CATHOLIC JUNIOR COLLEGE
General Certificate of Education Advanced Level
Higher 2
JC1 Promotional Examination

CANDIDATE NAME	

CLASS		INDEX NUMBER	

MATHEMATICS **9758/01**

Paper 1

8 Oct 2025
3 hours

Additional Materials: Printed Answer Booklet
List of Formulae (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** the questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **7** printed pages and 1 blank page.

1 A sequence T_n is given by $T_n = \ln\left(n^a\right) + bn^2 + c$ for $n \in \mathbb{Z}^+$. The first three terms of the sequence are 9, 17.306 and 31.901.

(a) Find the values of a, b and c, correct to the nearest integers. [3]

(b) Using the integer values found in part **(a)**, find the sum of the first hundred terms of T_n. [1]

2 Differentiate $\tan^{-1}\left(\dfrac{x}{\sqrt{1-x^2}}\right)$ with respect to x. You should simplify your answer as a single fraction in its simplest form. [4]

3 Without using a calculator, solve the inequality $\dfrac{x}{x-2} < \dfrac{3}{(5x+2)(2-x)}$. [3]

Hence, solve the following inequalities.

(a) $\dfrac{e^x}{e^x-2} < \dfrac{3}{\left(5e^x+2\right)\left(2-e^x\right)}$ [2]

(b) $\dfrac{x}{x-4} > \dfrac{12}{(5x+4)(4-x)}$ [2]

4 Aenicillin, a type of bacteria, is grown in a petri dish. Its population doubles every six hours. Denoting the amount of Aenicillin at the beginning of the first day as x_0,

(a) write down a recurrence relation for x_n, $n \geq 1$, the amount of Aenicillin in the petri dish at the end of n^{th} day, [1]

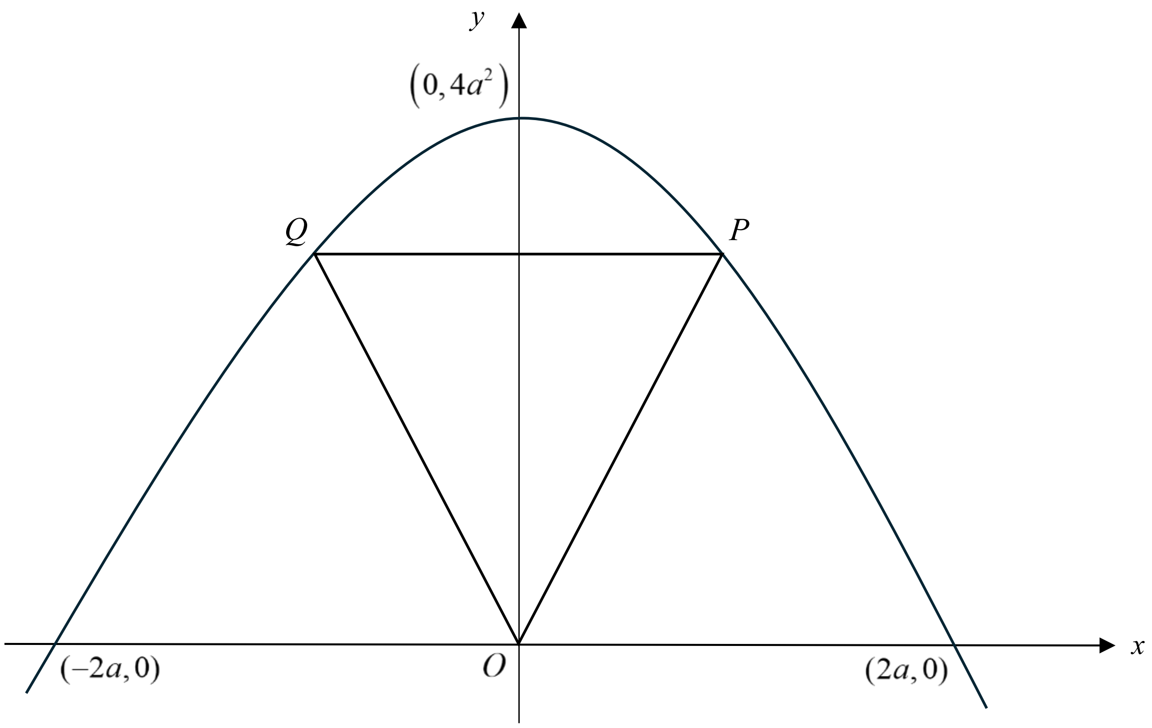
(b) find x_n, expressing your answer in the form of $x_n = \mathrm{f}(n)$, $n \geq 1$. [1]

The growth rate of another bacteria, Benicillin, is known to follow the recurrence relation $u_n = u_{n-1} + n$, $n \geq 1$ where u_n is the amount of Benicillin at the end of n^{th} day. The amount of Benicillin in the petri dish at the beginning of the first day, u_0, is 1 unit.

(c) Write down $u_1 - u_0$, $u_2 - u_1$ and $u_3 - u_2$. Hence find a quadratic expression for $\left(u_1 - u_0\right) + \left(u_2 - u_1\right) + \ldots + \left(u_{n-1} - u_{n-2}\right) + \left(u_n - u_{n-1}\right)$ in terms of n. [3]

(d) Show that $\sum_{r=1}^{n}\left(u_r - u_{r-1}\right) = u_n - u_0$. Using your results in part **(c)**, find u_n expressing your answer in the form of $u_n = \mathrm{f}(n)$, $n \geq 1$. [3]

5 The diagram below shows a parabola which cuts the x-axis at $(-2a,0)$, $(2a,0)$ and has a maximum point at $(0,4a^2)$, where a is a positive constant. A triangle OPQ is inscribed in the parabola such that the points P and Q lie symmetrically on the parabola at $x=\pm k$ (where $0<k<2a$) and O is the origin.

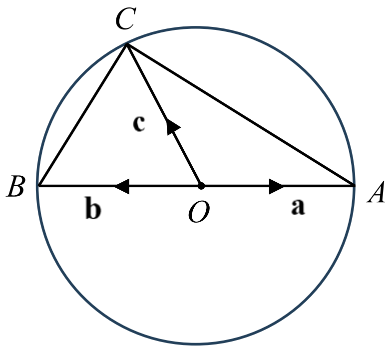


(a) Write down an equation of the parabola in terms of a. [1]

(b) By considering the coordinates of P, show that A, the area of triangle OPQ, can be written as $A=k\left(4a^2-k^2\right)$. [2]

(c) Hence using differentiation, find the maximum value of A in terms of a as k varies. [5]

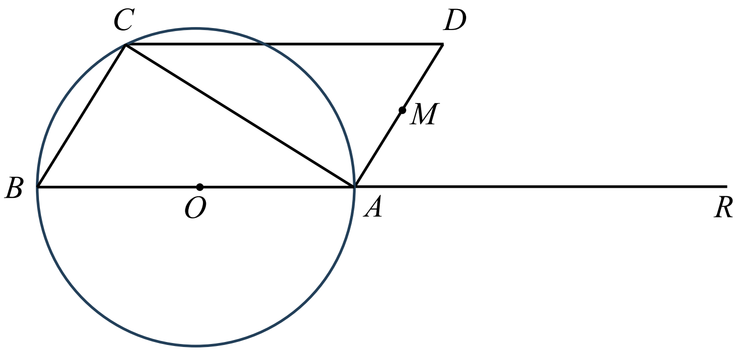
6 The diagram shows a unit circle with centre O and diameter AB. The point C lies on the circumference of the circle. The position vectors of A, B and C are $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively.



(a) By considering an appropriate scalar product, show that AC and BC are perpendicular. [4]

(b) Give a geometrical interpretation of $|\mathbf{a} \cdot (\mathbf{c} - \mathbf{a})|$. [1]

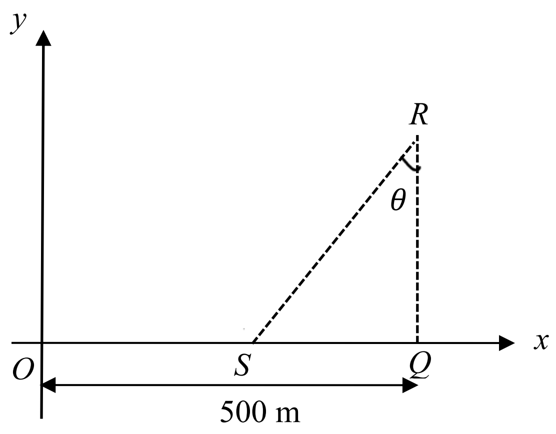
The point D is such that $ABCD$ is a parallelogram. The point M is the mid-point of AD and the point R lies on OA produced such that $OA : AR = 1 : 2$.



(c) By considering $\overrightarrow{CM} \times \overrightarrow{CR}$ or otherwise, show that the points C, M and R are collinear. [3]

7 **(a)** David flies a drone with a flight path C, modelled by the equation $y = 2^x$, where $x \geq 0$ and $y > 0$. He stands at the origin, O, and sees the drone at point $P(a, b)$. Assuming David's line of sight is the tangent to the curve C at point P which passes through the origin, find the exact coordinates of P. [4]

(b) In another instance, David stands at the origin O and flies the drone vertically upwards from a fixed point Q, 500 m away from O. The drone, represented by R, is ascending upwards at a constant speed of 20 m/s. At the same time, David walks along the horizontal line segment OQ at a constant speed of 5 m/s. At any time t seconds, David's position is point S on the horizontal line segment OQ.



(i) Given that the angle θ, in radians, is the angle SRQ, show that $\tan\theta = \dfrac{100 - t}{4t}$. [1]

(ii) By implicit differentiation or otherwise, find $\dfrac{\mathrm{d}\theta}{\mathrm{d}t}$ in terms of t. [4]

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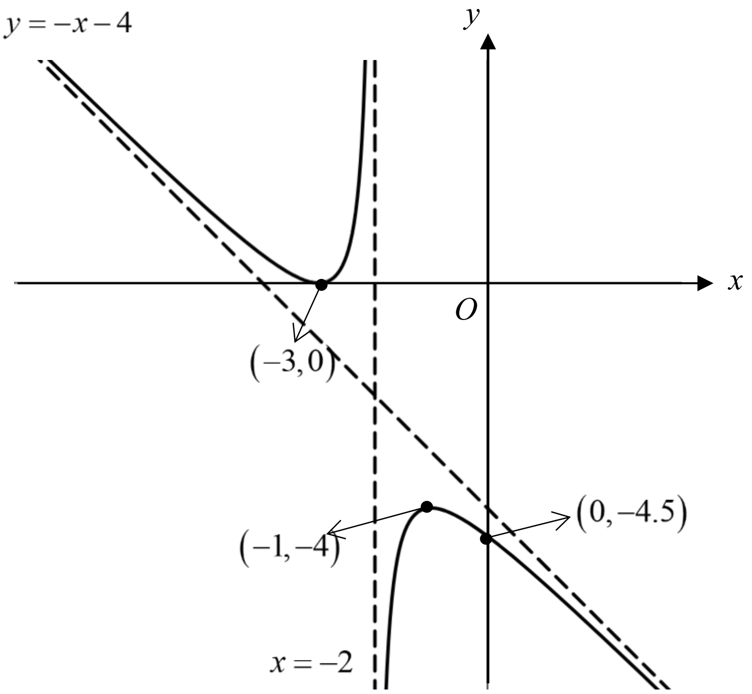

The diagram shows the curve $y=\mathrm{f}(x)$. The curve crosses the x-axis at its minimum point $(-3,0)$. It has a maximum turning point $(-1,-4)$ and crosses the y-axis at the point $(0,-4.5)$. The asymptotes of the curve are $x=-2$ and $y=-x-4$.

On separate diagrams, sketch the graphs of

(a) $y=\mathrm{f}'(x)$, [3]

(b) $y=\dfrac{1}{\mathrm{f}(x)}$, [3]

(c) $y=|\mathrm{f}(x)|$, [3]

indicating the equations of any asymptotes, turning points and intersections with the axes.

9 **(a)** The curve C has equation

$$y=\frac{x^2+4x-5}{x-5}, \quad x\in\mathbb{R}, \quad x\neq 5.$$

(i) Sketch C, stating the equations of the asymptotes, axial intercepts and the coordinates of the turning points, if any. [4]

(ii) Hence, by sketching another suitable graph on the same axes as in part **(a)(i)**, solve the inequality $\dfrac{x^2+4x-5}{2x-10}\geq \ln(x+5)$. [3]

(b) Describe a pair of transformations which would transform curve C onto the graph of $y=\dfrac{4x^2+8x-5}{2x-5}-2$. [2]

10 The functions f and g are defined by

$$\mathrm{f}: x \mapsto \left|\frac{2x+2\lambda}{x-2}\right|, \quad x \in \mathbb{R},\ -\lambda < x < 2$$

$$\mathrm{g}: x \mapsto x^2 - \lambda, \quad x \in \mathbb{R},\ -\sqrt{\lambda} < x < \sqrt{\lambda}$$

where λ is a positive constant.

(a) Show that f has an inverse and find an expression for $\mathrm{f}^{-1}(x)$. [4]

(b) Find the range of values of x such that $\mathrm{f}^{-1}\mathrm{f}(x) = \mathrm{ff}^{-1}(x)$. [1]

(c) **(i)** Explain why fg does not exist. [1]

(ii) By restricting the domain of g to $\left(0,\ \sqrt{\lambda}\right)$, find the rule of fg and its range. [4]

11 The plane Π_1 contains the points $A(1,4,2)$, $B(1,0,5)$ and $C(0,8,-1)$.

(a) Find a cartesian equation of Π_1. [3]

(b) Find the shortest distance between the point P (2,1,2) and the plane Π_1. [2]

A second plane Π_2 contains the point $D(2,2,3)$ and is perpendicular to the vector $\mathbf{i}+2\mathbf{j}+2\mathbf{k}$. The point $(p,0,q)$ lies in both planes.

(c) Find p and q. [3]

(d) Hence find an equation of the line of intersection of the two planes in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$, where λ is a real constant. [2]

(e) Find the acute angle between the line AC and the plane Π_2. [2]

12 **(a)** A team of cyclists plans to ride from Singapore to Penang and back over 10 days. The distance for the route taken from Singapore to Penang and back is 1600 km.

(i) Alex suggests that the team cycle 120 km for the first day and increase the distance covered by 8 km each subsequent day. Show that the team will not be able to complete the route in 10 days. [2]

(ii) Belle suggests changing the distance covered on the first day so that the team will reach Singapore on the tenth day. Given that the increment of distance covered for each subsequent day remains at 8 km, and the distance covered on the last day is 196 km, find the distance covered on the first day. [2]

(b) On 2 January 2025, Candy saved \$8000 in a bank account which gives a compound interest of 1% per annum at the end of each year. Withdrawals of \$1200 are made at the beginning of each subsequent year.

(i) Find the amount in Candy's bank account at the beginning of the third year. [2]

(ii) Show that the amount in Candy's bank account at the beginning of the $(n + 1)$th year is $\$\left[120000 - 112000(1.01)^n\right]$. [3]

(iii) In which year would Candy's bank account first have less than \$1200? [3]

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