC \times BD$



(intersecting chord theorem)
$AX \times XC = BX \times XD$