MA1132 Notes

Foundations in Mathematics IB

Chapters 1 thru 7

This is a compilation of topics taught in MA1132.
Made by LQ

Links of All LQ Notes: LQ Notes Links Document

Author's Note:

This set of notes were mainly made as a "dump" and was not really intended for viewing, so I must admit the standard (as compared to the other notes available) is really just not there.

Since this was a "dump", I also used some fancy notation that you are not required to know, including the functional notation f(x), g(x), as well as sigma notation Σ. Apologies for this.

Note: This module has since been renamed to MA1134[confirm].

MA1132 Chapter 1 : Simultaneous Linear Equations

System of Equations

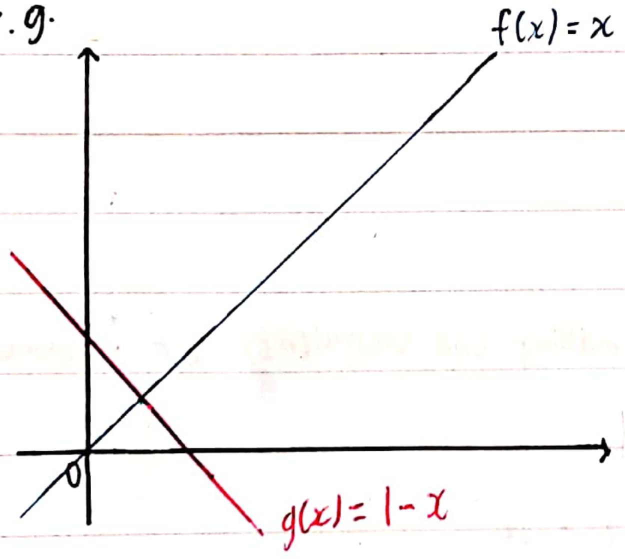
1. Any collection of two or more equations is a System of Equations.

2. If the equations of a system involve 2 variables, then the set of ordered pairs that satisfy all the equations is the solution set of the system.

3. A system of equations with 2 equations is known as Simultaneous Equations.

Given linear equations $f(x)$ and $g(x)$, how do we find their intersection?

E.g.



Here, we can solve for $x = 1 - x$.

$$+x \downarrow \qquad \downarrow +x$$

$$2x = 1$$

Hence, $x = \frac{1}{2}$ is where the functions intersect.

Plugging $x = \frac{1}{2}$, $f(x) = \frac{1}{2}$; $g(x) = 1 - \frac{1}{2}$
$= \frac{1}{2}$

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The Substitution Method

E.g. Solve for x and y in $\begin{matrix} 2x+3y=4 \\ y+3x=6 \end{matrix}$.

Since $y+3x=6$, $y=6-3x$.

Now, we can substitute $y=6-3x$ into $2x+3y=4$, getting

$2x+3(6-3x)=4 \Rightarrow 2x+18-9x=4$

$\Rightarrow 18-7x=4$

$\Rightarrow -7x=-14$

$\Rightarrow x=2$

Now, we substitute $x=2$ into $y+3x=6$, getting $y+3\cdot 2=6$.

$\Rightarrow y+6=6$

$\Rightarrow y=0$

Hence $x=2$, $y=0$ is the answer.

The Elimination Method

The Elimination Method involves making the coefficients of a variable the same in both equations.

E.g. Solve for x and y in $\begin{matrix} 2x-3y=-13 \\ 5x-12y=-46. \end{matrix}$

We want to eliminate y by making both coefficients -12.

We multiply Eq. 1 by 4, obtaining $8x - 12y = -52$

Now we subtract Eq. 1 from Eq. 2, obtaining $3x = -6 \Rightarrow x = -2$,

Next, we substitute $x = -2$ into $2x - 3y = -13$.

$2(-2) - 3y = -13$

$\Rightarrow -4 - 3y = -13$

$\Rightarrow \quad -3y = -9$

$\Rightarrow \quad y = 3$

Hence the solution is $x = -2$, $y = 3$.

MA1132 Chapter 2: Properties of Triangles

Basic Definitions

1. Point: An element in geometry with a position but no magnitude.

2. Line: A geometrical object that is straight, infinitely long and infinitely thin.

3. Line Segment: A straight line which links two points without extending beyond them.

4. Ray: A straight line extending infinitely in one direction from a fixed point.

5. Curve: A line, either straight or continuously bending without angles

6. Plane: A surface such that any line that joins two of the points of the surface lies on the surface

7. Angle: A figure formed by two line segments or rays that extends from a common point — the vertex, or by regions of two planes

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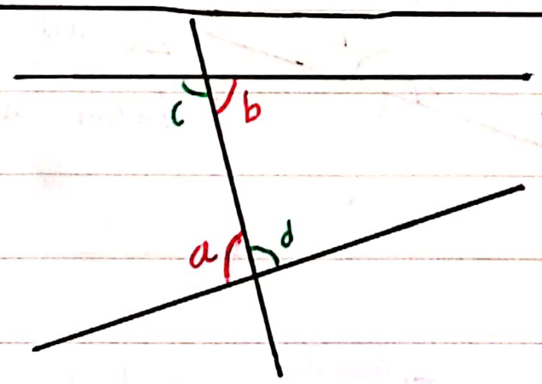
that extend from a common line.

Different types of angles

1. Acute angle — Between 0° and 90°.
2. Right angle — 90° exactly
3. Obtuse angle — Between 90° and 180°
4. Straight angle — 180° exactly
5. Reflex angle — Between 180° and 360°
6. Adjacent angles — Angles that lie next to each other
7. Angles at a point — The sum of angles at a point is 360°.
8. Complementary angles — Two angles with a sum of 90°.
9. Supplementary angles — Two angles with a sum of 180°
10. Alternate angles — Both pairs of angles between two given lines and a transversal, and lying on opposite sides of the transversal.

* Angles are equal if and only if the given lines are parallel

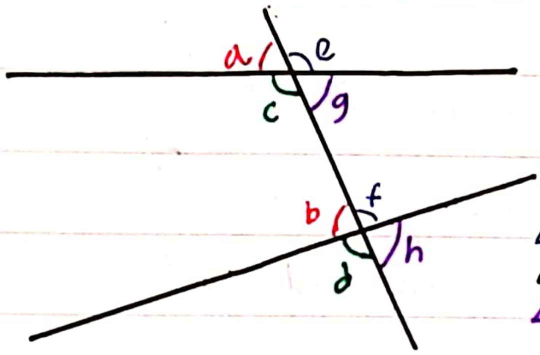


∠a and ∠b are alternate angles.

∠c and ∠d are alternate angles.

11. Corresponding angles — Angles that are on the same side of a transversal that cuts two lines, and on the same side of each line.

* Angles are equal if and only if the lines are parallel.



$\angle a$ and $\angle b$ are corresponding angles.

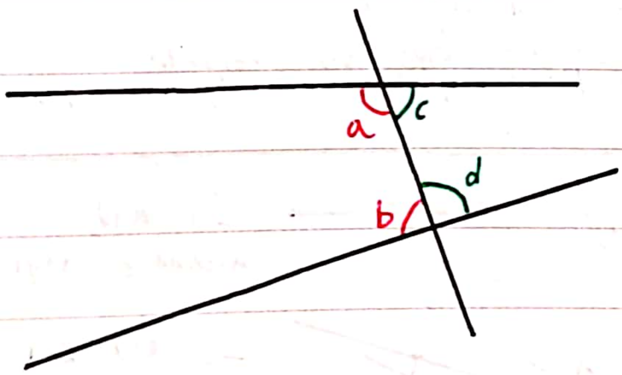
$\angle c$ and $\angle d$ are corresponding angles.

$\angle e$ and $\angle f$ are corresponding angles.

$\angle g$ and $\angle h$ are corresponding angles.

12. Interior angles — Both pairs of angles that lie on the same side of a transversal and between the other two lines.

* Angles are supplementary if and only if the lines are parallel.



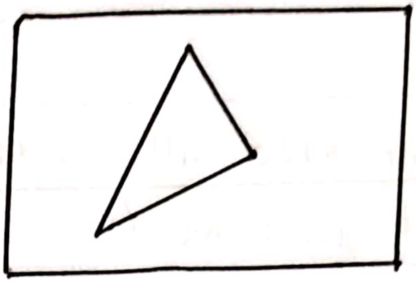
$\angle a$ and $\angle b$ are Interior angles.

$\angle c$ and $\angle d$ are Interior angles.

13. Vertically opposite angles — The pair of angles between two intersecting straight lines.

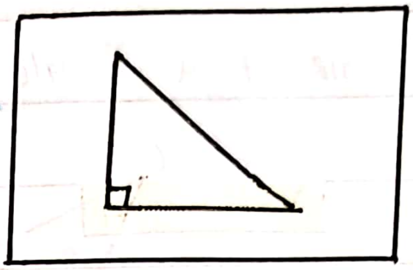
Classification of Triangles by Angles

1. Acute-angled Triangle



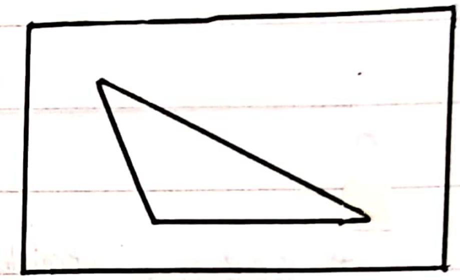
Triangle with three acute angles.

2. Right-angled Triangle



Triangle with a 90° angle.

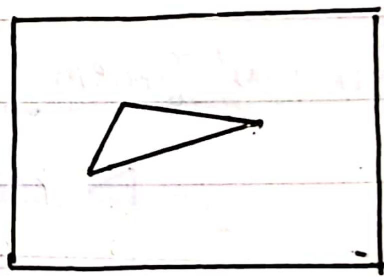
3. Obtuse-angled Triangle



Triangle with an angle larger than 90°.

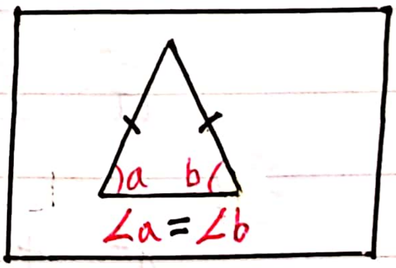
Classification of Triangles by Sides

1. Scalene Triangle



Triangle where all sides are of different length.

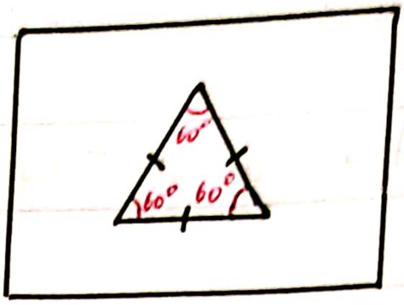
2. Isosceles Triangle



Triangle where two sides are equal in length.

— Its base angles are equal.

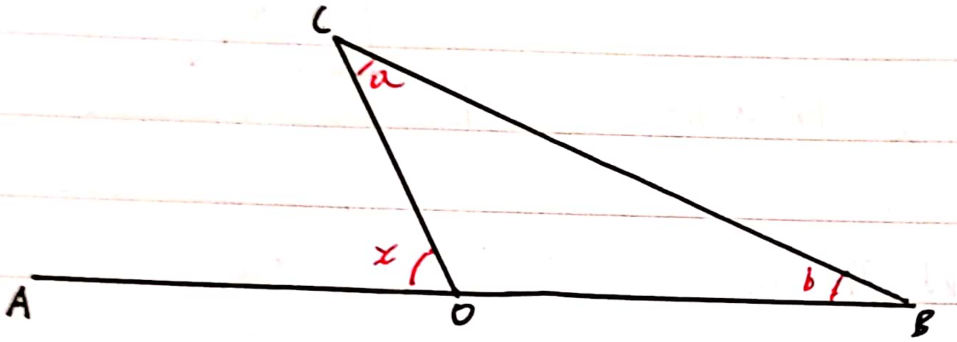
3. Equilateral Triangle



Triangle where all sides are equal in length.

- All angles are 60°.

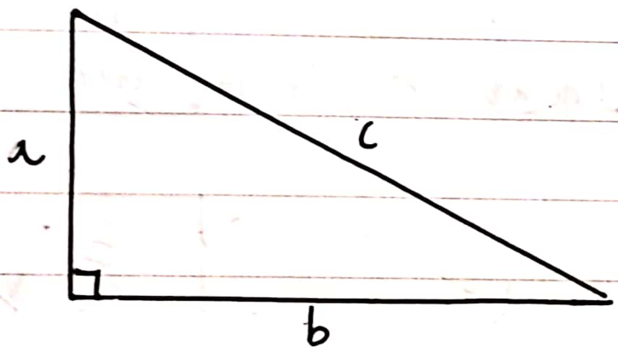
Exterior Angle of a Triangle



* AOB is a straight line.

$\angle a + \angle b = \angle x$

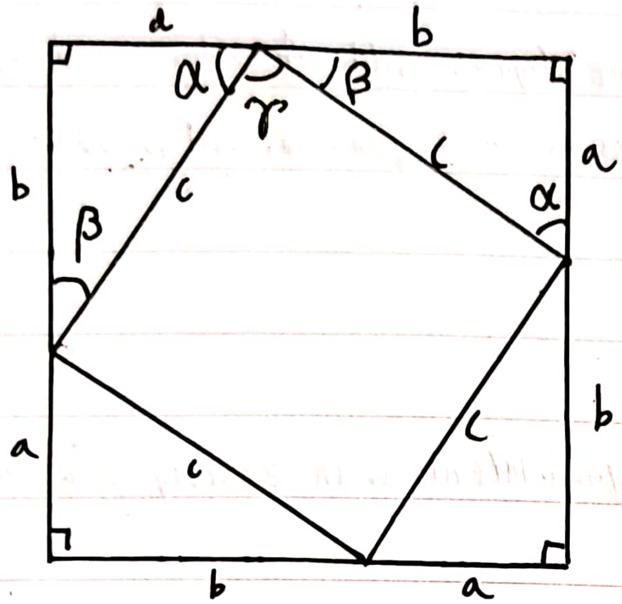
Pythagoras' Theorem



$a^2 + b^2 = c^2$

A Proof of Pythagoras' Theorem

Create 4 congruent right-angled triangles and arrange like so:



Proof $\gamma = 90°$: $\alpha + \beta + 90° = 180°$

$\alpha + \beta = 90°$

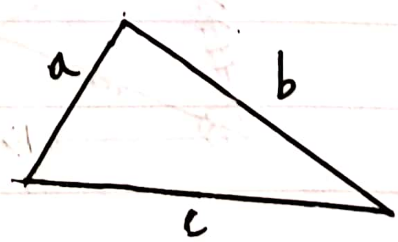
$180° - 90° = 90°$ (γ)

Now, we obtain $(a+b)^2 = 4 \cdot \frac{1}{2} \cdot ab + c^2$

$\Rightarrow a^2 + 2ab + b^2 = 2ab + c^2$

$\Rightarrow a^2 + b^2 = c^2$ (proven)

Triangle Inequality



Given triangle with sides a, b and c:

$a + b > c$ always holds.

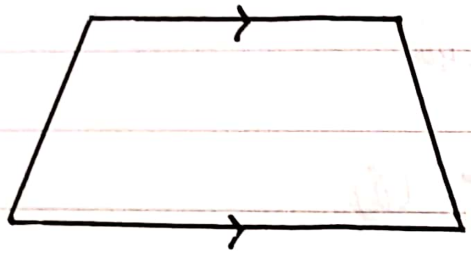
MA1132 Chapter 3: Quadrilaterals and Polygons

Quadrilaterals

Quadrilaterals are closed plane figures with 4 sides and 4 vertices.
The sum of interior angles in a quadrilateral is 360°.

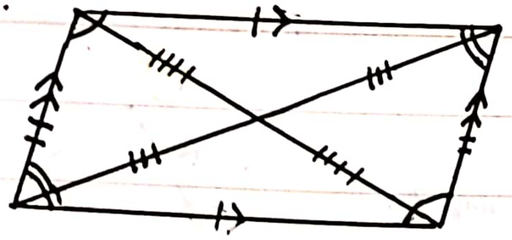
Trapezium

A trapezium is a quadrilateral with exactly one pair of parallel lines.



Parallelogram

A parallelogram is a quadrilateral with two pairs of parallel lines.



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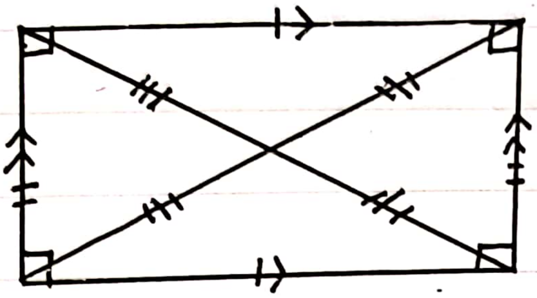
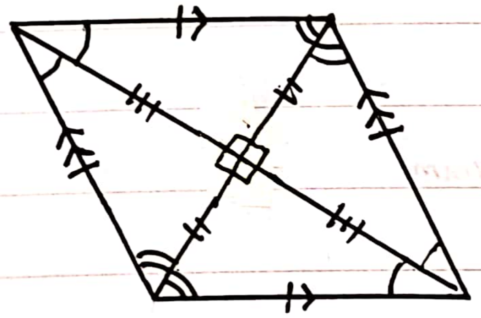
Rectangle

Rectangles are parallelograms with 4 right angles.



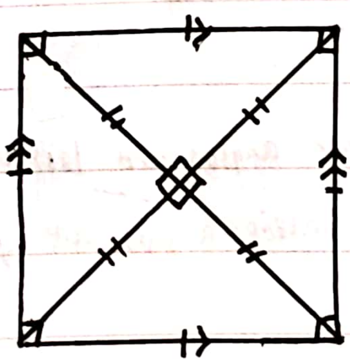
Rhombus

Rhombuses are parallelograms with 4 sides with equal length.



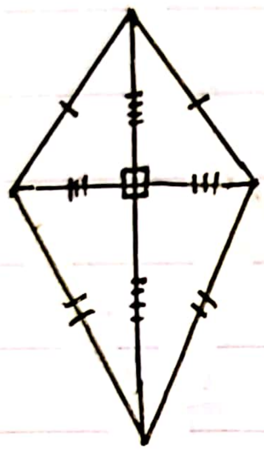
Square

Squares are both rectangles and rhombuses and have their traits.

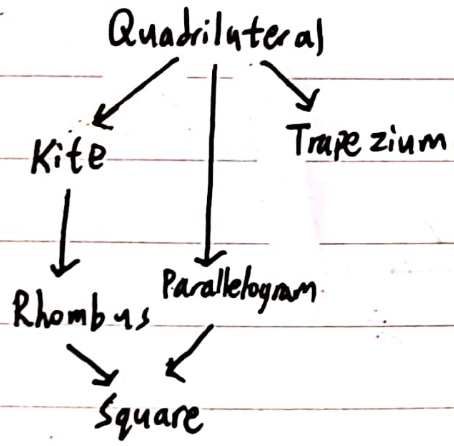


Kite

Kites are quadilateals with 2 pairs of equal adjacent sides.



Summary



Polygon

1. Convex Polygon: A polygon where all interior angles are less than 180°.
2. Concave Polygon: A polygon where one or more interior angles are greater than 180°.

3. Regular Polygon : A Polygon with all equal angles and all equal sides.

Types of Polygons

Name of Polygon	No. of sides
Triangle	3
Quandiilateral	4
Polygon	5
Hexagon	6
Heptagon / Septagon *	7
Octagon	8
Nonagon *	9
Decagon	10
Hendecagon / Undecagon *	11
Dodecagon	12

* Not in notes

Angle Properties of Polygons

Sum of interior angles of a Polygon with n sides = $180^\circ(n-2)$

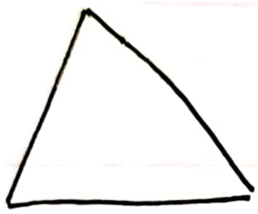
Sum of exterior angles of a Polygon is always 360°.

MA 1132 Chapter 4: Symmetry, Construction and Loci

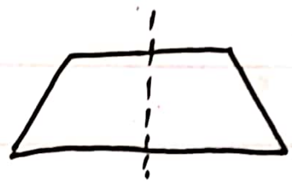
Line of Symmetry

The line of symmetry is a line in a figure such that the opposite sides of the figures are reflections of each other.

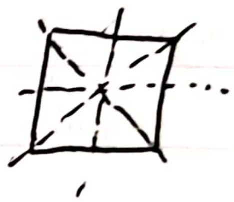
A figure can have more than 1 line of symmetry.



0 Lines of Symmetry



1 Line of Symmetry



4 Lines of Symmetry

Rotational Symmetry

If a figure can be rotated about a centre — the point of rotation through an angle other than 360°, without changing its original position, the figure can be said to have rotational symmetry about that point.

The order of rotational symmetry is the number of times the figure can be rotated without changing its original position.

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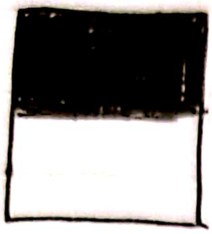
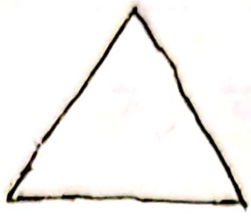
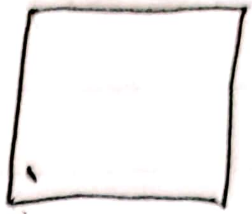


Order of rotational Symmetry 1
(No rotational Symmetry)

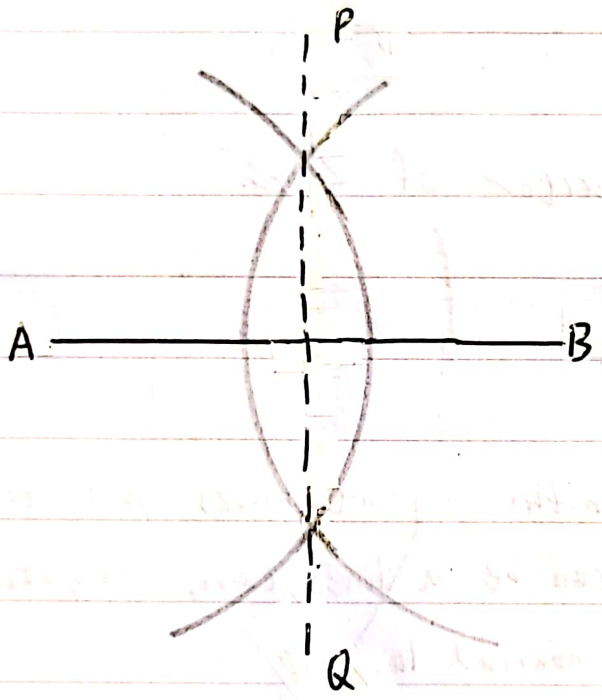


Order of rotational Symmetry 3

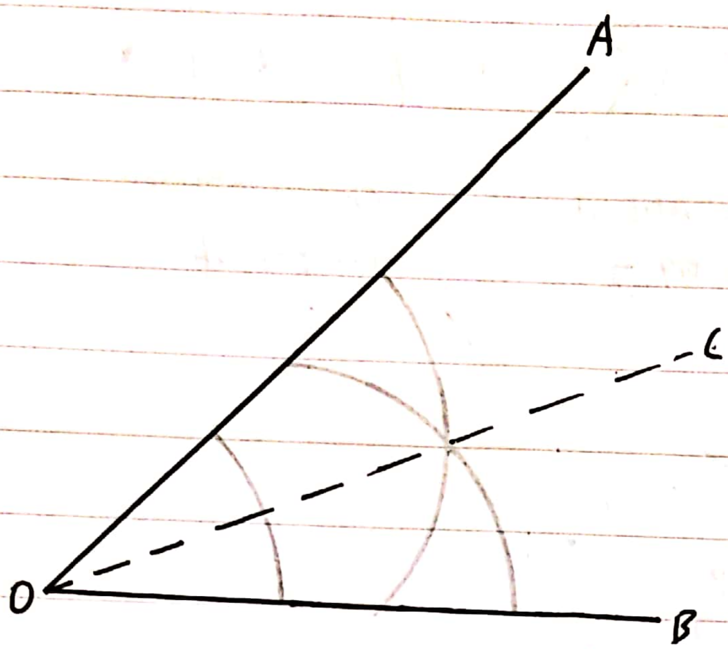


Order of rotational Symmetry 4

Construction of Perpendicular Bisector



Construction of Angle Bisector



OC is the angle bisector of $\angle AOB$.

Loci

A locus is the path along which a point moves so as to satisfy some given conditions. A locus can be a line, curve, plane etc. The plural of locus is loci (Pronounced lo-sai).

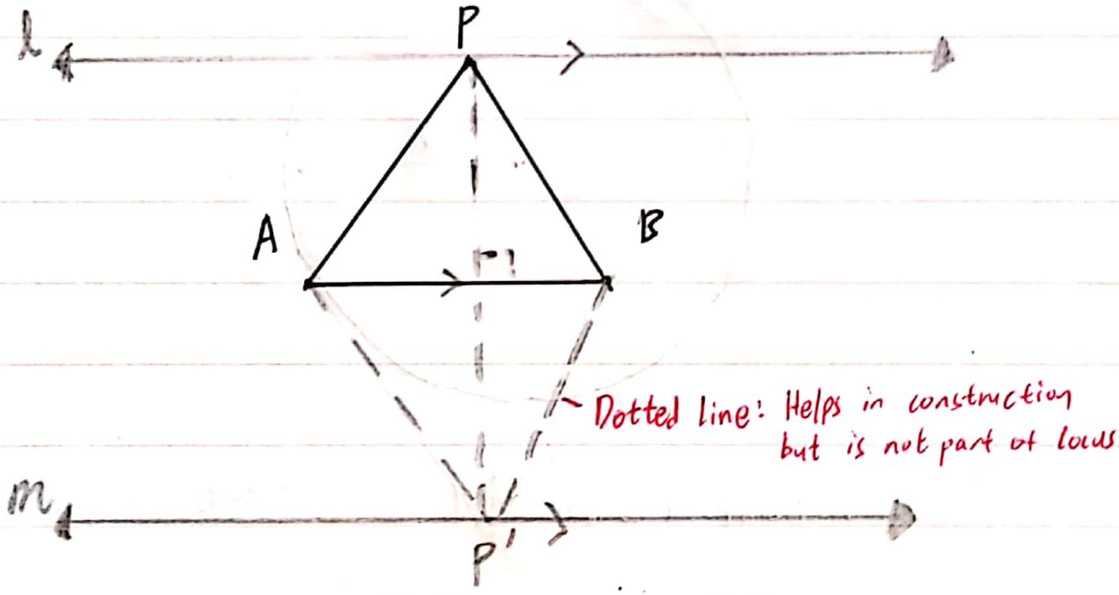
Locus of a point equidistant from two points

The locus would be the perpendicular bisector of the line between the two points.

Locus of a point equidistant from two intersecting lines

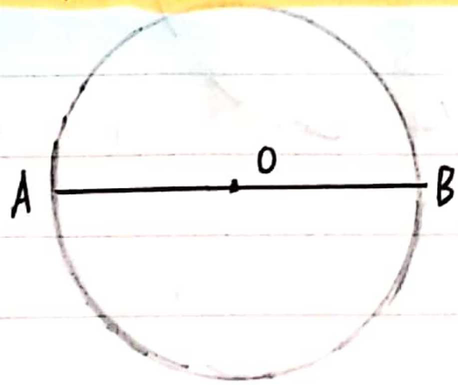
The locus would be the angle bisector of the angle formed by the intersecting lines.

Locus of a point P such that $\triangle ABP$ remains constant



The locus of P is the lines l and m, parallel to and equidistant from line AB.

Locus of a point P such that $\angle APB$ is $90°$



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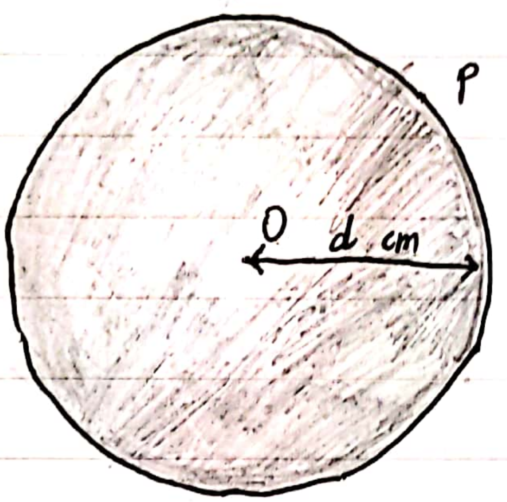
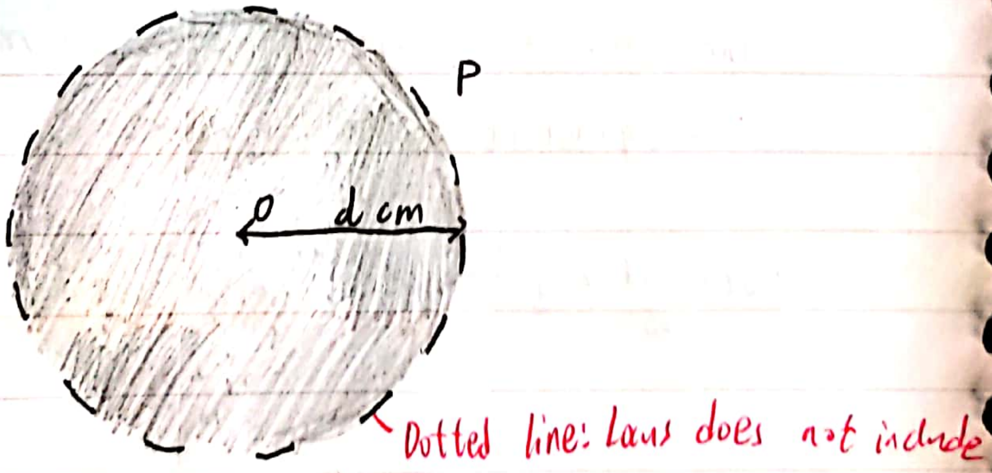
This is because of the angle in a semi-circle property: If AB is a diameter of the circle APB, then $\angle APB$ is $90°$. * Y2 chpt 6

Locus of point P such that the distance from $O \leq d$ cm



Locus of point P such that the distance from $O < d$ cm



MA1132 Chapter 5 : Mensuration

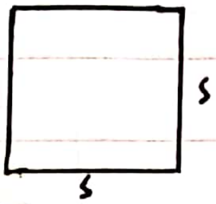
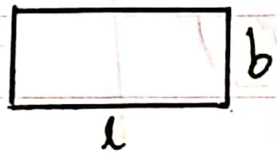
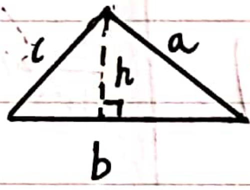
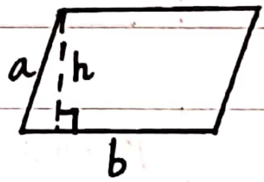
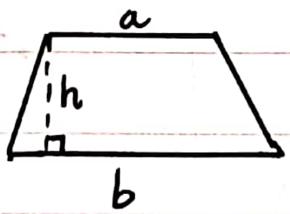
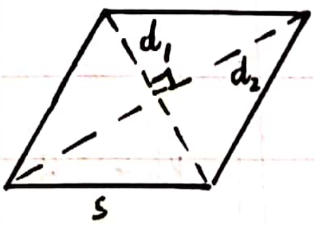
Figure	Diagram	Area	Perimeter
Square		s^2	$4s$
Rectangle		lb	$2(l+b)$
Triangle		$\frac{1}{2}bh$	$a+b+c$
Parallelogram		bh	$2(a+b)$
Trapezium		$\frac{1}{2}(a+b)h$	–
Rhombus		$\frac{1}{2}d_1d_2$	$4s$
	*d_1 and d_2 are the diagonals of the rhombus		

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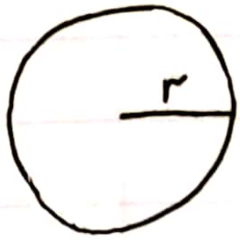
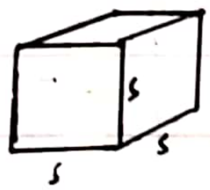
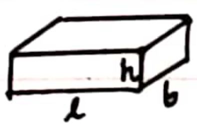
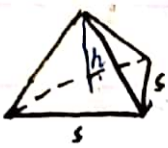
Figure	Diagram	Area	Perimeter
Circle		πr^2	$2\pi r$ (Circumference)
Annulus		$\pi(r_2^2 - r_1^2)$	—
Sector	 * $0° < \theta° < 360°$	$\frac{\theta°}{360°}(\pi r^2)$	$\frac{\theta°}{360°}(2\pi r) + 2r$

Solid	Diagram	Volume	Surface area
Cube		s^3	$6s^2$
Cuboid		lbh	$2(lb + bh + lh)$
Prism	—	Bh * where B = Cross section Area	$Ph + 2B$ * where B is cross section area and P is Perimeter of base
Cylinder		$\pi r^2 h$	$2\pi r(h+r)$ or $2\pi rh + 2\pi r^2$

Solid	Diagram	Volume	Surface area
Square-Based Pyramid		$\frac{1}{3}s^2h$	$s^2+\sqrt{3}s^2$ *
Triangle-Based Pyramid		$\frac{\sqrt{3}}{12}s^2h$ *	$\sqrt{3}s^2$ *
Pyramid (general)	—	$\frac{1}{3}Bh$ where B is the base area and h is the height of the pyramid	$B+L$ where B is the base area and L is the sum of area of lateral faces.
Cone		$\frac{1}{3}\pi r^2h$	$\pi r(l+r)$ OR $\pi rl+\pi r^2$
Sphere		$\frac{4}{3}\pi r^3$	$4\pi r^2$

* Not in notes

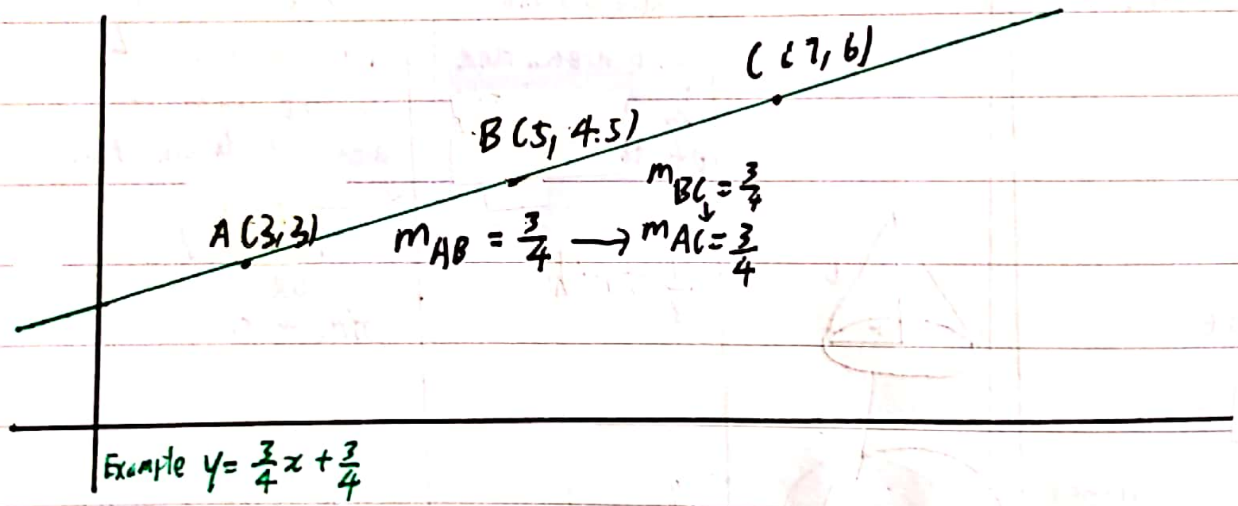
MA1132 Chapter 6: Concept of Gradient and y-intercept

Gradient

The gradient of a straight line joining A at (x_1, y_1) and B at (x_2, y_2) is $\frac{y_2 - y_1}{x_2 - x_1}$. This is denoted m_{AB}.

Collinear Points

Points A, B, C are collinear if and only if $m_{AB} = m_{AC} = m_{BC}$.



Equation of a line

The equation of a line can be defined as $y = mx + c$, where m is the gradient and c is the y-intercept. This is called the gradient-intercept form of a straight line.

x-intercept and y-intercept

The x and y intercepts are the x and y values where $mx+c$ intersects the x and y axes respectively.

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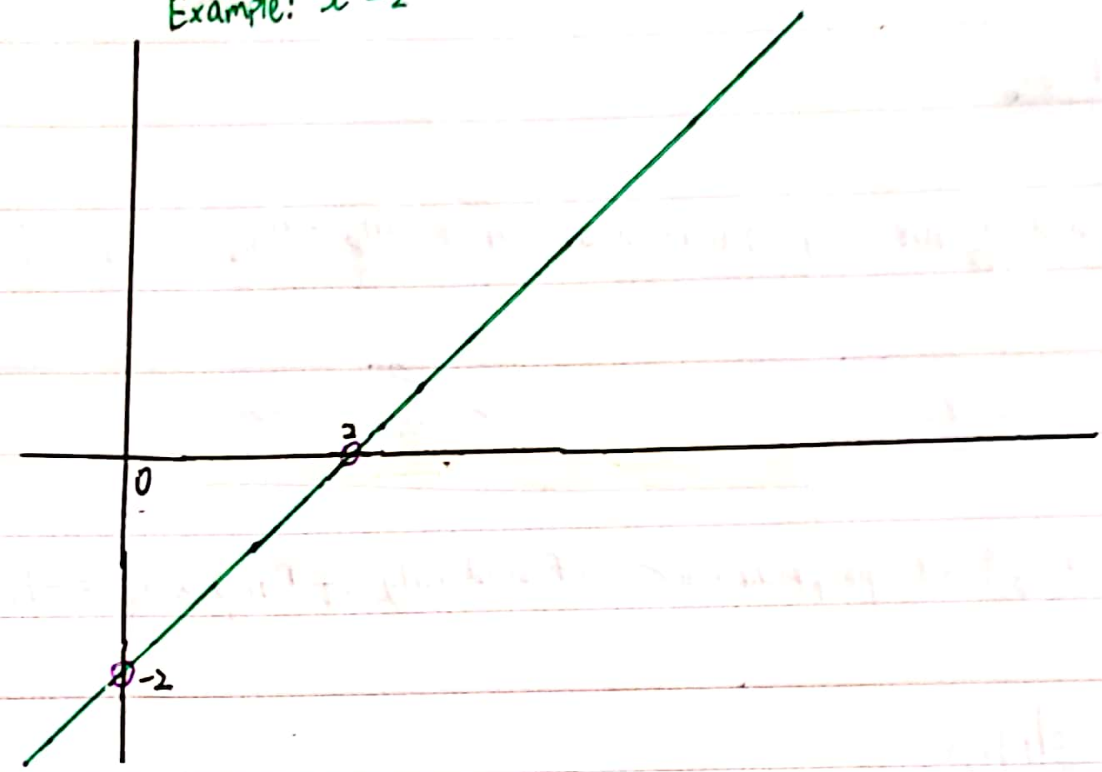
Example: $x-2$



Here, the x-intercept is 2 and the y-intercept is -2.

Different forms of Equations for Straight Lines

Gradient-Intercept Form

$y = mx + c$ where m is the gradient and c is the y-intercept.

Intercept Form

$\frac{x}{a} + \frac{y}{b} = 1$ where a and b are the x-intercept and y-intercept respectively.

General Form

$ax + by + c = 0$ where a, b, c are constants

Parallel Lines

Lines l_1 and l_2 are parallel if and only if $m_{l_1} = m_{l_2}$ and $l_1 \neq l_2$

Perpendicular Lines

Lines l_1 and l_2 are perpendicular if and only if $m_{l_1} \times m_{l_2} = -1$.

Area of Polygon

Given a polygon with vertices (x_1, y_1), (x_2, y_2), (x_3, y_3)... (x_n, y_n) taken in the anticlockwise direction with n sides, its area is

$$\frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & \cdots & x_n & x_1 \\ y_1 & y_2 & y_3 & \cdots & y_n & y_1 \end{vmatrix}.$$

Now, we take products from the above:

$$\frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & \cdots & x_n & x_1 \\ y_1 & y_2 & y_3 & \cdots & y_n & y_1 \end{vmatrix}.$$

We add each product with a red arrow and subtract each product with a green arrow.

* Not in notes

Written out, it would be:

$$\frac{1}{2}\left(\sum_{k=1}^{n-1}(x_k y_{k+1} - x_{k+1} y_k) - x_1 y_n + y_1 x_n\right)$$

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MA1132 Chapter 7: Variations and Graphs

Direct Variation

When two variables, x and y, vary in such a way that $\frac{y}{x}$ is a constant, they are said to be in direct proportion.

Given $k = \frac{x}{y}$, $y = kx$ is a straight line passing through the origin.

Direct Proportion is also known as Direct Variation. We can express it as $y \propto x$.

Inverse Proportion

When two variables, x and y, vary in such a way that xy is a constant, they are said to be in inverse proportion.

Given $k = xy$, $y = \frac{k}{x}$ is a straight line passing through the origin.

We can express it as $y \propto \frac{1}{x}$.

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Joint Variation

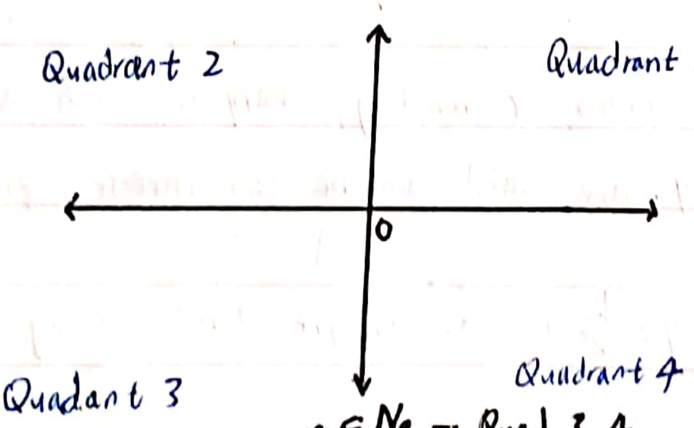
Consider the equation to calculate area of a triangle: $A = \frac{1}{2}bh$.

A varies directly with both b and h. Hence, $A \propto bh$, or A varies jointly as b and h.

Consider the equation to calculate pressure of a gas: $P = \frac{kT}{V}$

P varies directly with T and inversely with V. Hence, $P \propto \frac{T}{V}$.

Graphs of ax^n



- ax^n
  - $a<0$
    - $n<0$
      - $n \in N_e$ — Quad. 3, 4
      - $n \in N_o$ — Quad. 2, 4
    - $n>0$
      - $n \in N_e$ — Quad. 1, 2
      - $n \in N_o$ — Quad. 1, 3
  - $a>0$
    - $n<0$
      - $n \in N_e$ — Quad. 1, 2
      - $n \in N_o$ — Quad. 1, 3
    - $n>0$
      - $n \in N_e$ — Quad. 3, 4
      - $n \in N_o$ — Quad. 2, 4

* N_e refers to even positive integers and N_o refers to odd positive integers