Algebra

A1: Quadratic

Factorisation: $(ax+b)(cx+d)=0$

Completing sq: $(x+h)^2+k=0 \Rightarrow 1.\ (x+h)^2=-k$

$\rightarrow$ 1. ax^2+bx+c 2. $x+h=\pm\sqrt{-k}$ (if k is +ve: no real roots)

2. $a(x^2+\frac{b}{a}x+\frac{c}{a})$ 3. $x=\pm\sqrt{-k}-h$

3. $a[(x+\frac{b}{2a})^2+\frac{c}{a}-(\frac{b}{2a})^2]$

4. $a(x+h)^2+k$

Formula: $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Graphs:

$a\geq 1$: $\rightarrow$ 2 real & distinct; 2 real & eql; no real

$a<0$: $\rightarrow$ 2 real & distinct; 2 real & eql; no real

Find y-intercept: $x=0$

Find x-intercept: $y=0$

line of sym: $x=\frac{x_1+x_2}{2}$

turning pt: $(-h,k)$

ins intersect curve: simultaneous eqns

A2: Inequalities

$>$: more than

$<$: less than

$\geq$: more than or eql

$\leq$: less than or eql

when $\div$ or $\times$ by -ve no., flip sign

Number line:

$\rightarrow \circ$: more/less than or eql

$\rightarrow o$: more/less than

overlap: soln

no overlap: no soln

A3: Indices

Laws: $a^m\times a^n=a^{m+n}$

$a^m\div a^n=a^{m-n}$

$(a^m)^n=a^{mn}$

$(ab)^n=a^nb^n$

$(\frac{a}{b})^n=\frac{a^n}{b^n}$

$a^{-n}=\frac{1}{a^n}$

$(\frac{a}{b})^{-n}=(\frac{b}{a})^n$

$a^{\frac{1}{n}}=\sqrt[n]{a}$

$a^{\frac{m}{n}}=\sqrt[n]{a^m}=(\sqrt[n]{a})^m$

Solve eqns: make same base

Simple interest: $I=\frac{Prt}{100}$

Compound interest: $A=P(1+\frac{r}{100})^n$

A4: Functions & Graphs

Power function: $y=ax^n$

- line: $y=ax^0$
- linear: $y=ax^1$
- Quadratic: $y=ax^2$
- Cubic: $y=ax^3$
- Reciprocal: $y=\frac{a}{x}=ax^{-1}$ (asymptote)
- Reciprocal squared: $y=\frac{a}{x^2}=ax^{-2}$

Tangent: str. line that intersect only 1 pt

Distance: horizontal line: rest; str line: constant speed; curve: varying speed; $m=\frac{d_2-d_1}{t_2-t_1}$ = speed

Speed: horizontal line: constant speed; str line: constant acceleration; curve: varying acceleration; $m=\frac{v_2-v_1}{t_2-t_1}$ = acceleration; area under graph = d

depth-time graph:

A5: Set Notation

$P=\{a,b,c\ldots\}$ (set, element)

If $X=\{a,b,c\}$ & $Y=\{c,a,b\}$, $X=Y$

empty set: no element; $\rightarrow \{\}$ or $\varnothing$

no. of element in n: k

Finite set: finite no. of element, $n(n)=k$

Infinite set: inf. no. of element, $n(y)=\infty$

$\in$: is an element of

$\notin$: is not an element of

proper subset $\subset$: smaller set within the set

subset $\subseteq$: smaller or eql set within the set

ξ: universal set; set with all elements

A': complement of A: element not in A

$\cup$: union: elements of the sets combined

$\cap$: intersect: elements in both sets

A6: Matrix

Order: row $\times$ column

Square matrix: order $n\times n$

Identity matrix: $\begin{pmatrix}1&0\\0&1\end{pmatrix}$

Addition: $\begin{pmatrix}a\\b\end{pmatrix}+\begin{pmatrix}c\\d\end{pmatrix}=\begin{pmatrix}a+c\\b+d\end{pmatrix}$ (element)

Subtraction: $\begin{pmatrix}a\\b\end{pmatrix}-\begin{pmatrix}c\\d\end{pmatrix}=\begin{pmatrix}a-c\\b-d\end{pmatrix}$

Scalar multiplication: $k\begin{pmatrix}a\\b\end{pmatrix}=\begin{pmatrix}ka\\kb\end{pmatrix}$

matrices A & B must be of same order

Matrix multiplication: $\begin{pmatrix}a&b\\c&d\end{pmatrix}\begin{pmatrix}e&f\\g&h\end{pmatrix}=\begin{pmatrix}ae+bg&af+bh\\ce+dg&cf+dh\end{pmatrix}$

$2\times\underline{2}$ $\quad$ $\underline{2}\times 2$ $\rightarrow 2\times 2$

n_{row} of 1st matrix = n_{column} of 2nd matrix, otherwise undefined

Geometry

G1: Coordinate Geometry

linear eqn: $y=mx+c$ (gradient, y-intercept); $m=\frac{y_2-y_1}{x_2-x_1}$; m: undefined, $m>0$, $m<0$

Parallel lines: $l_1\parallel l_2 \Leftrightarrow m_1=m_2$

length $=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Collinear: same gradient; $m_{AB}=m_{BC}=m_{AC}$

G2: Congruence & Similarity

Congruent: $\equiv$ (corr. $\angle$s & sides eql)

same shape & size, diff. orientation

Congruence tests:

1. SSS: 3 sides of $\Delta_1\equiv$ 3 corr. sides of Δ_2
2. SAS: 2 sides & 1 $\angle$ of Δ_1 = 2 corr. sides & corr $\angle$ of Δ_2
3. ASA: 1 side & 2 $\angle$s of Δ_1 = 1 corr. side & 2 corr $\angle$s of Δ_2
4. RHS: hyp & 1 side of right-angled Δ_1 = corr. hyp & corr. side of right-angled Δ_2

Similar: same shape, diff. size (all same corr. $\angle$s, all same ratio of corr. sides)

Similarity tests:

1. AA similarity test: 2 $\angle$s = 2 corr. $\angle$s
2. SSS similarity test: 3 ratio of corr. sides are eql
3. SAS similarity test: 2 ratio of corr. sides & included $\angle$s are eql

Ratio of areas of similar fig: $\frac{A_1}{A_2}=(\frac{l_1}{l_2})^2$

Ratio of areas of non-similar Δ: $\frac{A_1}{A_2}=\frac{l_1}{l_2}$ (Δs same height)

Ratio of vol. of similar solids: $\frac{V_1}{V_2}=(\frac{l_1}{l_2})^3$

Mass & vol.: $\frac{m_1}{m_2}=\frac{V_1}{V_2}$

Perpendicular bisector: line that bisect $\perp$ line into 2 eql parts

Angle bisector: line that bisect $\angle$ into 2 eql parts

G3: Further Trigonometry

For obtuse $\angle$: $\sin(180°-\theta)=\sin\theta$

$\cos(180°-\theta)=-\cos\theta$

Scalene Δ: $\frac{1}{2}ab\sin C$

Sine rule: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$

when 2 sides & non included $\angle$: ambiguous case: $\sin A=\frac{a\sin B}{b}$, $A=180°-\sin^{-1}(\frac{a\sin B}{b})$

Cosine rule: $a^2=b^2+c^2-2bc\cos A$

$\cos A=\frac{b^2+c^2-a^2}{2bc}$

G4: Arc Length

major segment, minor segment, chord; major sector, minor sector, radius, major arc, minor arc

Deg: Arc length: $\frac{\theta}{360°}\times 2\pi r$; $1°=\frac{\pi}{180}$ rad

Sector area: $\frac{\theta}{360°}\times\pi r^2$; $180°=\pi$ rad

Rad: Arc length: $r\theta$; 1 rad $=\frac{180°}{\pi}$

Sector area: $\frac{1}{2}r^2\theta$

G5: Circle Properties

Symmetry:

1. $\perp$ bisector of chord
2. chords with equal length
3. tan $\perp$ rad
4. tan from an ext. pt.

$\angle$s:

1. $\angle$ at centre = 2$\angle$ at circumference
2. $\angle$ in semicircle
3. $\angle$s in the same segment
4. $\angle$s in opp. segments

G6: Vectors

Vector: $\underline{v}$ or $\overrightarrow{AB}$ or $\begin{pmatrix}a\\b\end{pmatrix}\Rightarrow(a,b)$

$\underline{a}+\underline{b}=\underline{c}$

Magnitude/length: $|\overrightarrow{AB}|=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

Translation from A to B: AB

$\overrightarrow{AB}=\overrightarrow{AO}+\overrightarrow{OB}$

$=\overrightarrow{OB}-\overrightarrow{OA}=-\overrightarrow{OA}+\overrightarrow{OB}$ (O: origin)

Collinear: $\overrightarrow{AB}=k\overrightarrow{BC}$

Statistics & Probability

S1: Statistics

Cumulative graph: percentile; Q_1: lower quartile $=P_{25}$; Q_2: median $=P_{50}$; Q_3: upper quartile $=P_{75}$

Range = max − min

IQR $=Q_3-Q_1$

Box-&-Whisker: min, Q_1, Q_2, Q_3, max

$SD\cdot\sigma=\sqrt{\frac{\Sigma(x_i-\bar{x})^2}{n}}$ or $\sqrt{\frac{\Sigma fx^2}{\Sigma f}-(\frac{\Sigma fx}{\Sigma f})^2}$

When comparing:

performance: median > range (higher better)

consistency: SD > IQR > range (lower better)

S2: Probability

$P(E)=1$: certainly happening

$P(E)=0$: certainly not happening

Probability Diagram; Listing; Probability Tree (1st, 2nd)

Multiplication law: if 2 events are independent, $P(A\cap B)=P(A)\times P(B)$

Addition law: if 2 events are mutually exclusive, $P(A\cup B)=P(A)+P(B)$

$\rightarrow$ they cannot be the same time, $P(A\cap B)=0$