

TEMASEK JUNIOR COLLEGE

2025 IP4 TERM 1 REVISION



CANDIDATE
NAME

CG

SUBJECT
TUTOR'S NAME

ADVANCED MATHEMATICS

23 February 2024

45 minutes

READ THESE INSTRUCTIONS FIRST

Write your name, CG and tutor's name on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

Write your answers in the spaces provided in the question paper.

You may request for additional writing materials if there is insufficient space. These should be attached to the back of the booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question. The calculator value for π should be used unless the question requires the answer in terms of π .

The use of an approved scientific and/or graphing calculator is expected where appropriate.

You are reminded of the need for clear presentation in your answers. Marks will be deducted for poor or unclear presentation. The number of marks is given in brackets [] at the end of each question or part question.

For Examiner's Use	
Q1	/10
Q2	/7
Q3	/7
Q4	/6
Presentation Deduction	-1
Total	/30

This document consists of **6** printed pages and **no** blank page.

MATHEMATICAL FORMULAE

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin P + \sin Q \equiv 2 \sin \frac{1}{2}(P+Q) \cos \frac{1}{2}(P-Q)$$

$$\sin P - \sin Q \equiv 2 \cos \frac{1}{2}(P+Q) \sin \frac{1}{2}(P-Q)$$

$$\cos P + \cos Q \equiv 2 \cos \frac{1}{2}(P+Q) \cos \frac{1}{2}(P-Q)$$

$$\cos P - \cos Q \equiv -2 \sin \frac{1}{2}(P+Q) \sin \frac{1}{2}(P-Q)$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of } \triangle ABC = \frac{1}{2}ab \sin C$$

[2020 WA1]

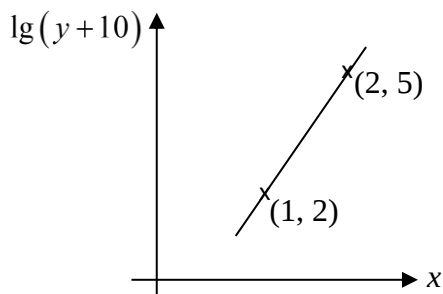
- 1 (i) State the coordinates of the centre and the radius of the circle with equation $(x+1)^2 + (y-2)^2 = 25$. [2]
- (ii) Find the exact coordinates of the points where the circle crosses the axes. [3]
- (iii) Hence, sketch the circle, labelling the coordinates of the centre and the points where the circle crosses the axes. [2]
- (iv) State the equation of a tangent to the circle that is parallel to the x -axis. [1]

[2018 CBA2: Modified]

- 2 Sketch the following graphs, stating the equation(s) of asymptotes (if any) and intersection(s) with the axes.
- (a) $(x-2)^2 + (y+3)^2 = 36$ [4]

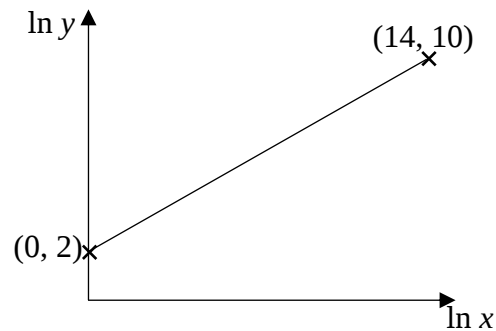
[2023 WA1]

- 3 The equation $y = ab^x - 10$, where a and b are constants, can be represented by a straight line when $\lg(y+10)$ is plotted against x , as shown in the diagram below. Find the value of a and of b . [6]



[2024 WA3]

- 4 The variables x and y are related in such a way that when $\ln y$ is plotted against $\ln x$, a straight line is obtained as shown in the figure below.

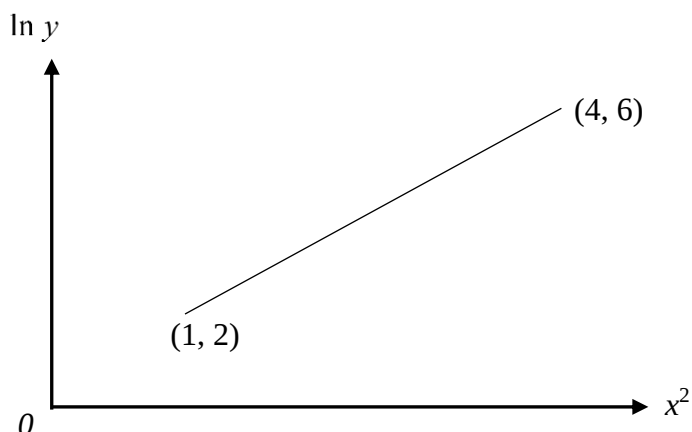


Given that this straight line passes through $(0, 2)$ and $(14, 10)$, find

- (i) the **exact** value of x when $\ln y = 6$, [3]
- (ii) the value of a and of n when the relationship between x and y is expressed in the form $y = ax^n$. [4]

[2022 WA1]

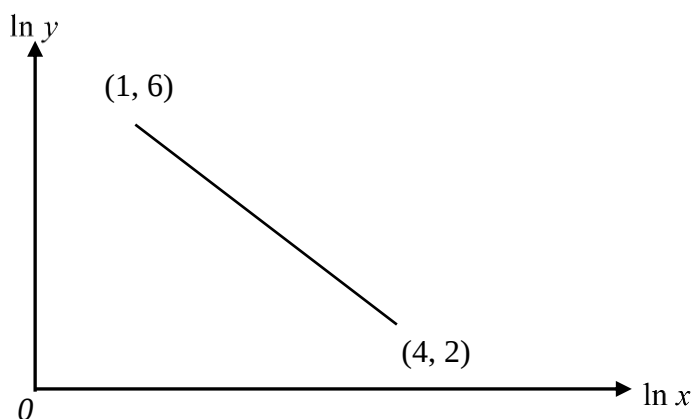
- 5 The diagram shows part of a straight line graph of $\ln y$ against x^2 , passing through the points (1, 2) and (4, 6).



- (i) Express y in terms of x . [3]
- (ii) Find the exact value of y when $x = 3$. [1]
- (iii) Determine, with clear reasoning, if the point (8, 10) lies on the graph above. [1]

[2019 WA1]

- 6 The diagram below (not drawn to scale), shows part of a straight-line graph drawn to represent the equation $x^n y = c$.



- (i) Calculate the values of n and c . [4]
- (ii) Using the values from **part (i)**, express y in terms of x . [1]
- (iii) A straight line $\ln y = 3 \ln x + 2$ is drawn on the same axes. Find the values of x and of y for which $cx^{-n} = e^2 x^3$. [3]

[2022 IP3 WA2]

7 (a) Solve the equation $e^{\sqrt{x}}(4e^{\sqrt{x}} - 1) = \frac{3}{2}$. [3]

(b) Solve the equation $1 + \log_3 y + \frac{1}{\log_{\frac{1}{27}} 3} = \log_3(2 - y)$. [4]

[2022 IP3 WA3]

8 Solve the equation $\log_2 x - 3\log_x 2 = \frac{1}{2}$. [6]

[2021 WA2]

9 Solve the equation $\log_8 x - \log_x 64 = 1$. [5]

[2020 IP3 WA2]

10 Solve the equation $\lg x = 24\log_x 10 - 5$. [6]

[2020 WA1]

11 Solve the equation $\log_4 x + \log_2 4x = \log_8 1 + 5$. [6]

[2017 IP3 CBA4]

12 (a) Solve the equation $\ln(2^x + 4) - x \ln 2 = \ln 3$. [4]

(b) Find the exact value of $\log_{\frac{1}{4}} 8$. [3]

~~~~~ End of Paper ~~~~~

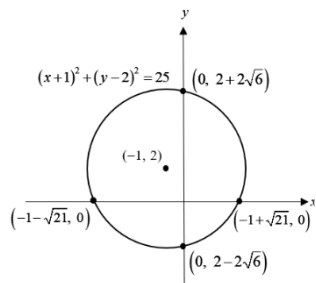
Answers

1 (i) Centre: $(-1, 2)$, Radius: 5

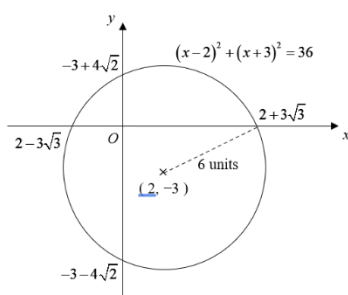
(ii) $(0, 2+2\sqrt{6})$, $(0, 2-2\sqrt{6})$, $(-1-\sqrt{21}, 0)$, $(-1+\sqrt{21}, 0)$

(iii)

(iv) $y = 7$ or $y = -3$



2



3 $a = 0.1$, $b = 1000$

4 (i) $x = e^7$ (ii) $n = \frac{4}{7}$, $a = e^2$

5 (i) $y = e^{\frac{4}{3}x^2 + \frac{2}{3}}$ (ii) $y = e^{\frac{38}{3}}$ (iii) The point does not lie on the line.

6 (i) $n = \frac{4}{3}$, $c = e^{\frac{22}{3}}$ or 1530 (3 s.f.) (ii) $y = e^{\frac{22}{3}} x^{-\frac{4}{3}}$

(iii) $x = e^{\frac{16}{13}}$ or 3.42 (3 s.f.), $y = 297$ (3.s.f.)

7 (a) No real number solution. (b) $y = \frac{9}{5}$

8 $y = 2^{-\frac{3}{2}}$ or $y = 4$

9 $x = 64$ or $x = \frac{1}{8}$

10 $x = 10^{-8}$ or $x = 1000$

11 $x = 4$

12 (a) $x = 1$ (b) $-\frac{3}{2}$