Temasek Junior College (IP)

2025 Year 4 Mathematics

AM

Unit 16 – Graphing with Graphing Calculator (GC)[1]

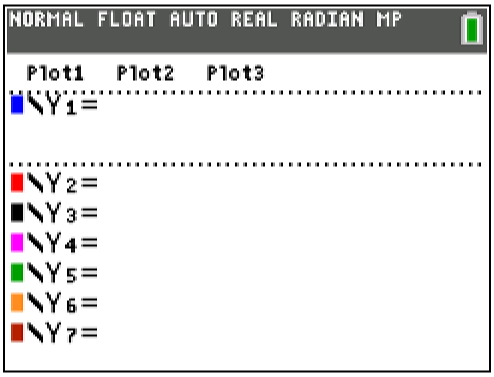
Lesson 2: Analysing Graphical Behaviour

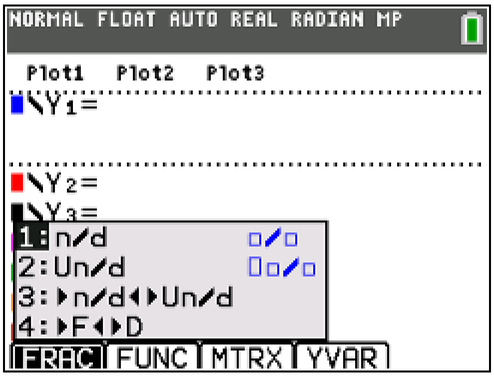
Learning Objectives
By the end of this lesson you should be able to:
1) Sketch the graphs of different functions with the aid of a graphing calculator (GC).
2) Analyse behaviour of the graphs of different functions with the aid of a GC.
3) Solve simple problems involving graphs using the GC.

2.1 Asymptotic Behaviour

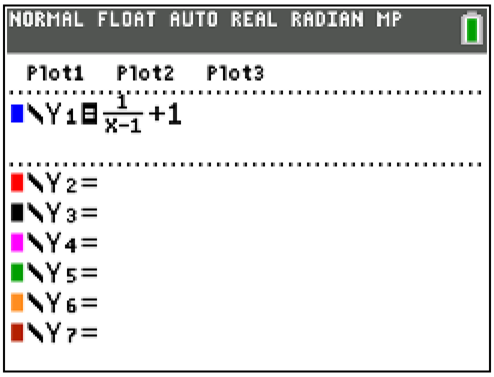
Consider graphing the function $y=\dfrac{1}{x-1}+1$ using the GC.

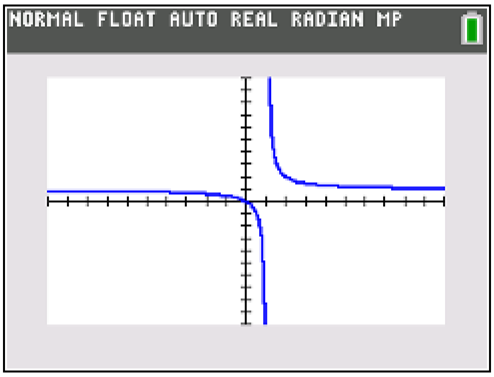
Recall:
- The $\boxed{\text{Y=}}$ button will bring you to the graphing interface.
- To enter fractions use $\boxed{\text{ALPHA}}$ $\boxed{\text{Y=}}$ $\boxed{1}$.





- To enter the variable (x), use $\boxed{\text{X,T,}\theta\text{,n}}$.
- To generate the graph of a function, use $\boxed{\text{GRAPH}}$ after entering the function.





The screen capture above is how the graph of $y=\dfrac{1}{x-1}+1$ will look like in the **ZStandard** ($\boxed{\text{ZOOM}}$ $\boxed{6}$) setting. Notice that the asymptotes are not displayed.

[1] *This set of notes is to be used in conjunction with the TI-84 Plus CE graphing calculator. Operation of other brands and models of graphing calculators may differ*

In fact, the **GC is not able to display asymptotes**. Any asymptotes will have to be drawn based on your understanding of the graph and the function it represents.

In other words, you **SHOULD NOT BE RELIANT** on the GC to show every feature of the graph of a function.

That being said, the GC can still be used to provide an idea of what the asymptotes might be. This needs to done in conjunction with:

- Your knowledge of the graph and the function it represents.
- The visual observations made on the graph displayed using appropriate WINDOW and ZOOM settings (we shall look at this at a later stage of the lesson).
- An understanding of the definition of different types of asymptotes.

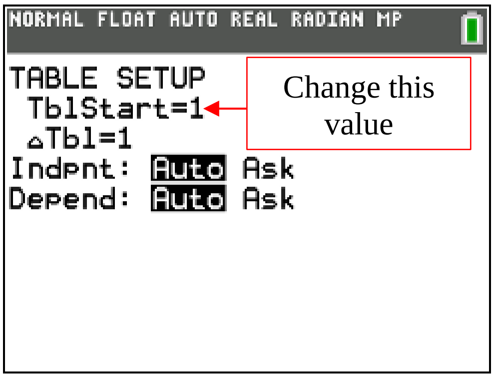
Analysing horizontal asymptotic behaviour

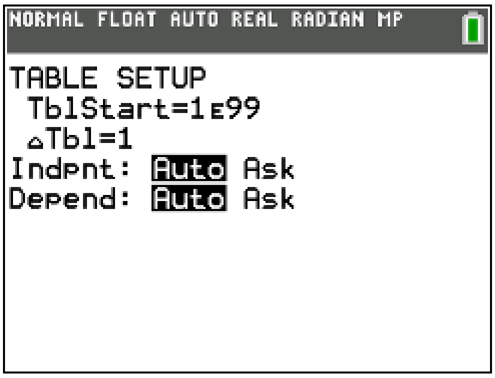
Recall that a horizontal asymptote is a line with equation $y = k$, where k is the value a function approaches as x approaches $+\infty$ and $-\infty$, and may or may not be the same at $+\infty$ and $-\infty$.

With this understanding, we can use the GC to observe what happens to the value of a function when x is an extremely large positive value and an extremely negative value. This will allow us to identify possible asymptotic behaviour(s).

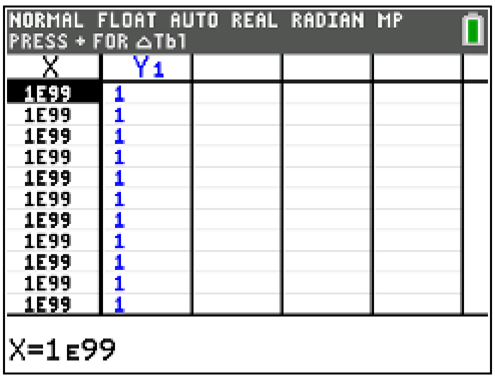
To determine what happens to $y = \dfrac{1}{x-1} + 1$ when x becomes an extremely large positive value:

- Press 2ND Window to access the TABLE SETUP of the GC.
- Enter an extremely large positive TBLSTART value of x for e.g. 1 2ND , 9 9, which is 1E99 i.e 10^{99}.





- Press 2ND GRAPH and observe the values.



- Observations
  - ✓ The values of x simulates the event where x approaches $+\infty$.
  - ✓ The values of y are observed to "constantly" be $y=1$, which is the value y approaches when x approaches $+\infty$.
  - ✓ There is a horizontal asymptote of $y=1$.

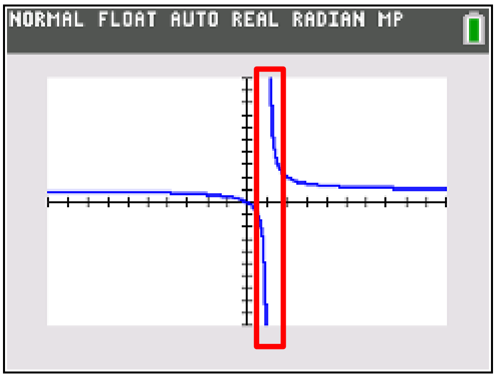
In fact, our current knowledge of the function should tell us that the values are approaching the value 1 and will never reach the exact value 1.

> ***Try it!***
>
> Use your GC to analyse the asymptotic behaviour of $y=\dfrac{1}{x-1}+1$ when x approaches $-\infty$.

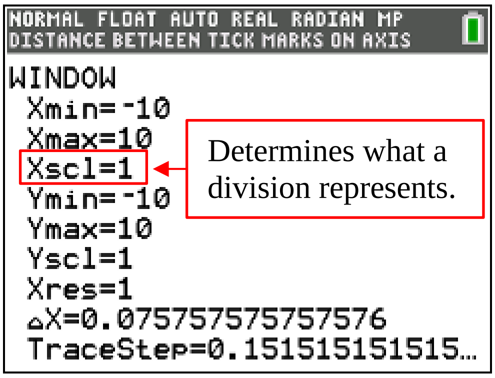
Analysing vertical asymptotic behaviour

Recall that a vertical asymptote is a line with equation $x=k$, where k represents the value(s) of x for which the function is undefined. With this understanding, we can use the GC to identify the vertical asymptotes.

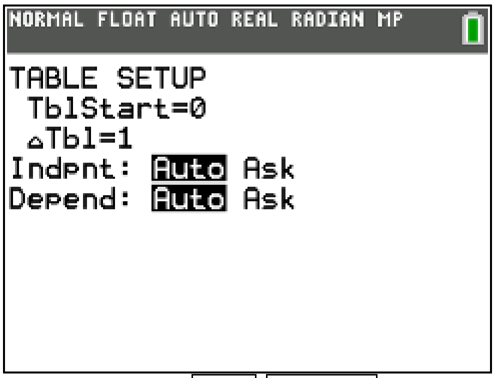
- From the graph, we know that there is possibly a vertical asymptote around $x=1$.



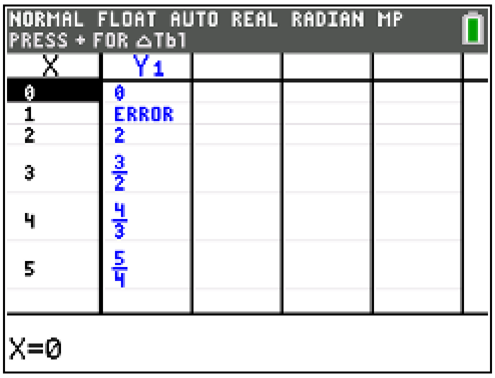
Each division on the x-axis is 1 (under ZStandard, $\boxed{\text{ZOOM}}$ $\boxed{6}$ settings). You can verify this using XSCL in the $\boxed{\text{WINDOW}}$. It may be different under different WINDOW and ZOOM settings.



- Under TABLE SETUP ($\boxed{\text{2ND}}$ $\boxed{\text{WINDOW}}$), enter a TBLSTART value close to 1 (or where the asymptote is observed to be located on the graph) e.g. 0.



- We then access the tabulated values ($\boxed{\text{2ND}}$ $\boxed{\text{GRAPH}}$) to identify value(s) of x for which the function is undefined.



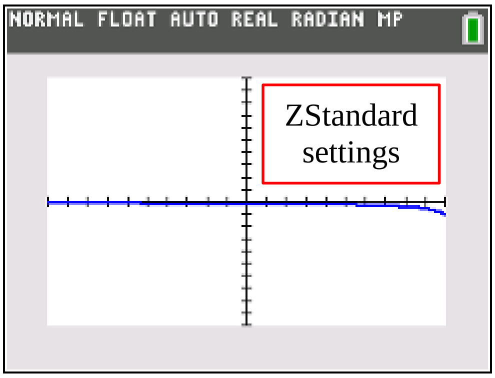
X	Y1
0	0
1	ERROR
2	2
3	$\frac{3}{2}$
4	$\frac{4}{3}$
5	$\frac{5}{4}$

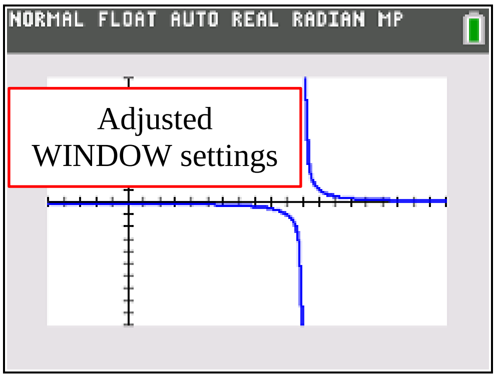
- Observations
  - ✓ The value of y is undefined when $x=1$ (indicated as ERROR).
  - ✓ There is a vertical asymptote of $x=1$.

<u>Important!</u>

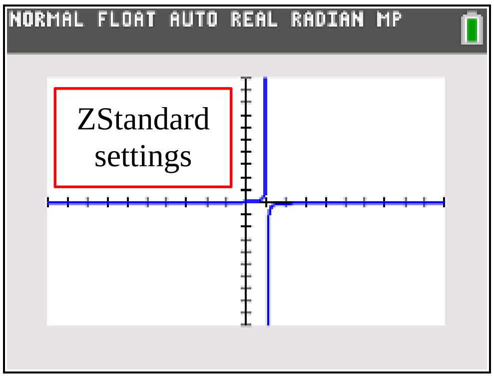
- The possible location(s) of the vertical asymptote(s) may not be displayed on the screen when the WINDOW and ZOOM settings are inappropriate.

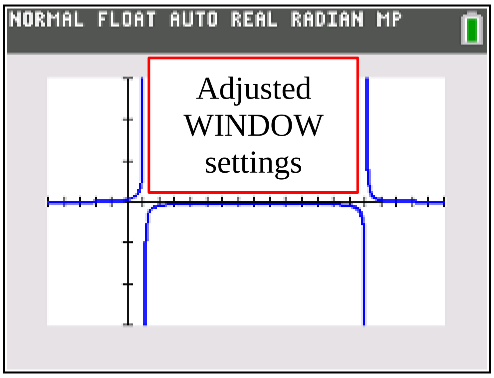
  For example, the possible location of the vertical asymptote of the graph of $y=\dfrac{1}{x-15}$, will not be displayed under ZStandard ($\boxed{\text{ZOOM}}$ $\boxed{6}$) settings.



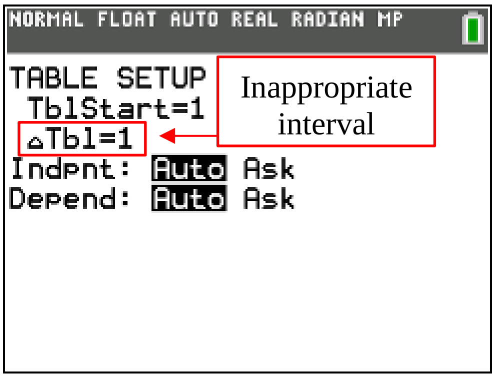


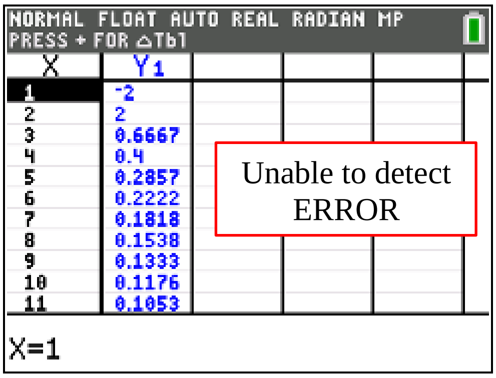
- A graph may have more than 1 vertical asymptote and not all may be displayed simultaneously e.g. $y=\dfrac{1}{(x-1)(x-15)}$ under ZStandard ($\boxed{\text{ZOOM}}$ $\boxed{6}$) settings.



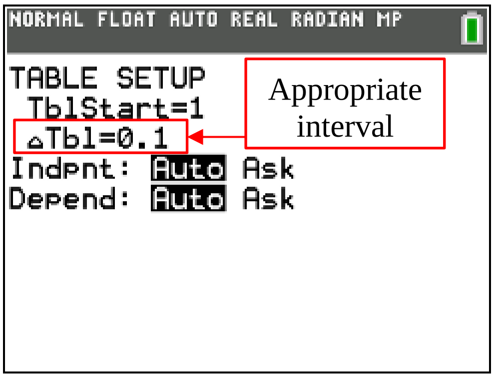


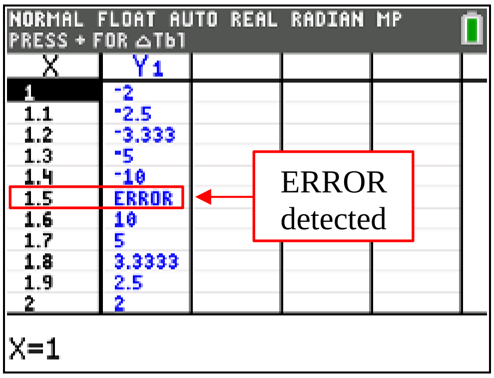
- A suitable incremental interval of x ($\triangle$ TBL) is required in the TABLE SETUP to subsequently identify the value(s) of x for which a function is undefined.
  For example, using an incremental interval of 1 for x ($\triangle \text{TBL} = 1$), we will not be able to identify the value of x for which $y = \dfrac{1}{x - 1.5}$ is undefined. Consequently, the vertical asymptote cannot obtained.





  In this case, an incremental interval of 0.1 is more appropriate.



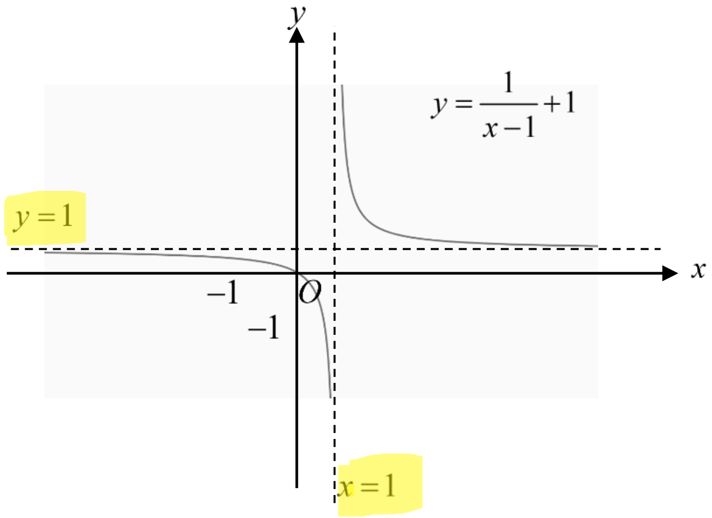


- While the GC can help us locate the vertical asymptotes, a lack of knowledge of the function will still not allow us to identify all of them, especially when they are not displayed. In this case, one will not be able to find the un-displayed asymptotes without good knowledge of the function.

- In other words, you **SHOULD NOT BE RELIANT** on the GC to show every feature of the graph of a function. Mastering the mathematical knowledge required is necessary.

Exercise 1

Sketch the graph of $y = \dfrac{1}{x-1} + 1$.



Discussion

(a) Write down the equation of the asymptote of the graph of $y = \ln x$.

(b) Without the use of a GC, sketch the graph of $y = \ln x$.

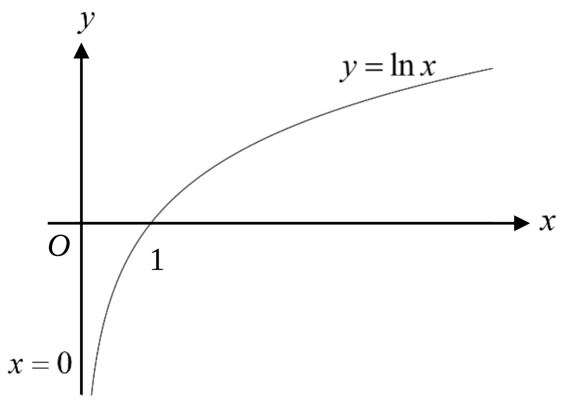
(c) **(i)** Now, graph $y = \ln x$ on your GC.

(ii) What do you observe has occurred? Why do you think this has occurred?

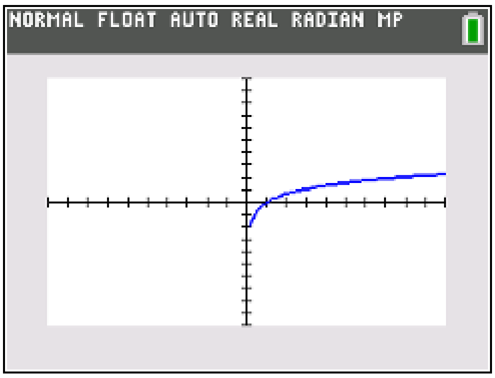
(iii) What lessons can you learn?

(a) $x = 0$

(b)



(c)(i)



(c)(ii) The graph of $y = \ln x$ seemed to have "truncated" in the GC display. This is due to the display limitations of the GC, which can be improved with different ZOOM settings.

(c)(iii) Again, we cannot be totally reliant on the GC to tell us all the features of the graph. In this case, a lack of knowledge of the graphs of logarithmic functions will result in us being misled by the display seen on the GC.

2.2 The ZOOM and WINDOW Settings

Example 1

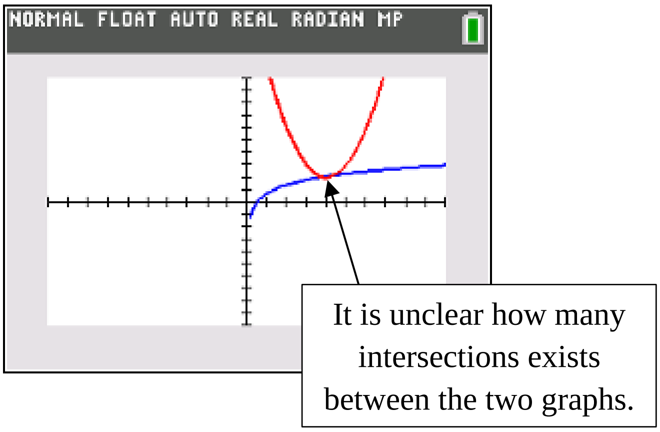
With the aid of a GC,

(i) sketch the functions $y=\ln(2x)$ and $y=x^2-8x+18$ on the same set of axes.

(ii) find all the solutions to the equation $\ln(2x)=x^2-8x+18$.

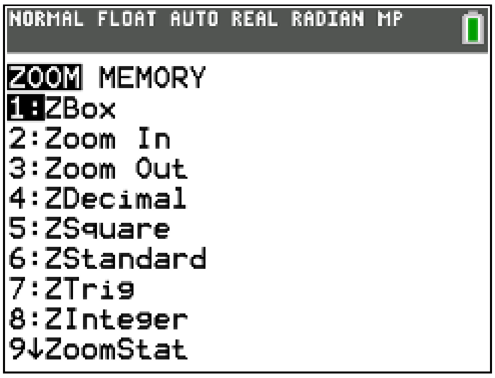
[Solution]

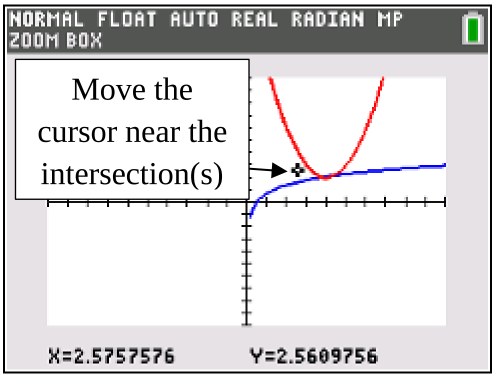
(i) This will probably be what you see on the GC using ZStandard ($\boxed{\text{ZOOM}}$ $\boxed{6}$) settings.



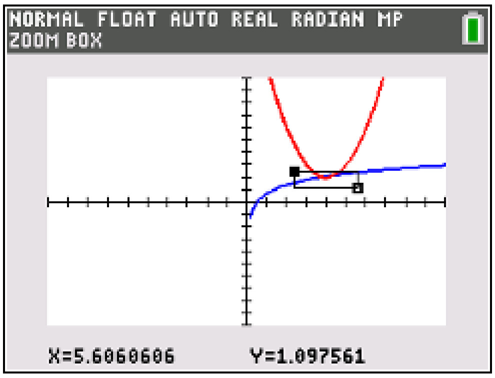
To get a better view of the intersections, we can use the ZBOX function ($\boxed{\text{ZOOM}}$ $\boxed{1}$).

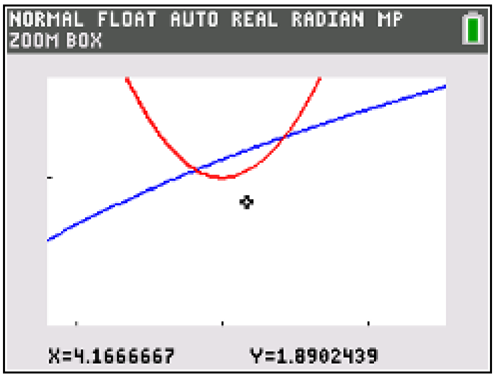
- Select $\boxed{\text{ZOOM}}$ $\boxed{1}$ to activate the ZBox function.
- Move the cursor to a position near the left (and slightly above) of the intersection point using the arrow keys.



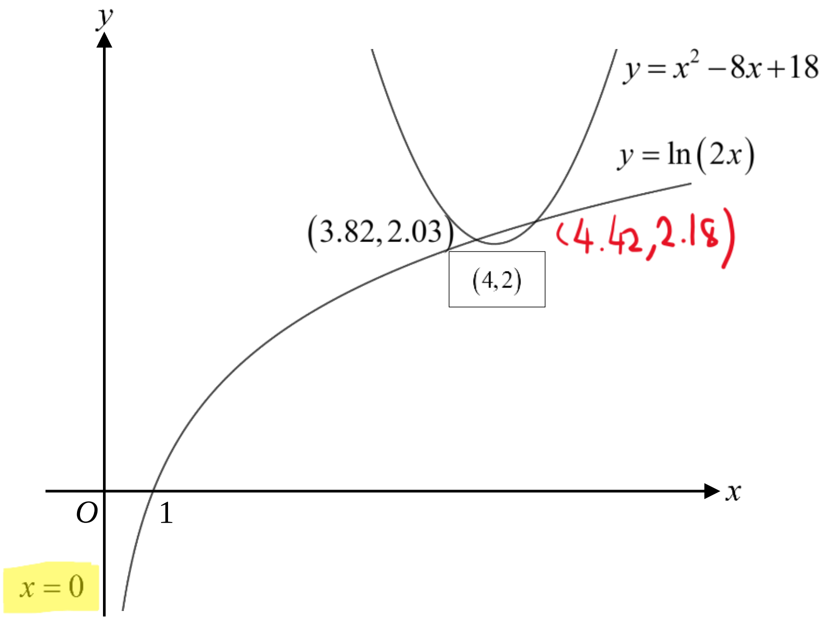


- Press $\boxed{\text{ENTER}}$, then scroll rightwards and downwards to form a box around the intersection point.
- Press $\boxed{\text{ENTER}}$ again to zoom into the boxed up portion of the graph.

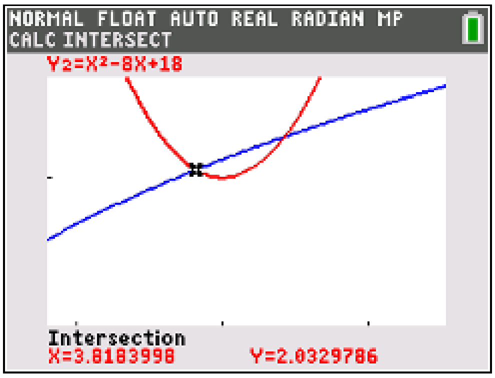


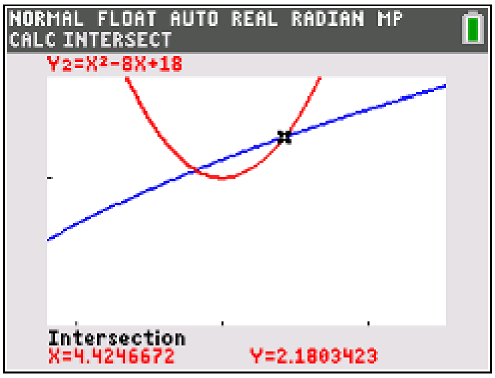


Hence the graphical sketch should be as follows:



(ii) Proceed to find the intersection points, hence solve the question.
(Recall: How do you find the intersection points between two graphs on a GC.)





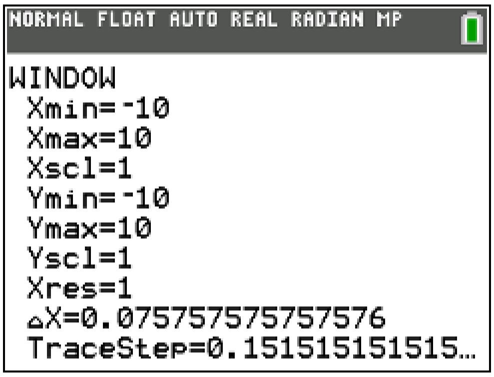
Hence for $\ln(2x) = x^2 - 8x + 18$, $x = 3.82$ or $x = 4.42$ (to 3 s.f.)

Try it!
Instead of using ZBox, attempt to solve this problem by exploring the use of the ZOOM IN ($\boxed{\text{ZOOM}}$ $\boxed{2}$) function instead.

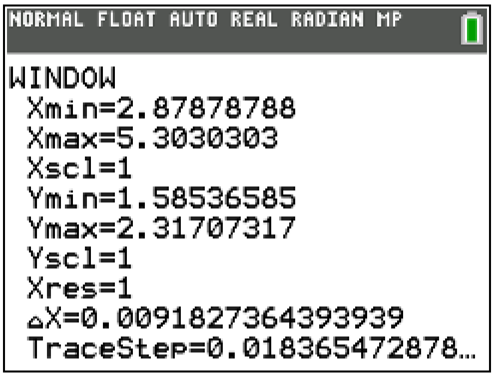
Ponder over the advantages/disadvantages of using the ZOOM IN function. Which do you prefer, ZBox or ZOOM IN?

More about the ZOOM function

- The default ZOOM used by the GC (after reset) is the ZStandard ($\boxed{\text{ZOOM}}$ $\boxed{6}$). Under this setting, the WINDOW settings will be as such:



- Changing the ZOOM setting will change the WINDOW settings consequently. For example, in using ZBox in Example 1, the WINDOW settings could become



- The commonly used ZOOM functions are as follows:

ZBox	ZOOM 1	Creates a region of focus for the user to zoom into.
ZStandard	ZOOM 6	The default ZOOM setting of the GC.
ZoomFit	ZOOM 0	Causes the GC to fit the window around the graph.
ZSquare	ZOOM 5	Causes the *x*- and *y*- axes to follow the same scale. (Useful when graphing functions to the correct proportions e.g. circles.)
ZTrig	ZOOM 7	Allows the GC to display trigonometric graphs in a better to view manner.
ZoomStat	ZOOM 9	For statistical plots.

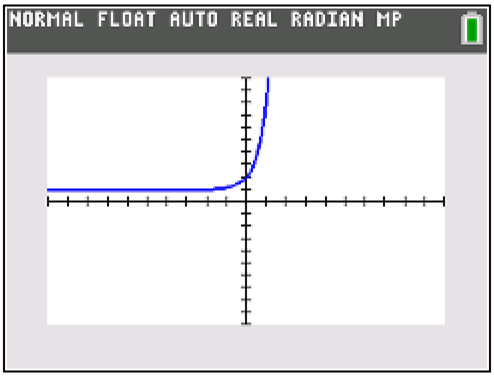
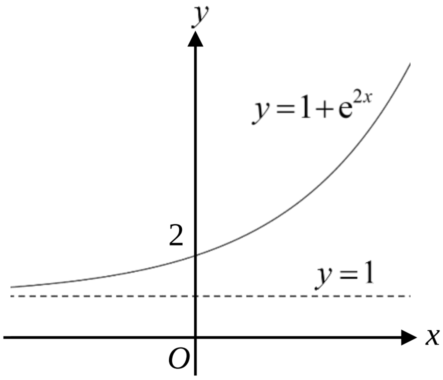
It may sometimes be more convenient to just adjust the WINDOW settings rather than using the ZOOM function.

<u>Example 2</u>
With the aid of a GC,

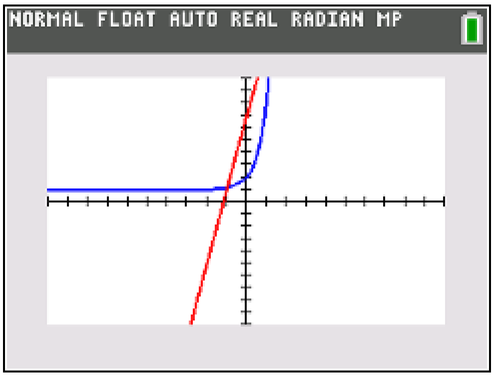
(i) sketch the graph of $y = 1 + e^{2x}$

(ii) find, using a graphical method, the solution(s) to the equation $1 + e^{2x} = 6x + 7$

[Solution]
(i)

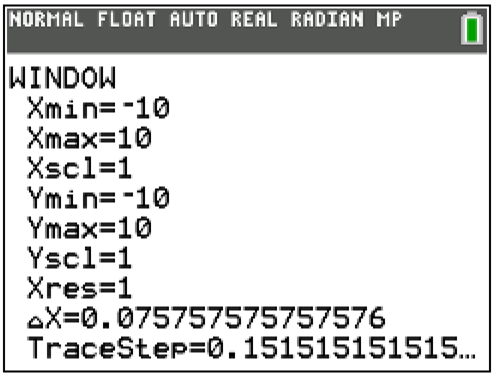
Graph of $y = 1 + e^{2x}$ on ZStandard settings. 	The actual graphical sketch should demonstrate understanding of the proper curvature. 

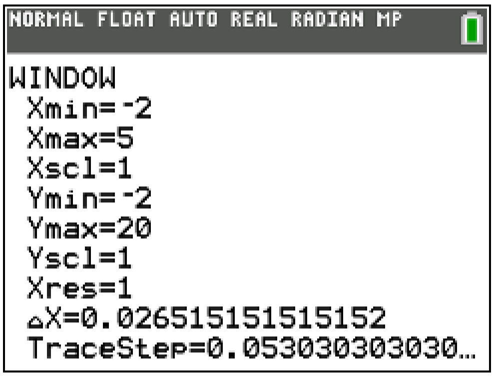
(ii) To solve the equation graphically, graph the line $y = 6x + 7$ on the GC. Under ZStandard settings, the graphs will be displayed as such.



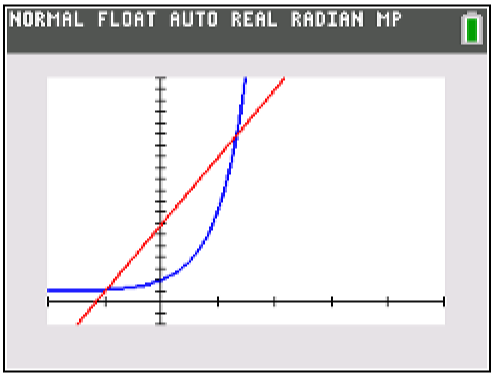
Observing the behaviour of the 2 graphs, it seems that a second intersection is not being displayed.

Hence adjust the WINDOW settings as such:



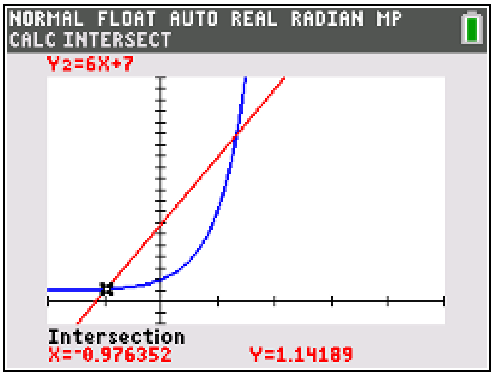


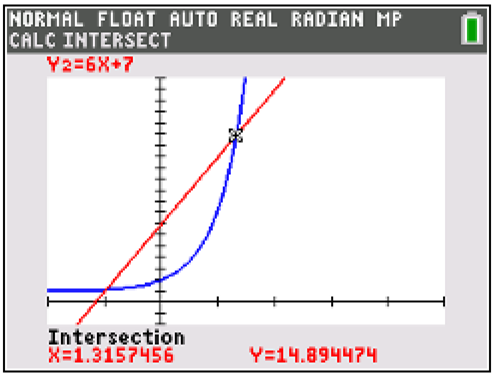
The graph will then be displayed as such:



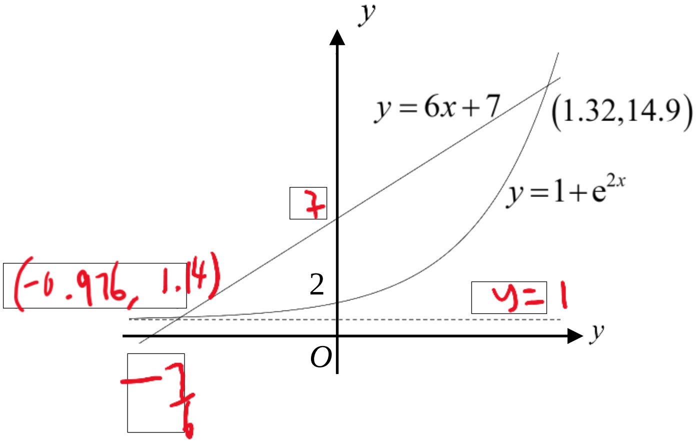
You might sometimes need to adjust the WINDOW settings a few times before obtaining the required display.

Proceed to find the intersection points, hence solve the question.
(Recall: How do you find the intersection points between two graphs on a GC.)





Hence for $1+e^{2x}=6x+7$, $x=-0.976$ or $x=1.32$ (to 3 s.f.)



Homework 16A

1 On separate sets of axes, sketch the graph of

(a) $y=x^3-2x-1$

(b) $y=x\left(e^x-3\right)$

with the aid of a GC. Label **all** the intersections with the axes, the maximum and minimum points, and the equations of the asymptotes (if any).

2 Solve the following simultaneous equations using a graphical method.

(a) $\begin{cases} y=2-e^x \\ y=x^2+x-1 \end{cases}$ **(b)** $\begin{cases} y=\ln(4-x) \\ y=\sqrt{2x+1} \end{cases}$

For each pair of simultaneous equations, sketch the graphs on the same set of axes, label **all** points of intersection, intersections with the axes, the maximum and minimum points, and the equations of the asymptotes (if any).