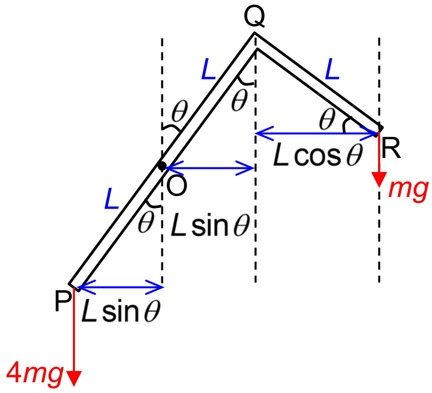
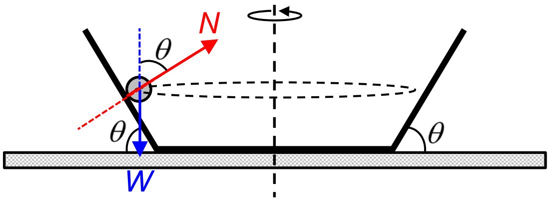
Raffles Institution
Year 5-6 Physics Department

2023 Year 5 H2 Physics Promotional Examination Solution

Section A

Qn	Ans	Solution
1	B	$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3$ $\frac{\Delta V}{V} = 3\frac{\Delta d}{V}$ Gradient of the graph is 3.
2	D	milli – 10^{-3} micro – 10^{-6} nano – 10^{-9}
3	C	$s_{\text{bus}} = u_{\text{bus}}t = 12 \times 30 = 360 \text{ m}$ $s_{\text{car}} = u_{\text{car}}t + \frac{1}{2}at^2$ $360 + 12 + 3.2 = 0 + \frac{1}{2}a(30)^2$ $a = 0.83378 = 0.83 \text{ m s}^{-2}$
4	A	from velocity-time graph, $\Delta v = -0.15 - 0.2$ $= 0.35 \text{ m s}^{-1}$ By Newton's third law, force on cart = –force on sensor impulse on cart = –area under force-time graph of sensor $m(\Delta v) = -\frac{1}{2}(0.08 - 0.02)(40)$ $m(-0.35) = -1.2$ $m = 3.4286 = 3.4 \text{ kg}$
5	D	Option A (incorrect): Upthrust on an object is equal to its weight only if the object is floating (i.e. in equilibrium). Option B (incorrect): Upthrust does not depend on the mass of the object. It depends on the mass and hence the volume of the liquid displaced by the submerged object. The volume of liquid displaced depends on the volume of the fully submerged object, which can be different even if the two objects have the same mass. Option C (incorrect): Upthrust is due to the difference in fluid pressure at the top and bottom surfaces of the object. Option D (correct): Upthrust is numerically equal to the weight of the liquid displaced by the submerged object. Hence upthrust depends on the density of the liquid and the volume of the liquid displaced. Since both spheres have the same dimensions, both displace the same volume of liquid when fully submerged and hence experience the same upthrust.

<table>
<tr><td>6</td><td>C</td><td>
Taking moments about O,
$$4mg\left(L\sin\theta\right)=mg\left(L\sin\theta+L\cos\theta\right)$$
$$3\sin\theta=\cos\theta$$
$$\tan\theta=\frac{1}{3}$$
$$\theta=18.435=18.4^\circ$$


</td></tr>
<tr><td>7</td><td>C</td><td>
Power is the rate of transfer of energy. Hence the gradient of the graph is power. Maximum power occurs between $t=4$ s to $t=6$ s, when the gradient is the largest.
$$\text{maximum power output to lift container}=\frac{40-10}{6-4}=15\text{ kW}$$
$$\text{Since efficiency}=\frac{\text{power output}}{\text{power input}},$$
$$\text{power input (supplied)}=\frac{\text{power output}}{\text{efficiency}}=\frac{15}{0.65}=23.077=23\text{ kW}$$
</td></tr>
<tr><td>8</td><td>B</td><td>
$$\begin{aligned}v&=\omega r\\&=\frac{2\pi}{T}r\\&=\frac{2\pi\left(1.50\times10^{11}\right)}{365\times24\times60\times60}\\&=29886=2.99\times10^{4}\text{ m s}^{-1}\end{aligned}$$
</td></tr>
<tr><td>9</td><td>A</td><td>


The centripetal force on the ball is the resultant of the contact force and weight. The centripetal force points horizontally towards the centre of the horizontal circle.
</td></tr>
<tr><td>10</td><td>C</td><td>
Work done by the gravitational force,
$$\begin{aligned}W_{F_g}&=-W_{F_{ext}}\\&=-m(\phi_f-\phi_i)\\&=-1\left[-\frac{3GM}{2R}-\left(-\frac{GM}{R}\right)\right]\\&=\frac{GM}{2R}\end{aligned}$$
</td></tr>
</table>

11	**A**	By conservation of energy, $$E_{\text{surface}} = E_{\text{infinity}}$$ $$-\frac{GMm}{R} + \frac{1}{2}mv_{escape}{}^{2} = 0$$ $$v_{escape} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G\rho\left(\frac{4}{3}\pi R^{3}\right)}{R}} = \sqrt{\frac{8}{3}\pi\rho G R^{2}} \Rightarrow v_{escape} \propto \sqrt{\rho R^{2}}$$ $$\frac{v_{escape,M}}{v_{escape,E}} = \sqrt{\frac{\rho_M R_M{}^{2}}{\rho_E R_E{}^{2}}}$$ $$v_{escape,M} = \left(\sqrt{\left(\frac{4}{5}\right)\left(\frac{1}{4}\right)^{2}}\right)(11.2)$$ $$= 2.5044 = 2.50 \text{ km s}^{-1}$$
12	**D**	Option A (incorrect): Maximum transfer of energy from the driving system to the oscillating system occurs only at resonance. Option B (incorrect): With a decrease in damping, less energy is lost from the oscillating system. The total energy of the oscillating system should increase. Option C (incorrect): With a decrease in damping, less energy is lost from the oscillating system. The amplitude at resonance should increase. Option D (correct): With a decrease in the degree of damping, the frequency at which resonance occurs increases. Since $T = 1/f$, the period of the driving force decreases.
13	**B**	The *x*-*t* graph shows the displacement from the equilibrium position of one particle at different times as it oscillates in simple harmonic motion. Option A (incorrect): At P, the particle is at maximum displacement from the equilibrium position. Its acceleration is maximum. Option B (correct): Since P and Q are $\frac{1}{4}T$ apart, their phase difference $\phi = \frac{(1/4)T}{T} \times 360° = 90°$. Option C (incorrect): Horizontal axis is time. OS represents two periods. Option D (incorrect): At P, the particle is at maximum displacement and is momentarily at rest. Hence its kinetic energy at P is zero. At R, the particle is at the equilibrium position and its speed is maximum. Hence its kinetic energy at R is maximum. The velocity of the particle at various times can also be determined from the gradient of the *x*-*t* graph.

14	**B**	Option A (incorrect): In a stationary wave, points R and S oscillate π rad out of phase with each other. The phase difference between them is π rad. Option B (correct): In a stationary wave, points Q and S oscillate in phase with each other with different amplitudes. Amplitude varies from zero at the nodes to a maximum at the antinodes. Option C (incorrect): Point P is a node which does not oscillate. At the node, the incident and reflected waves always meet π rad out of phase with each other such that the resultant displacement is always zero. Option D (incorrect): The stationary wave is formed by the superposition of the incident wave and the reflected wave. Hence the amplitude of the incident wave is half the amplitude of point Q.
15	**D**	The third order maxima in the interference pattern are missing as they coincide with the first order minima of the single slit diffraction envelope. single slit first order minima: $b\sin\theta = \lambda$ ----- (1) double slit maxima: $a\sin\theta = n\lambda$ ----- (2) $\frac{(2)}{(1)}$: $\frac{a}{b} = n$ $\frac{0.60}{b} = 3$ $b = \frac{0.60}{3} = 0.20$ mm

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Section B

1 **(a)** $s_x = u_x t$

$$u_x = \frac{s_x}{t} = \frac{20.0}{1.40} = 14.286 = 14.3\text{ m s}^{-1}$$

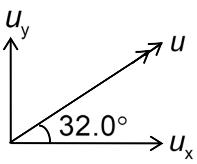
*Final answer to 3 s.f.

(b)

$$\tan 32.0° = \frac{u_y}{u_x}$$

$$u_y = u_x \tan 32.0° = 14.286 \tan 32.0° = 8.9269 = 8.93\text{ m s}^{-1}$$

*Final answer to 3 s.f.



(c) Consider vertical motion of the ball from starting point to point of maximum height.

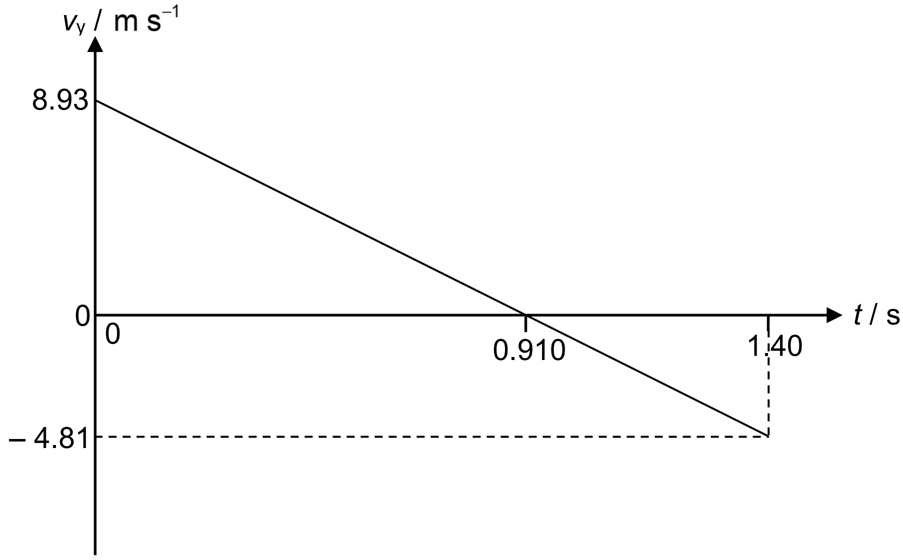
$$v_y = u_y + at$$

$$0 = 8.9269 + (-9.81)t$$

$$t = \frac{8.9269}{9.81} = 0.90998 = 0.910\text{ s}$$

*Final answer to 3 s.f.

(d) **(i)**



vertical velocity when the ball hits the wall:

$$v_y = u_y + at = 8.9269 + (-9.81)(1.40) = -4.8071 = -4.81\text{ m s}^{-1}$$

OR

$$\text{gradient, } \frac{v_y - 0}{1.40 - 0.90998} = -9.81 \quad \Rightarrow \quad v_y = -4.8071 = -4.81\text{ m s}^{-1}$$

*Straight line with negative gradient, positive and negative v_y
*Correct label values

(ii) Height of the wall is the area under the graph from $t = 0$ s to $t = 1.40$ s.
OR
Height of the wall is the difference in the absolute areas from $t = 0$ s to $t = 0.910$ s and $t = 0.910$ s to $t = 1.40$ s.

(e) (i) With air resistance, there is now a resultant force in the horizontal direction that causes the horizontal component of the ball's velocity to decrease continuously / gradually / at a decreasing rate.

(ii) **Preferred answer (comparing the rate at which vertical component of velocity changes with and without air resistance) :**

With air resistance, when the ball is moving upwards towards its highest point, the vertical component of its velocity decreases to zero at a faster rate compared to the rate when there is no air resistance.
When the ball is moving downwards from its highest point, the vertical component of its velocity increases at a slower rate compared to the rate when there is no air resistance.

Alternative answer (describing the effect of air resistance on the variation of vertical component of velocity):

With air resistance, the acceleration in the vertical direction is no longer constant.
When the ball is moving upwards, the vertical component of its velocity decreases at a decreasing rate.
When the ball is moving downwards, the vertical component of its velocity increases at a decreasing rate.

Comments

(a) (b) (c)
- A number of students stated the answer in terms of the initial velocity *u*. The question stated 'calculate' which means numerical answers are expected.
- Students should present their worked solutions clearly. Defining equations with appropriate symbols should be stated first before substituting numerical values. For example in part (c), students could have written $v_y = u_y + at$ first, where subscript *y* represents the vertical components of the vectorial physical quantities in the equation.

(d) (i)
- The area under the graph above the horizontal axis is larger than that below the horizontal axis. This is because the former represents the vertical distance of the ball from the ground to the highest point and the latter is the vertical distance of the ball from the highest point to the top of the wall.
- A number of students did not indicate the final vertical velocity of the ball.

(ii)
- It is important to state the time interval which the area under the graph should be considered, or state 'TOTAL area under the graph'.
- Phrasing needs to be improved. The area under the graph from 0 s to 0.910 s is positive while the area under the graph from 0.910 s to 1.40 s is negative, since the area under the graph is displacement, a vector quantity. So the sum of these two areas would actually give the height of the wall. Answers that suggest subtracting the areas should rightfully state 'subtract the absolute areas'
- Some students labelled the two areas with 'A' and 'B' and used these symbols to explain how to determine the height of the wall. This was clear and accepted.

(e) (i) (ii)
- Many students misread the question and described the effect of air resistance on the components of displacement instead of velocity for both parts.
- For (ii), some gave long answers that described acceleration and made no reference to velocity. Such answers were not given credit.
- It should be stated clearly which direction the ball is moving, and to consider both upward and downward motions.

2 **(a)** The normal contact force / resultant force always points towards the centre of the circular track and is thus always perpendicular to the instantaneous velocity of the ball. This only changes the direction of the velocity of the ball while the magnitude of its velocity remains constant. Hence the speed and kinetic energy of the ball remain constant.

OR

The normal contact force / resultant force always points towards the centre of the circular track and is thus always perpendicular to the instantaneous velocity of the ball. Since there is no displacement in the direction of the resultant force, work done by the resultant force is zero. This does not cause a change to the kinetic energy of the ball and kinetic energy remains constant.

(b) **(i)** **1.** At point P, normal contact force on ball and weight of ball provides the centripetal force.

$$N + mg = m\frac{v^2}{r}$$

$$N = m\frac{v^2}{r} - mg$$

For ball to complete the vertical circle, it cannot lose contact with the track.

$$N \geq 0$$

$$m\frac{v^2}{r} - mg \geq 0$$

$$v \geq \sqrt{rg}$$

minimum speed at P,

$$v_{\text{min}} = \sqrt{rg}$$

$$= \sqrt{\left(\frac{0.60}{2}\right)(9.81)}$$

$$= 1.7155 = 1.72 \text{ m s}^{-1}$$

OR explain

The normal contact force on the ball decreases (since speed decreases) as it moves from the bottom of the track to point P. Hence for the ball to just complete the vertical circle without losing contact, the minimum speed at point P is when $N = 0$ at P. This means that the ball will not lose contact with the track at any other points along the vertical circle.

2. By conservation of energy from just before spring is released to point P,
decrease in EPE = increase in GPE + increase in KE

$$\text{EPE}_\text{i} - \text{EPE}_\text{f} = (\text{GPE}_\text{f} - \text{GPE}_\text{i}) + (\text{KE}_\text{f} - \text{KE}_\text{i})$$

$$\frac{1}{2}kx^2 - 0 = mg(h_\text{f} - h_\text{i}) + \left(\frac{1}{2}mv_{\text{min}}{}^2 - 0\right)$$

$$x = \sqrt{\frac{2m}{k}\left[g(h_\text{f} - h_\text{i}) + \frac{1}{2}v_{\text{min}}{}^2\right]}$$

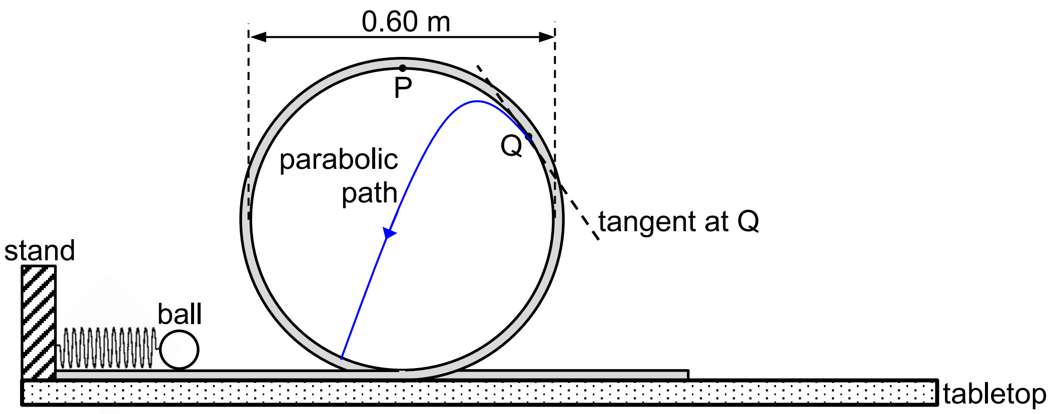
$$= \sqrt{\frac{2(45\times10^{-3})}{80}\left[(9.81)(0.60) + \frac{1}{2}(1.7155)^2\right]}$$

$$= 0.090979 = 0.0910 \text{ m}$$

(ii) **1.** At minimum speed, the centripetal acceleration at P is always the acceleration of free fall and is independent of mass. Hence the minimum speed remains the same.
OR
The minimum speed is only dependent on the diameter of the circular track and the acceleration of free fall. Since it is independent of mass, the minimum speed at P remains the same.

2. With a greater mass, the increase in gravitational potential energy and kinetic energy needed to reach P is greater. Hence the initial compression of the spring must be greater.
OR
With a greater mass, the total energy at P is greater. Hence the initial compression of the spring must be greater.
OR
With a greater mass, a greater force is needed to project the ball to the same minimum initial speed and consequently the same minimum speed at P. Hence the initial compression of the spring must be greater.

(iii)



*Parabolic path in the correct direction, initial part of the path tangential to track at point Q, path within circumference of track

Comments

For all problems on circular motion, students have to distinguish if they are analysing a body moving in a horizontal or vertical circular motion and if its speed is uniform or not.

Is this question, (a) is on horizontal circular motion with uniform speed while (b) is on vertical circular motion with non-uniform speed (since there is no constant input of energy to keep the speed constant).

(a) Students have to distinguish the different quantities and terms and use them precisely in the explanations. For example in this question, many students use the terms 'velocity' and 'speed' interchangeably, which is incorrect. It is fundamental to understand that velocity is a vector quantity while speed is a scalar quantity.

For a body in uniform circular motion, it's speed (or the magnitude of its velocity) is constant but it's velocity is changing (due to the changing direction of its velocity). It is incorrect to state that the linear velocity or velocity of the body is constant. Rather, it is the linear speed (which is the magnitude of the liner velocity) that is constant.

Likewise, acceleration and force are vector quantities. Students have to consider the magnitudes and directions of these vector quantities and be very conscious when dealing with them.

Students need to sequence their explanations in a logical flow to show a good understanding of the concepts that lead to the observations.

(b) **(i)** **1.** Students are expected to show the following steps clearly:

- $N + mg = m\dfrac{v^2}{r}$

  This equation shows the forces that provide the centripetal force and is an essential concept of circular motion.
- Explain why $N \geq 0$.
- Determine the minimum speed of the ball at the top of the vertical circular motion, using the inequality.

A substantial number of students misread the question and determined the minimum speed of the ball at the bottom of the circular motion instead. Do read the questions carefully.

2. When applying the conservation of energy, students are expected to

- be clear which body and which part of the motion they are considering.
- state clearly how the conservation of energy is applied
  EITHER initial energy at (position) = final energy at (position)
  OR increase in (energy)… = decrease in (energy)…
- state the different forms of the energies considered and substitute them in the word equation above.
- substitute numerical values into the equation.

(ii) **1.** Students should focus their answers on point P as required by the questions.

2. In all explanation questions, Physics concepts are often required. Students should not just quote the equations or formulae and rely on mathematics when trying to explain. If equations or formulae are used, these should be accompanied with clear explanations of the concepts beind these equations that lead to the conclusions made.

(iii) Students are expected to draw as accurately as possible, showing the important features of the path. Students can always accompany their drawings with labels to describe the path e.g. tangential to track (label part of path at point Q), parabolic path (label part of path when the ball is in free fall), to avoid any misinterpretation of their drawings.

The ball will not drop vertically down when it loses contact at point Q as the radial component of the ball's weight that acts towards the centre of the circular motion, will not allow the speed of the ball to decrease to zero. So when the ball leaves the track at point Q, it has a tangential velocity that is pointing upwards at an angle to the horizontal. After which, the ball is in free fall (projectile motion).

Since the question asked to draw the path of the ball after it leaves the track, do draw the entire path and not just the initial part of the path.

3 **(a)** **(i)** Gravitational force provides the centripetal force.

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

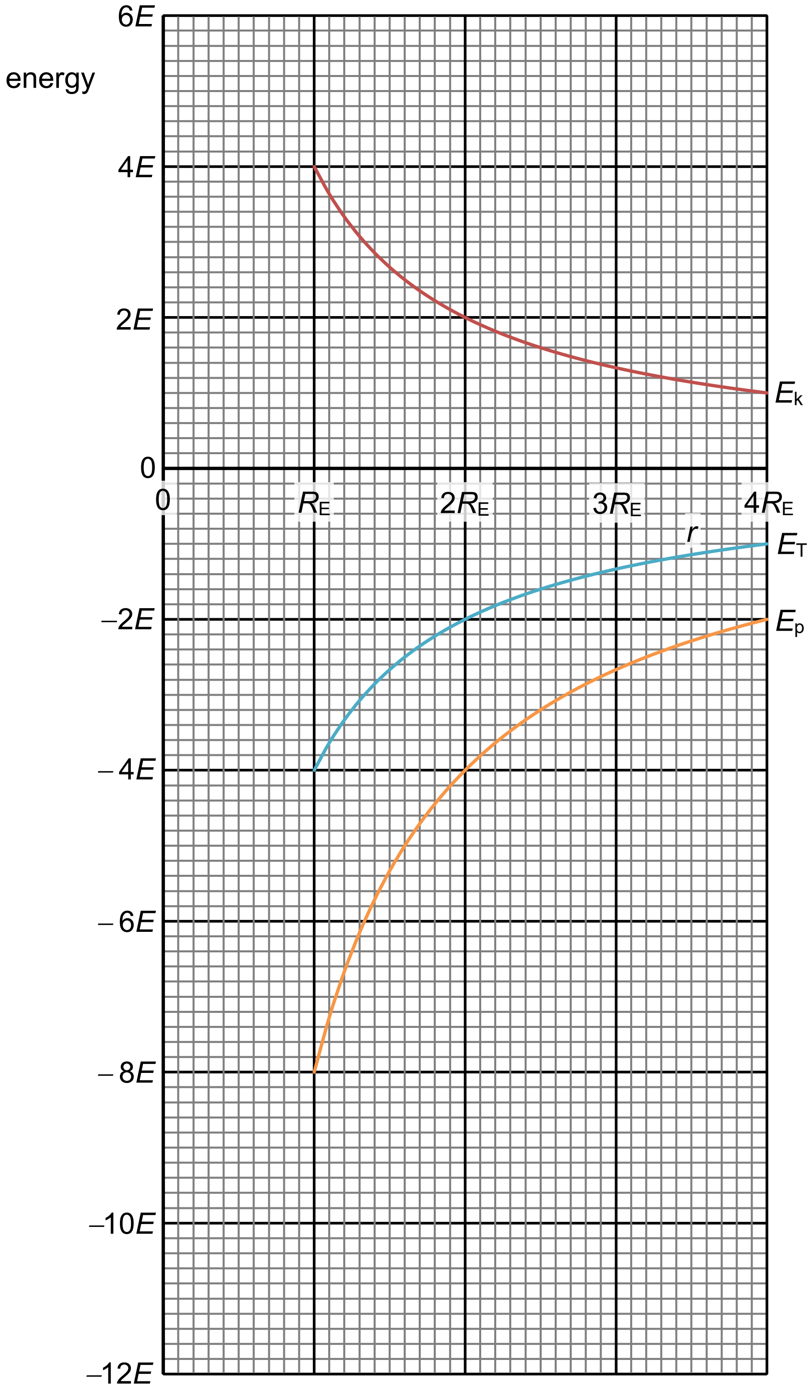
$$v = \sqrt{\frac{GM}{r}}$$

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{GM}{r}\right) = \frac{GMm}{2r}$$

$$E_p = -\frac{GMm}{r} = -2\left(\frac{GMm}{2r}\right) = -2E_k$$

Hence $E_p = -2E_k$ (shown)

(ii)



*Graphs have to pass through the correct points
*Labels, smooth curves, graphs start after $r = R_E$, positions of graphs relative to one another is correct

(b) (i) Gravitational force provides the centripetal force.

$$\frac{GMm}{r^2} = mr\omega^2$$

$$\frac{GM}{r^3} = \omega^2 = \left(\frac{2\pi}{T}\right)^2$$

$$T = \sqrt{\frac{4\pi^2 r^3}{GM}}$$

$$= \sqrt{\frac{4\pi^2\left(6.37\times10^6 + 408\times10^3\right)^3}{\left(6.67\times10^{-11}\right)(5.98\times10^{24})}}$$

$$= 5551.6 \text{ s}$$

$$= \frac{5551.6}{60\times60} = 1.54 \text{ h}$$

The period of geostationary satellites is 24 h. Since the period of the ISS is not 24 h, it is not in geostationary orbit.

(ii) The ISS has to do work against atmospheric drag which causes the total energy of the ISS to decrease with time.

Since total energy is $E_T = -\frac{GMm_{ISS}}{2r}$, its radius of orbit will decrease with time as total energy decreases (becomes more negative).

Hence engine thrusters need to be fired such that positive work is done on the ISS to increase its total energy and return it to its desired orbit.

Comments

(a) (i) Since this is a show question, students are reminded to be systematic and sequential in their presentation. It is important to be explicit when stating that centripetal force is provided by the gravitational force with the variables properly defined before using them.

The formula for the speed or kinetic energy of the satellite must be derived.

(ii) Many students were careless in calculating the corresponding energies as a function of orbital radius or made no attempt to plot the curves accurately. Since grid lines are provided, students are strongly encouraged to mark out the coordinates within the required range before sketching the required curves.

Many students made the mistake of extending the energy curves for $r < R_E$ or assumed that there is an asymptote at $r = R_E$ or $r = 0$. Students should take care to distinguish between point masses and uniform spheres when sketching such graphs.

Some students are of the misconception that the total energy of a satellite in orbit is a constant, independent of its orbital radius.

(b) (i) A significant number of students wrongly assumed that the period of a geostationary satellite is 365 days.

When making comparisons, students should present the calculations/data for the period/angular velocity e.t.c. using the same units for both ISS and geostationary satellite.

(ii) Quite a number of answers made no reference to the fact that the atmosphere was part of the context of the question and students were not able to correctly identify the source of the resistive force.

Two common misconceptions:

- Many students wrongly conclude that work done against resistive forces leads to a decrease in kinetic energy and hence the ISS's orbital radius decreases. Students need to be careful to frame and sequence their explanation in the context of the question. In this case, work done against resistive forces leads to a decrease in total energy. It is also not meaningful to discuss energy conversions between potential and kinetic energies when the orbital radius is changing.
- The attractive gravitational force by the earth on the ISS which students identified as the net force acting on the ISS was responsible for its orbital decay, which is incorrect.

4 **(a)**

$$\text{intensity at dectector} = \frac{P}{4\pi d^2} = \frac{480}{4\pi(5.0)^2}$$

$$\text{power at dectector} = \frac{P}{4\pi d^2} \times (\pi r^2) = \frac{480}{4\pi(5.0)^2} \times \pi\left(\frac{6.0\times10^{-2}}{2}\right)^2$$

$$\text{energy at dectector} = \frac{P}{4\pi d^2} \times (\pi r^2) \times t$$

$$= \frac{480}{4\pi(5.0)^2} \times \pi\left(\frac{6.0\times10^{-2}}{2}\right)^2 \times (4.0\times60)$$

$$= 1.0368 = 1.04 \text{ J}$$

(b) **(i)** Let I_1, I_2 and I_3 be the intensity of the light after passing through P_1, P_2 and P_3 respectively.

$$I_1 = \frac{I_0}{2}$$

Using Malus' law,

$$I_2 = I_1\cos^2\theta = \frac{I_0}{2}\cos^2\theta$$

$$I_3 = I_2\cos^2\theta$$

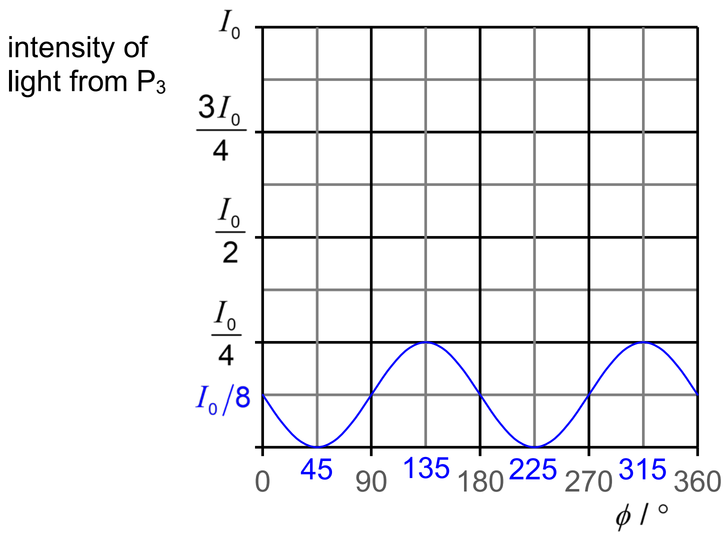
$$\frac{I_0}{8} = \frac{I_0}{2}\cos^4\theta$$

$$\theta = \cos^{-1}\left(\frac{2}{8}\right)^{1/4} = 45° \text{ (shown)}$$

*Last line with $\cos^{-1}$ to determine θ has to be shown

(ii) **1.** When $\phi = 45°$, the <u>transmission axis of P_3 is 90° (perpendicular) to the transmission axis of P_2</u>. Hence no light from P_2 will pass through P_3.

2.



$$I_3 = I_2\cos^2(45° + \phi) = \left(\frac{I_0}{2}\cos^2 45°\right)\cos^2(45° + \phi) = \frac{I_0}{4}\cos^2(45° + \phi)$$

*Shape of graph (2 cycles of cos squared), smooth turning points
*Position of graph (correct intensity and angle values)

Comments

(a) The intensity of a wave is defined as the rate of energy transfer per unit area normal to the direction of energy transfer of the wave. For a point source that radiates energy uniformly in all directions, energy is spread over a spherical wavefront. The intensity of the wave at a distance *d* from the source is thus $I_{\text{dist. d fr source}} = \frac{P_{\text{source}}}{S_{\text{dist. d fr source}}}$ where $S_{\text{dist. d fr source}} = 4\pi d^2$. The *d* refers to the distance from the source and should not be confused with the diameter or radius of the source or detector.

Since the intensity is a constant at a fixed distance *d* from the source, the amount of light energy that is received by the detector distance *d* away can be found using $E = P_{received} \times t$, where $P_{\text{received}} = I_{\text{dist. d fr source}} \times S_{\text{detector}}$. S_{detector} refers to the flat circular area of the dectector on which light falls, hence it should be πr^2 instead of $4\pi r^2$.

Students should commit to memory the mathematical formulae for the surface areas and volumes of common 3D geometric shapes such as cylinders and spheres. Students should also read the question carefully and highlight key words such as 'diameter' and take note of the units of the quantities given.

(b) (i) Since the question requires students to show how $\theta = 45°$ can be derived using Malus' law, working should be presented <u>neatly and sequentially</u> to <u>demonstrate how Malus' law was applied at each polariser P_2 and P_3</u>. Students should also ensure that any symbols used that are not already given in the question, should be explained or annotated in the diagram (e.g. I_2 and I_3). Common mistakes in this question were:

- Not recognising that intensity I_0 of an unpolarised beam of light will be halved after it passes through the first polariser.
- Intensity varies with *cos²* and not *cos* of the angle.
- The angle in Malus' law is the angle between the transmission axes of P_2 and P_3 which is $(2\theta - \theta)$ and not 2θ.

(ii) **1.** The question asked to 'explain', hence students should attempt to answer qualitatively (with quantitative substantiation as required) instead of showing only mathematical equation(s). Students should use precise terminologies (i.e. transmission axes) defined in the question to explain how the orientation of the P_2 and P_3 will cause the emerging light from P_3 to be zero. Students should also exercise caution when quoting $I = I_0 \cos^2 \theta$ for this part question because I_0 is defined as the intensity of unpolarised light before polariser P_1 in (b).

2. When $\phi = 0°$, emerging light from P_3 would be $\frac{I_0}{8}$ as defined in (b). As ϕ increases from $0°$ to $360°$, intensity will vary according to $I_3 = \frac{I_0}{4}\cos^2(45° + \phi)$. As such, students are expected to draw 2 cycles of a smooth $\cos^2$ graph within the defined range, with minimum intensity at $\phi = 45°$ and maximum intensity at $\phi = 135°$.

5 **(a)** The principle of superposition states that when two or more waves of the same kind meet at a point in space, the resultant displacement at that point is equal to the vector sum of the displacements of the individual waves at that point.

(b) **(i)** Coherent sources are sources that emit waves with a constant phase difference.

(ii) 0 rad since at point P, where the path difference is zero, it is a maximum.

(iii) For $\Delta x = \frac{\lambda D}{d}$ to be valid, *D* must be much larger than *d* (*D* >> *d*).
(OR waves / wave trains from S_1 and S_2 must be almost parallel OR *D* should be at least 2 orders of magnitude larger than *d*.)

Since *D* and *d* are of the same order of magnitude (85 cm and 50 cm) in this setup, the fringe separation formula cannot be used.

OR

For $\Delta x = \frac{\lambda D}{d}$ to be valid, the angle of diffraction must be small.
(OR the angle should be less than 6° to be considered small, so that small angle approximation is valid.)

From Fig. 5.1, the angle of diffraction for the first order high intensity signal at Q is $\tan^{-1}\left(\frac{25}{85}\right) = 16.4°$, which is quite large. Hence, the fringe separation formula cannot be used.

(iv) Since first order maximum is at point Q, path difference at Q is

$$S_2Q - S_1Q = n\lambda$$

$$\sqrt{85^2 + 50^2} - 85 = 1\lambda$$

$$\lambda = 13.615 = 13.6 \text{ cm}$$

(v) From Q to S_1, receiver will detect a high intensity signal whenever the path difference is $n\lambda$, where n is an integer.

At Q, $n=1$.

At S_1, $\dfrac{\text{path difference}}{\lambda}=\dfrac{50}{13.6}=3.676$

Hence, receiver will detect $n=2$ and $n=3$ high intensity signals as it is moved from Q to S_1.

number of high intensity signals detected, excluding at Q $= 2$

(c) Since only five bright spots (maxima) are observed, they must correspond to the zeroth order maximum and the two first and two second order maxima.

The third order and higher orders maxima cannot be observed on the screen because their diffraction angle $\theta \geq 90°$.

Hence, the minimum possible wavelength corresponds to the third order maxima having a diffraction angle of exactly $90°$.

$$d\sin\theta = n\lambda$$
$$d\sin 90° = n\lambda$$
$$\lambda = \frac{d}{n}$$

The range of wavelengths where only five bright spots can be observed is from $n=3$ to $n=2$.

$$\lambda_{\text{min}} = \frac{d}{n_{\text{max}}} = \frac{1\times10^{-3}/833}{3}$$
$$= 400.16\times10^{-9} = 400 \text{ nm}$$

Comments

(a) Students are advised to commit to memory some important definitions and be able to recall and write them word for word so as to maintain the accuracy of the meaning of the terms.

Many students stated 'resultant amplitude' rather than 'resultant displacement'. Amplitude refers to the maximum displacement the wave can have and is one particular value that is constant with time. Principle of superposition is the vector sum of the displacements of the waves at a particular instant in time and hence the resultant displacement, which need not be the amplitude, varies with time.

A number of students mistakenly described the formation of a stationary wave instead.

(b) (i) Coherent sources produce waves that have a constant phase difference. While this means that the waves have the same frequency, the reverse is not true – two waves having the same frequency does not necessarily mean they (or the sources) are coherent.

Students should note that the phase difference between coherent waves can be any value, it does not have to be exactly zero or π, as long as this difference is constant with time.

(ii) Phase angles and phase differences should be given in units of radians (rad).

Students should learn to differentiate between phase difference and path difference. Path difference is the difference in the lengths of the paths taken by two waves, whereas phase difference is a measure of the fraction of a cycle which one wave is ahead of the other at some point.

(iii) For $\Delta x = \dfrac{\lambda D}{d}$ to be true, there are two different assumptions that need to be made. Firstly, we must assume $D >> d$ so that the rays can be assumed to be parallel, which allows us to equate the path difference to be $d \sin\theta$. Secondly, we must assume that $d >> \lambda$ so that small angle approximation can be used to reduce $d \sin\theta = n\lambda$ to $\Delta x = \dfrac{\lambda D}{d}$.

Since we do not know the exact wavelength of the radio waves used yet, we need to approach this question by tackling whether $D >> d$ is true, or whether the anglular separation of the maxima from the centre is small.

In order to make a proper comparison between D and d, an order of magnitude argument should be used in order to determine if $D >> d$. This is how scientists conventionally compare two different quantities to see how different they are, or if the assumptions in their theory are valid.

(iv) As stated in part (iii), because $D >> d$ is not true, using $d \sin\theta$ as the path difference between the sources is not correct. Instead, students are expected to find the path difference geometrically.

(v) As $D >> d$ is still not true in this case, using $d \sin\theta$ as the starting point to find the path difference between the sources is not correct.

(c) The larger the wavelength, the greater the angle of diffraction. Hence, the minimum wavelength here is for the situation where the $n = 3$ maxima is just out of view at $90°$, so that only five maxima can be seen on the screen.

Do not apply formulae blindly – instead, learn to visualise what the question is asking for, before coming to a logical conclusion as to which formulae to use, and how they should be used to solve the problem.

Section C

6 **(a)** Equilibrium position at $x = 0.10$ m.

(b) From the graph, maximum K.E. is 3.2 J.

$$\tfrac{1}{2} m v_0^2 = E_{k,\max}$$

$$v_0 = \sqrt{\frac{2E_{k,\max}}{m}} = \sqrt{\frac{2 \times 3.2}{2.6}} = 1.5689 = 1.57 \text{ m s}^{-1}$$

(c) Since $\tfrac{1}{2} m\omega^2 x_0^2 = E_{k,\max}$,

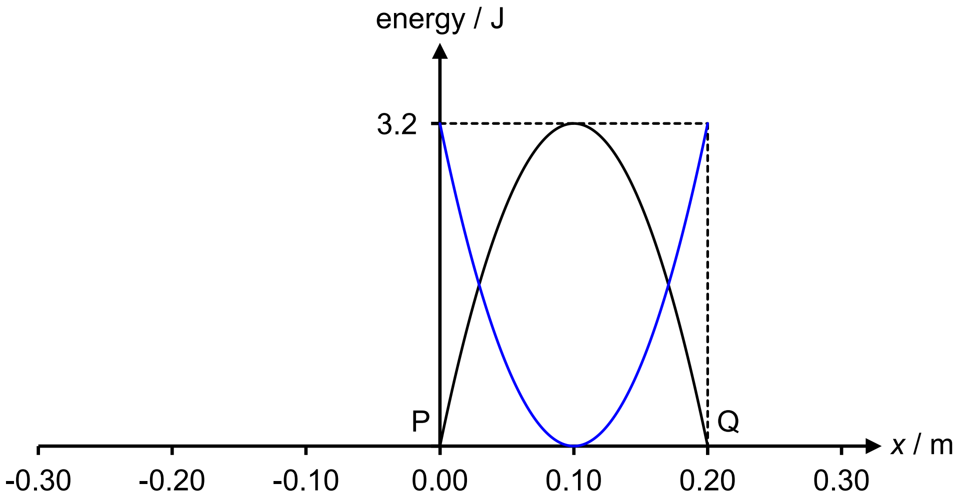
$$a_0 = \omega^2 x_0 = \frac{2E_{k,\max}}{m x_0} = \frac{2 \times 3.2}{2.6 \times 0.10} = 24.615 = 24.6 \text{ m s}^{-2}$$

(d) Since $v_0 = \omega x_0$,

$$\omega = \frac{v_0}{x_0} = \frac{1.5689}{0.10} = 15.689 \text{ rad s}^{-1} \quad \text{OR} \quad \omega = \sqrt{\frac{a_0}{x_0}} = \sqrt{\frac{24.615}{0.10}} = 15.690 \text{ rad s}^{-1}$$

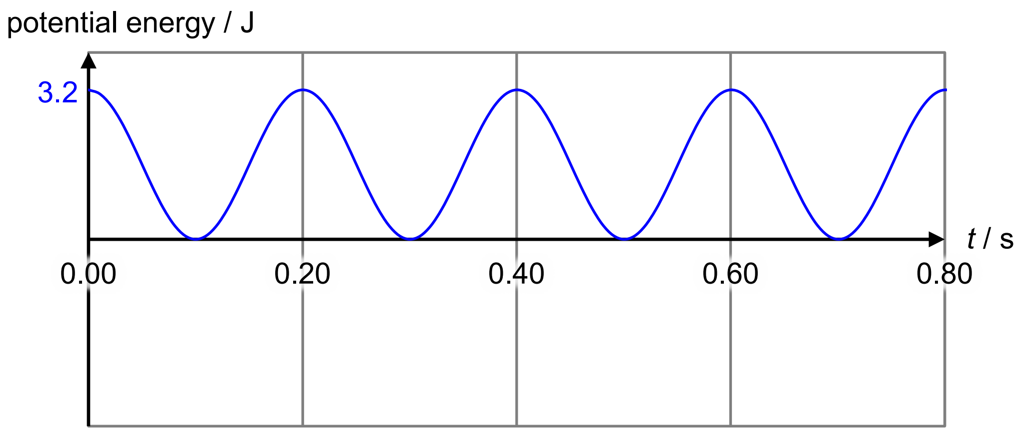
period, $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{15.689} = 0.40048 = 0.400 \text{ s}$

(e)



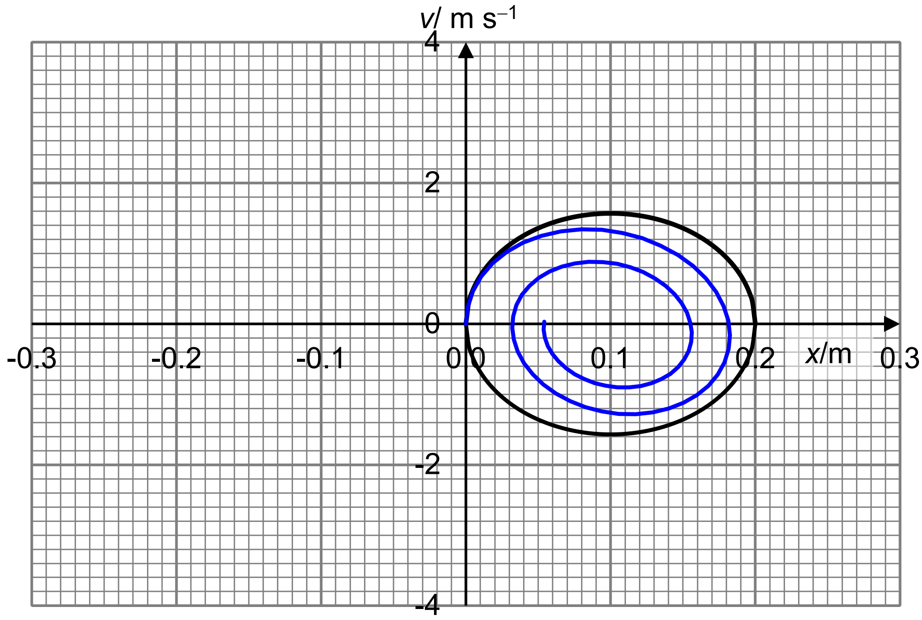
*Parabolic curve, smooth, symmetrical

(f)



*Shape of graph ($\cos^2 2\omega t$), non-zero starting point, smooth turning points
*Period, number of cycles (4 cycles)

(g)



*Smooth ellipse, symmetrical about horizontal axis
*Position of ellipse (correct *x* and *v* values)

(h) **(i)** See Fig. 6.3 in (g).
*Clockwise spiral from P ($x = 0$), 2 cycles end at $v = 0$ on the left of $x = 0.10$ m
*Within ellipse drawn in (g)

(ii) When the body is released and accelerates towards the equilibrium position, the restoring force towards equilibrium decreases while the damping force towards P increases. When the two forces are equal in magnitude, the resultant force on the body is zero and the speed is maximum.

Hence the body will attain maximum speed before it reaches the equilibrium position from the time it is released.

Comments

(a) Mark is lost if unit "m" is not stated.

(b) Note that an acceptable notation for maximum velocity is v_0.

(c) Maximum acceleration a_0 is a positive value. Mark is lost for students who quoted $a_0 = -\omega^2 x_0$. Marks are lost if students obtained the numerical answers by assuming either constant speed (zero), using mass-spring system (lose 1 mark) or using circular motion formula $a = r\omega^2$ (zero).

(d) Incorrect formula was used, e.g. $v = f\lambda$ or $v = r\omega$ to obtain either *f* or ω.

(e) Some students carelessly omitted to answer this part.
Use of ruler as guides are strongly encouraged. A smooth, symmetrical parabola is expected.

(f) Similarly, a smooth $\cos^2 2\omega t$ graph is expected. The number of cycle in 0.80 s must agree with the calculated value of *T* in (d), i.e. 2 cycles in one period.

(g) Many students are unable to draw a smooth ellipse (symmetrical about the *x*-axis and about $x = 0.10$ m).

(h) **(i)** Many did not realise that the graph is a clockwise spiral starting from (0,0) and some did not read instructions to draw 2 cycles.

(ii) When water was introduced, many wrong answers referred to object bobbing vertically in water under the influence of upthrust, weight and drag. Some also introduced Hooke's law in their answers. As there is no reference to the types of forces in the question, students should just refer to 'restoring force' and 'drag force' without going into specifics. Weaker answers would refer to the body undergoing 'acceleration' by the original force and 'deceleration' by the drag force, without being aware that since there is only body, it cannot be both accelerating and decelerating at the same time!

7 **(a)** When the gases are ejected, there is a rate of change of momentum of the gases. By Newton's second law, this is due to the force on the gases by the rocket.

By Newton's third law, the gases exert an equal and opposite force on the rocket. This force is the thrust that the rocket experiences.

(b)
$$F = I_s \times m_f$$
$$= 3250 \times 620$$
$$= 2015000 = 2.02 \times 10^6 \text{ N}$$

(c) Since specific impulse is the amount of thrust produced per unit mass of gas in each second, the larger the specific impulse, the more efficient is the engine.

Since the specific impulse in vacuum is larger, the engine works more efficiently in vacuum.

(d) Some fuel is required to operate the engines to have a controlled landing so that Super Heavy can be reused for future missions.

(e) **(i)** total thrust, $T = 3560 \times 620 \times 33$
$$= 72837600 = 7.28 \times 10^7 \text{ N}$$

(ii) total mass of fuel, $m = 620 \times (2.0 \times 60) \times 33$
$$= 2455200 = 2.46 \times 10^6 \text{ kg}$$

(iii)
$$T - (M - m)g = (M - m)a$$
$$72837600 - (5.0 \times 10^6 - 2455200)(9.81) = (5.0 \times 10^6 - 2455200)a$$
$$a = 18.812 = 18.8 \text{ m s}^{-2}$$

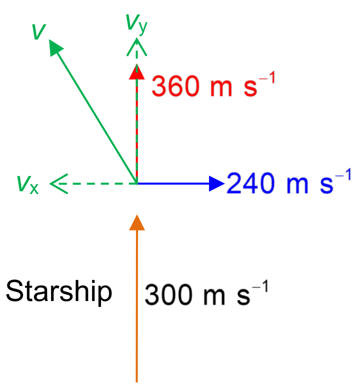
(iv) Distance of 30 km is small coompared to the radius of the Earth (6371 km). So the gravitational field strength will not have decreased by much at that altitude since $g = \dfrac{GM_E}{(R_E + h)^2}$.

(f) Let v be the speed of the third fragment.

Consider the conservation of momentum in the horizontal and vertical directions.
Take direction upwards and to the right as positive.

Horizontally,
$$0 = (\frac{M}{3})(240) + (\frac{M}{3})v_x$$
$$v_x = -240 \text{ m s}^{-1}$$

Vertically,
$$M(300) = (\frac{M}{3})(360) + (\frac{M}{3})v_y$$
$$v_y = 540 \text{ m s}^{-1}$$



$$v = \sqrt{(240)^2 + (540)^2}$$
$$= 590.93 = 591 \text{ m s}^{-1}$$

Comments

(a) Time, marks and space were lost by many students in regurgitating Newtons Laws of Motion without context. Many students who quoted the laws correctly were unable to apply them, and they often gave answers that contradicted the laws that they had written due to lack of internalisation / understanding.

Wrong applications of Newton's third law included: 'rocket exert a force on the surrounding air, which in turn exerts a force on the ground – hence by Newton's third law, the ground exert a force on the rocket as thrust'! Many referred to the fuel gas acting on the surrounding air and ground when they do not exist in space and yet the rocket can have thrust in space!

This question clearly highlights the fact that students are memorising Physics without actually understanding it.

(b) Wrong answers either used the specific impulse in vacuum and included a factor 33 in their working. Students must learn to read the question and data more carefully.

(c) Since the question asked for the more efficient engine, students need to clearly explain why specific impulse is a measure of engine efficiency and how a larger magnitude indicates higher efficiency.

Some students considered the air drag but this affects the resultant force acting on the whole rocket and is not applicable in the discussion of the efficiency of the engine.

(d) Students need to get the answer from the first paragraph on page 6. Some students did not show understanding of 'controlled landing' and thought that it is just to change the direction of flight.

(e) **(i)** The altitude of 30 km is reached 2.0 minutes after take-off so the specific impulse in vacuum should be used. Some students did not multiply by 33 to account for all engines in operation.

(ii) Some students did not multiply by 33 to account for all engines in operation.

(iii) Students did not understand the concept that the acceleration is caused by the resultant force. The mass and thus the weight of the Starship system after 2.0 minutes need to be taken into account for this question.

Some students tried to solve using kinematics equations but this is not possible as the acceleration of the Starship is changing over time.

Note that 'error carried forward' will only be applied if all terms are stated correctly in setting up the equation for the resultant force on the Starship.

(iv) Students need to compare the distance of 30 km to the radius of the Earth before concluding that the change in distance is negligible.

Some students incorrectly stated that the gravitational field strength is too weak to be considered. It is not true and a calculation will show that it is not negligible.

(f) Students did not consider the initial vertical momentum of the Starship before exploding. The principle of conservation of momentum should be applied separately in both the vertical and horizontal directions.