General Certificate of Education Ordinary Level 2023
Mathematics 4038/01
Syllabus 4038 Paper 1
SUGGESTED Solutions & Marking Scheme Mark Allocation
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1	(i)	Angle $ABX = 180° - 120° = 60°$ (adj. angles on a st. line)			
		$\sin 60° = \frac{AX}{4} \Rightarrow AX = 4\left(\frac{\sqrt{3}}{2}\right)$	M1	$\frac{\sqrt{3}}{2}$ s.o.i.	
		$AX = 2\sqrt{3}$ cm	A1		
	(ii)	By Pythagoras' Theorem, $BX = \sqrt{4^2 - \left(2\sqrt{3}\right)^2} = 2$ cm	M1		
		$\tan \angle ACB = \frac{2\sqrt{3}}{4}$	M1		
		$\angle ACB = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (Shown)	A1 AG		
					[5]
2		$9^x\left(\sqrt{27}\right)^y = 1$ $\left(3^2\right)^x\left(3^{\frac{3}{2}}\right)^y = 3^0$ $8^y \div \left(\sqrt{2}\right)^x = 16\sqrt{2}$ $\left(2^3\right)^y \div \left(2^{\frac{1}{2}}\right)^x = 2^4\left(2^{\frac{1}{2}}\right)$	M1	Attempt to change to a common base in either equation	
		$3^{2x+\frac{3}{2}y} = 3^0$ $2x + \frac{3}{2}y = 0$ $x = -\frac{3}{4}y \quad --(1)$ $2^{3y-\frac{1}{2}x} = 2^{\frac{9}{2}}$ $3y - \frac{1}{2}x = \frac{9}{2}$ $6y - x = 9 \quad --(2)$	M1	Either equation obtained through equating powers s.o.i.	
		Substitute (1) into (1) $6y - \left(-\frac{3}{4}y\right) = 9$	M1	Substitution or elimination s.o.i.	
		$\frac{27}{4}y = 9$ $y = \frac{4}{3}$	A1	o.e.	
		$x = -1$	A1		
					[5]
3		**NOT** within the 4049 syllabus	E5		
					[5]

4	(i)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(x^3 \ln x\right) = x^3\left(\frac{1}{x}\right) + 3x^2 \ln x$	M1	Apply the product rule correctly	
		$= x^2 + 3x^2 \ln x$	A1		
	(ii)	$\int x^2 + 3x^2 \ln x \ \mathrm{d}x = x^3 \ln x + c_1$	M1	Use result in (i)	
		$3\int x^2 \ln x \, \mathrm{d}x = x^3 \ln x + c_1 - \int x^2 \, \mathrm{d}x$ $= x^3 \ln x + c_1 - \frac{x^3}{3} + c_2$	M1	Correct integration	
		$\int x^2 \ln x \, \mathrm{d}x = \frac{x^3}{3} \ln x - \frac{x^3}{9} + d$, where $d = c_1 + c_2$	A1	Final answer with arbitrary constant	
					[5]
5	(i)	Let $\frac{8x-46}{(x-5)(x+1)} = \frac{A}{x-5} + \frac{B}{x+1}$	M1		
		$8x - 46 = A(x+1) + B(x-5)$			
		When $x = -1$, $-54 = -6B$ $B = 9$ When $x = 5$, $-6 = 6A$ $A = -1$	M1	Find A or B correctly	
		$\frac{8x-46}{(x-5)(x+1)} = \frac{9}{x+1} - \frac{1}{x-5}$	A1		
	(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{9}{(x+1)^2} + \frac{1}{(x-5)^2}$	M2	M1 for each term	
		At $x = 2$, $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{8}{9}$.	A1		
					[6]

6	(i)	12 seconds	B1	c.a.o.	
	(ii)	Distance $= \int_0^{12} 6t - \frac{1}{2}t^2 \, \mathrm{d}t$	M1	Integral seen	
		$= \left[3t^2 - \frac{1}{6}t^3\right]_0^{12}$	M1	Correct integration	
		$= 3(12)^2 - \frac{1}{6}(12)^3 = 144 \text{ m}$	A1		
	(iii)	$a = \frac{\mathrm{d}v}{\mathrm{d}t} = 6 - t$	M1		
		-2 m/s^2	A1	c.a.o.	
					[6]
7		$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(2-\cos x)(\cos x) - \sin^2 x}{(2-\cos x)^2}$	M2	M1 Attempt to use quotient rule M1 for any correct differentiation	
		$= \frac{2\cos x - \cos^2 x - \sin^2 x}{(2-\cos x)^2}$	M1	Simplify	
		$\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow \frac{2\cos x - \cos^2 x - \sin^2 x}{(2-\cos x)^2} = 0$	M1	s.o.i.	
		$2\cos x - \left(\cos^2 x + \sin^2 x\right) = 0$ $2\cos x - 1 = 0$	M1	Convert using $\sin^2 x + \cos^2 x = 1$	
		$\cos x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}$	A1		
					[6]

8	(i)	$\sin 3x + \sin x$ $= \sin(2x + x) + \sin x$ $= \sin 2x \cos x + \cos 2x \sin x + \sin x$	M1	Compound angle formula s.o.i.	
		$= 2\sin x \cos^2 x + \sin x(2\cos^2 x - 1) + \sin x$	M1	Double angle formula s.o.i.	
		$= 2\sin x \cos^2 x + 2\sin x \cos^2 x - \sin x + \sin x$ $= 4\sin x \cos^2 x$ (Shown)	A1 AG		
	(ii)	$4\sin x \cos^2 x = 2\cos^2 x$ $4\sin x \cos^2 x - 2\cos^2 x = 0$ $\cos^2 x(4\sin x - 2) = 0$	M1	Factorise o.e.	
		$\cos^2 x = 0$ $\cos x = 0$ $x = \frac{\pi}{2}$ $4\sin x - 2 = 0$ $\sin x = \frac{1}{2}$ $x = \frac{\pi}{6}, \frac{5\pi}{6}$	A2	A1 for any value A1 for all values	
					[6]
9		Let Ann and Betty's age be x and y years respectively.			
		$x^2 - 2y^2 = 6(x - y)$ ---(1)	M1		
		$x + y = 5(x - y)$ ---(2)	M1		
		From (2) $x + y = 5x - 5y$ $3x = 6y$ $x = \frac{3}{2}y$ ---(3)			
		Substitute (3) into (1) $\left(\frac{3}{2}y\right)^2 - 2y^2 = 6\left(\frac{3}{2}y - y\right)$	M1	Substitution s.o.i.	
		$\frac{9}{4}y^2 - 2y^2 = 6\left(\frac{1}{2}y\right) \Rightarrow \frac{1}{4}y^2 - 3y = 0$ $y\left(\frac{1}{4}y - 3\right) = 0$ $y = 0$ (reject) *or* $y = 12$	M1	Must reject	
		When $y = 12$, $x = 18$.			
		Ann's age is 18 years and Betty's age is 12 years.	A2	Conclude	
					[6]

10	(i)	Discriminant <0 $5^2-4a(2)<0$	M1	s.o.i.	
		$25-8a<0$ $8a>25 \Rightarrow a>\dfrac{25}{8}$	M1	o.e.	
		Smallest integer $a=4$	A1		
	(ii)	Discriminant <0 $b^2-4(-5)(-2)<0$	M1	s.o.i.	
		$b^2-40<0$ $\left(b+\sqrt{40}\right)\left(b-\sqrt{40}\right)<0$ $-2\sqrt{10}<b<2\sqrt{10}$	M1	o.e.	
		Smallest integer $b=-6$	A1		
					[6]
11	(i)	$T_{r+1}=\binom{7}{r}x^{7-r}\left(kx^{-1}\right)^r$ $=\binom{7}{r}k^r x^{7-r}\left(x^{-r}\right)=\binom{7}{r}k^r x^{7-2r}$	M1	Attempt to simplify the general term o.e.	
		For x^3, $7-2r=3 \Rightarrow r=2$. For x, $7-2r=1 \Rightarrow r=3$.	M1	Find values of r	
		$\binom{7}{2}k^2=\binom{7}{3}k^3$	M1	Form equation	
		$21k^2=35k^3$ $35k=21 \quad (\because k>0, \text{ then } k\neq 0)$ $k=\dfrac{3}{5}$	A1	o.e.	
	(ii)	For x^5, $7-2r=5 \Rightarrow r=1$. For x^7, $7-2r=7 \Rightarrow r=0$.	M1	Find values of r	
		Coefficient of $x^5=\dfrac{21}{5}$ Coefficient of $x^7=1$	M1	Either one s.o.i.	
		Coefficient of $x^7=1+(-5)\left(\dfrac{21}{5}\right)=-20$	A1		
					[5]

12	(i)	$y = kb^x$ $\lg y = \lg\left(kb^x\right)$ $= \lg k + \lg b^x$ $= \lg k + x\lg b$	M1	Attempt to take lg on both sides to rewrite equation	
		Gradient $= \dfrac{1.3-0.8}{0-11} = -\dfrac{1}{22}$	M1	s.o.i.	
		$\lg b = -\dfrac{1}{22}$ $b = 10^{-\frac{1}{22}}$	M1	Convert back to index form	
		$b = 0.9006280202 = 0.90$ (2 s.f.)	A1		
		$\lg k = 1.3$ $k = 10^{1.3}$	M1	Convert back to index form	
		$k = 19.95262315 = 20$ (2 s.f.)	A1		
	(ii)	$y = 10^{1.3}\left(10^{-\frac{1}{22}}\right)^8$	M1	FT from (i)	
		$y = 8.637014255 = 8.64$ (3 s.f.)	A1		
					[8]

13	(i)	$2(5x)+2h+6x=360$ $16x+2h=360$ $h=180-8x$	M1	Expression for h in terms of x s.o.i.	
		Height of triangle $QRS=\sqrt{(5x)^2-(3x)^2}$	M1	Use Pythagoras' Theorem s.o.i.	
		$=\sqrt{25x^2-9x^2}$ $=\sqrt{16x^2}=4x$	M1		
		$A=h(6x)+\frac{1}{2}(6x)(4x)$ $=6x(180-8x)+12x^2$ $=1080x-36x^2$ (Shown)	A1 AG		
	(ii)	$\frac{\mathrm{d}A}{\mathrm{d}x}=1080-72x$	M1		
		$\frac{\mathrm{d}A}{\mathrm{d}x}=0\Rightarrow 1080-72x=0$	M1	s.o.i.	
		$72x=1080$ $x=15$	M1		
		When $x=15$, $A=1080(15)-36(15)^2=8100$	A1		
	(iii)	$\frac{\mathrm{d}^2A}{\mathrm{d}x^2}=-72<0$ Hence, this value of A is a maximum value.	B1		
					[9]

END OF MARKING SCHEME

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