1 A sequence T_n is given by $T_n = \ln\left(n^a\right) + bn^2 + c$ for $n \in \mathbb{Z}^+$. The first three terms of the sequence are 9, 17.306 and 31.901.

(a) Find the values of a, b and c, correct to the nearest integers. [3]

(b) Using the integer values found in part **(a)**, find the sum of the first hundred terms of T_n. [1]

Solution

(a)	$T_1 = \ln\left(1^a\right) + b\left(1\right)^2 + c$ $9 = b + c$ ----------(1) $T_2 = \ln\left(2^a\right) + b\left(2\right)^2 + c$ $17.306 = a\ln 2 + 4b + c$ ----------(2) $T_3 = \ln\left(3^a\right) + b\left(3\right)^2 + c$ $31.901 = a\ln 3 + 9b + c$ ----------(3) Solving, $a = -1$, $b = 3$ and $c = 6$ (nearest integers)
(b)	Required sum = $\sum_{n=1}^{100} T_n = \sum_{n=1}^{100}\left(\ln\left(n^{-1}\right) + 3n^2 + 6\right)$ $= 1015286.261$ $= 1020000$ (3 s.f.)

Examiners' Report

(a)	This part question was generally well done. The majority of the students were able to write down the 3 equations involving 3 unknowns and used G.C. to solve directly. A small percentage of students solved the 3 questions algebraically at the expense of their time which could have been used for other questions. A small percentage of students did not read the question carefully that the final answers should be rounded off to the nearest integer or mindlessly wrote rounded off to the nearest integers, but the final answers were still rounded off to 3 significant figures. A small percentage of students did not know how to formulate the 3 equations (by substituting the values of *n* and values of terms) or had wrongly substituted the values of the terms (T_1, T_2, T_3) as *n*.
(b)	This part question was not well done. Many students applied the sum formulae of A.P./G.P. even though there was no common difference/common ratio. Some students did not recognize that the question asked for sum of first 100 terms, not the 100th term. A small percentage of students spilt up the summation and used G.C. to evaluate the 3 summations separately, which was unnecessary. A small percentage of students keyed in the summation wrongly (e.g. the wrong placement of brackets) which resulted in the wrong value.

2 Differentiate $\tan^{-1}\left(\dfrac{x}{\sqrt{1-x^2}}\right)$ with respect to x. You should simplify your answer as a single fraction in its simplest form. [4]

Solution [~2.5m]

$$\frac{\mathrm{d}}{\mathrm{d}x}\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

$$=\frac{1}{1+\left(\dfrac{x^2}{1-x^2}\right)}\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

$$=\left(\frac{1-x^2}{1-x^2+x^2}\right)\left(\frac{\sqrt{1-x^2}(1)-x\left[\frac{1}{2}\left(1-x^2\right)^{-\frac{1}{2}}(-2x)\right]}{1-x^2}\right)$$

$$=\left(\frac{1}{\sqrt{1-x^2}}\right)$$

Examiners' Report

- Many students erroneously differentiated $\tan^{-1}\left(\mathrm{f}\left(x\right)\right)$ into $\dfrac{1}{1+x^2}\mathrm{f}'\left(x\right)$ instead of $\dfrac{1}{1+\left[\mathrm{f}\left(x\right)\right]^2}\mathrm{f}'\left(x\right)$.
- A notable number of students curiously assumed that $\dfrac{\mathrm{d}}{\mathrm{d}x}\tan^{-1}\left(\dfrac{x}{\sqrt{1-x^2}}\right)=\dfrac{\mathrm{d}}{\mathrm{d}x}\tan^{-1}\left(x\right)-\dfrac{\mathrm{d}}{\mathrm{d}x}\tan^{-1}\left(\sqrt{1-x^2}\right)$, a confusion that is probably spawned by the quotient law of logarithm.
- Most students who struggle with this question bungled up the application of quotient rule in one way or another. But mistakes appear to be mostly careless slips.
- Some students erroneously assumed that chain rule involves addition of derivatives instead of multiplication of derivatives.

3 Without using a calculator, solve the inequality $\dfrac{x}{x-2}<\dfrac{3}{(5x+2)(2-x)}$. [3]

Hence, solve the following inequalities.

(a) $\dfrac{e^x}{e^x-2}<\dfrac{3}{(5e^x+2)(2-e^x)}$ [2]

(b) $\dfrac{x}{x-4}>\dfrac{12}{(5x+4)(4-x)}$ [2]

Solution

$$\frac{x}{x-2}<\frac{3}{(5x+2)(2-x)}$$

$$\frac{x}{x-2}-\frac{3}{(5x+2)(2-x)}<0$$

$$\frac{x}{x-2}+\frac{3}{(5x+2)(x-2)}<0$$

$$\frac{x(5x+2)+3}{(5x+2)(x-2)}<0$$

$$\frac{5x^2+2x+3}{(5x+2)(x-2)}<0$$

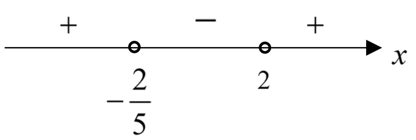
$$\frac{5\left[x^2+\frac{2}{5}x+\frac{3}{5}\right]}{(5x+2)(x-2)}<0$$

$$\frac{5\left[\left(x+\frac{1}{5}\right)^2+\frac{3}{5}-\left(\frac{1}{5}\right)^2\right]}{(5x+2)(x-2)}<0$$

$$\frac{5\left[\left(x+\frac{1}{5}\right)^2+\frac{14}{25}\right]}{(5x+2)(x-2)}<0$$

Since numerator $5\left(x+\frac{1}{5}\right)^2+\frac{14}{5}$ is always positive for all real values of x,

So only need to consider $\dfrac{1}{(5x+2)(x-2)}<0$



Hence, $-\frac{2}{5}<x<2$

Alternative Method

(for showing numerator is always positive) $\frac{5x^2+2x+3}{(5x+2)(x-2)}<0$ Consider $5x^2+2x+3=0$ $b^2-4ac=2^2-4(5)(3)$ $\quad <0$ Additionally, $a=5>0$. Hence $5x^2+2x+3$ is always positive for all values of x.
(a) Replace x with e^x $-\frac{2}{5}<e^x<2$ $0<e^x<2$ (since $e^x>0$ for all real values of x) $x<\ln 2$
(b) Substitute x by $\frac{x}{2}$ into $\frac{x}{x-2}>\frac{3}{(5x+2)(2-x)}$ we get $\frac{\frac{x}{2}}{\frac{x}{2}-2}>\frac{3}{\left(5\left(\frac{x}{2}\right)+2\right)\left(2-\frac{x}{2}\right)}$ Multiplying by $\frac{2}{2}$ to both sides of inequality, $\frac{x}{x-4}>\frac{6}{(5x+4)\left(2-\frac{x}{2}\right)}$ $\frac{x}{x-4}>\frac{12}{(5x+4)(4-x)}$ Hence, using above answer, $\frac{x}{2}<-\frac{2}{5}$ or $\frac{x}{2}>2$ $x<-\frac{4}{5}$ or $x>4$

Examiners' Report

Generally, students brought all the terms to one side before making the expression a single fraction. However, students had trouble explaining why the expression $5x^2+2x+3$ is always positive. Students either • totally ignore $5x^2+2x+3$ and proceed to solve the remaining inequality • attempting to use discriminant to show that $5x^2+2x+3$ is always positive without stating that the coefficient of x^2 is 5 (which is positive) • attempting to complete the square without explaining that it is always positive In the above situations, students will never be awarded full credit due to omission of essential working. A number of students also made errors in completing the square. This is an assumed knowledge that H2 Math students are expected to have.
(a) Students were able to identify the correct replacement. However, a number of them struggled to arrive at the final answer due to errors made in the earlier part or in solving inequalities involving exponential function.
(b) Many students struggled to identify the correct replacement. This was a *hence* question, therefore, no marks were awarded if students solved the inequality from scratch.

4 Aenicillin, a type of bacteria, is grown in a petri dish. Its population doubles every six hours. Denoting the amount of Aenicillin at the beginning of the first day as x_0,

(a) write down a recurrence relation for x_n, $n \geq 1$, the amount of Aenicillin in the petri dish at the end of n^{th} day, [1]

(b) find x_n, expressing your answer in the form of $x_n = \mathrm{f}(n),\ n \geq 1$. [1]

The growth rate of another bacteria, Benicillin, is known to follow the recurrence relation $u_n = u_{n-1} + n,\ n \geq 1$ where u_n is the amount of Benicillin at the end of n^{th} day. The amount of Benicillin in the petri dish at the beginning of the first day, u_0, is 1 unit.

(c) Write down $u_1 - u_0$, $u_2 - u_1$ and $u_3 - u_2$. Hence find a quadratic expression for $(u_1 - u_0) + (u_2 - u_1) + \ldots + (u_{n-1} - u_{n-2}) + (u_n - u_{n-1})$ in terms of n. [3]

(d) Show that $\sum_{r=1}^{n} (u_r - u_{r-1}) = u_n - u_0$. Using your results in part **(c)**, find u_n expressing your answer in the form of $u_n = \mathrm{f}(n),\ n \geq 1$. [3]

Solution

(a) $x_n = 2^4 x_{n-1} \quad n \geq 1$

Or, $x_n = 16 x_{n-1} \quad n \geq 1$

(b) $x_n = x_0 \left(2^4\right)^n$

Or, $x_n = x_0 (16)^n$

(c) When n =1, $u_1 - u_0 = 1$

When n =2, $u_2 - u_1 = 2$

When n =3, $u_3 - u_2 = 3$

Hence

$$(u_1 - u_0) + (u_2 - u_1) + (u_3 - u_2) + \ldots + (u_{n-1} - u_{n-2}) + (u_n - u_{n-1})$$
$$= 1 + 2 + 3 + \ldots + (n-1) + n$$
$$= \frac{n}{2}(1+n)$$

OR

Hence $(u_1 - u_0) + (u_2 - u_1) + \ldots + (u_{n-1} - u_{n-2}) + (u_n - u_{n-1}) = an^2 + bn + c$

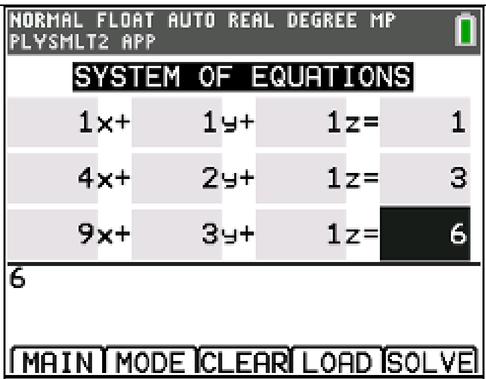
When n = 1, $u_1 - u_0 = 1 = a + b + c$

When n = 2, $(u_2 - u_2) + (u_2 - u_1) = 1 + 2 = 4a + 2b + c$

When n = 3, $(u_3 - u_2) + (u_2 - u_1) + (u_1 - u_0) = 1 + 2 + 3 = 9a + 3b + c$

From GC, $a = \frac{1}{2}, b = \frac{1}{2}, c = 0$

$$(u_1 - u_0) + (u_2 - u_1) + \ldots + (u_{n-1} - u_{n-2}) + (u_n - u_{n-1}) = \frac{1}{2}n^2 + \frac{1}{2}n$$



(d) $\sum_{r=1}^{n}\left(u_r - u_{r-1}\right)$

$= \left(u_1 - u_0\right) + \left(u_2 - u_1\right) + \left(u_3 - u_2\right) + \ldots + \left(u_{n-1} - u_{n-2}\right) + \left(u_n - u_{n-1}\right)$

$= -u_0 + u_n$

OR

$\sum_{r=1}^{n}\left(u_r - u_{r-1}\right) = \sum_{r=1}^{n} u_r - \sum_{r=1}^{n} u_{r-1}$

$= u_1 + u_2 + u_3 + \ldots + u_{n-1} + u_n - \left(u_0 + u_1 + u_2 + \ldots + u_{n-2} + u_{n-1}\right)$

$= u_n - u_0$

$1+2+3+4+\ldots+n = -u_0 + u_n$

$\frac{n}{2}(n+1) = -1 + u_n$ [from part (c)]

$u_n = \frac{n}{2}(n+1) + 1$

Examiners' Report

(a)

- Many students gave part (b) answer here, writing x_n in terms of x_0 instead of x_{n-1}, as they did not know what is a recurrence relation.
- Some students did not read the questions carefully and gave $x_{n+1}=2^4 x_n$ instead of $x_n=2^4 x_{n-1}$ for $n\geq 1$.
- Some even wrote the general term of Geometric Progression.
- A few students had poor notations (wrote superscript n instead of subscript).

(b)

- Many students left this blank or copied their answer in part (a) here as they did the working earlier.
- Some wrote the general term of Geometric Progression, or were confused and gave $x_n=x_0\left(2^4\right)^{n-1}$ or $x_n=x_0\left(2^{n-1}\right)$ instead of $x_n=x_0\left(2^4\right)^n$.

(c)

- Many students did not know how to use the given recurrence relation $u_n=u_{n-1}+n,\ n\geq 1$, and ended up with:

  $u_1=u_0+n,\ u_2=u_1+n,\ u_3=u_2+n,\ldots$

  $u_1-u_0=u_2-u_1=u_3-u_2=\ldots=n$

  $\left(u_1-u_0\right)+\left(u_2-u_1\right)+\left(u_3-u_2\right)+\ldots+\left(u_{n-1}-u_{n-2}\right)+\left(u_n-u_{n-1}\right)$

  $=\underbrace{n+n+n+\ldots+n+n}_{n\text{ times}}$

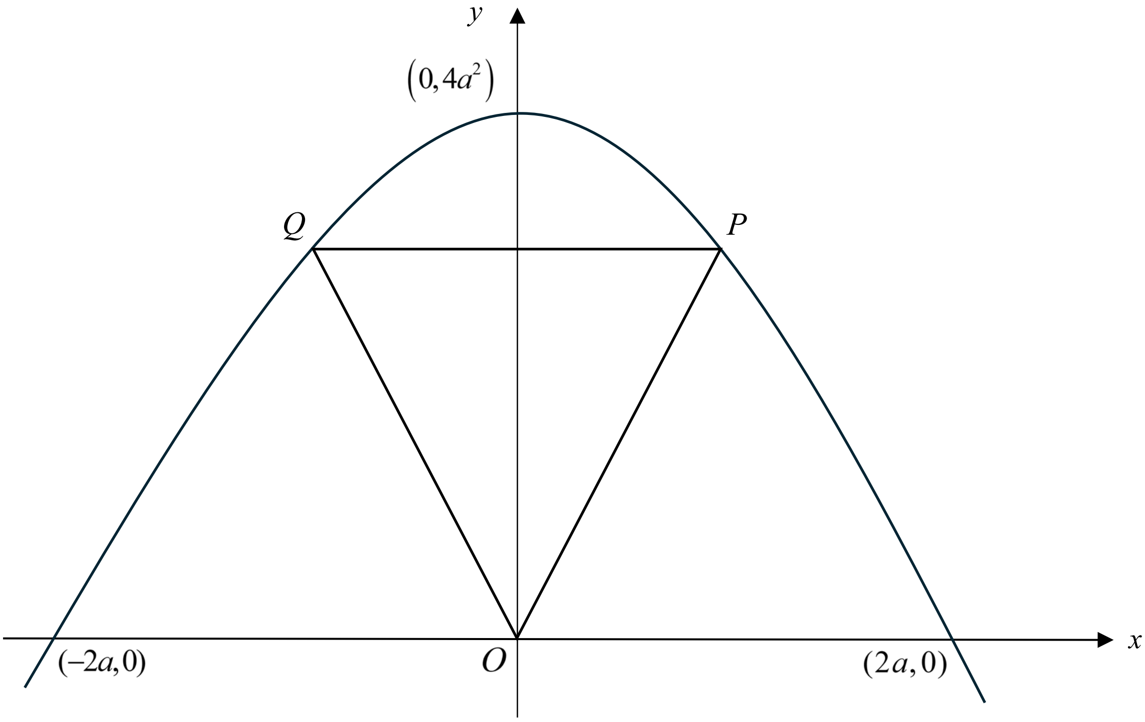
  $=n\left(n\right)=n^2$
- For those who got the first mark, did not know $u_{n-1}-u_{n-2}=n-1$ and $u_n-u_{n-1}=n$, or only state the sum of an Arithmetic Progression and thus not given full credit for this part.
- Some students ended up with what was asked to be shown in the next part $\sum_{r=1}^{n}\left(u_r-u_{r-1}\right)=u_n-u_0$.
- Only 1 student used the alternative method but fail to get all 3 equations correct and thus could not the answer.

(d)

- Many students did not show their working clearly for the first part. Some stopped at $\sum_{r=1}^{n}\left(u_r-u_{r-1}\right)=\sum_{r=1}^{n}u_r-\sum_{r=1}^{n}u_{r-1}$, and a few even continue to make the mistake

  $\sum_{r=1}^{n}\left(u_r-u_{r-1}\right)=\sum_{r=1}^{n}u_r-\sum_{r=1}^{n}u_{r-1}=u\sum_{r=1}^{n}r-u\sum_{r=1}^{n}\left(r-1\right)\ldots$
- Students who used the given expression to proceed with the 2nd part obtained full or method marks if they got part (c) wrong. Many did not do so.

5 The diagram below shows a parabola which cuts the x-axis at $(-2a,0)$, $(2a,0)$ and has a maximum point at $(0,4a^2)$, where a is a positive constant. A triangle OPQ is inscribed in the parabola such that the points P and Q lie symmetrically on the parabola at $x=\pm k$ (where $0<k<2a$) and O is the origin.



(a) Write down an equation of the parabola in terms of a. [1]

(b) By considering the coordinates of P, show that A, the area of triangle OPQ, can be written as $A=k\left(4a^2-k^2\right)$. [2]

(c) Hence using differentiation, find the maximum value of A in terms of a as k varies. [5]

Solution

(a)	$y=4a^2-x^2$
(b)	At P, $x=k$ and $y=4a^2-k^2$ $A=\frac{1}{2}(2k)\left(4a^2-k^2\right)$ $=k\left(4a^2-k^2\right)$
(c)	At maximum A, $\frac{\mathrm{d}A}{\mathrm{d}k}=0$. $k(-2k)+\left(4a^2-k^2\right)=0$ $3k^2=4a^2$ $k=\frac{2a}{\sqrt{3}}$ (-ve k rejected) $\frac{\mathrm{d}^2A}{\mathrm{d}k^2}=-2(2k)-2k=-6k$ When $k=\frac{2a}{\sqrt{3}}$,

$$\frac{d^2A}{dk^2} = -6\left(\frac{2a}{\sqrt{3}}\right) = -\frac{12a}{\sqrt{3}} \quad (<0)$$

Alternatively,

k	$\left(\frac{2a}{\sqrt{3}}\right)^-$	$\frac{2a}{\sqrt{3}}$	$\left(\frac{2a}{\sqrt{3}}\right)^+$
Sign of $\frac{dA}{dk}$	>0	0	<0
Tangent	/	—	\

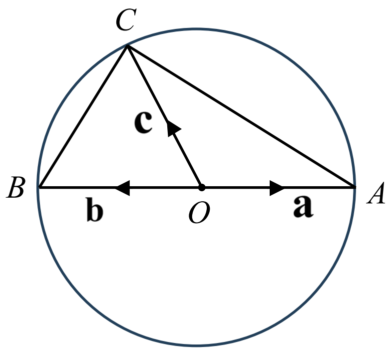
Thus,

$$\text{Maximum } A = \frac{2a}{\sqrt{3}}\left(4a^2 - \left(\frac{2a}{\sqrt{3}}\right)^2\right) = \frac{16a^3}{3\sqrt{3}}$$

Examiners' Report

(a) Some students observed that the x-intercepts are $2a$ and $-2a$, so $y=(x+2a)(x-2a)$, but by doing this, they miss out the negative sign for x^2. It would be correct if they put as $y=-(x+2a)(x-2a)=-x^2+4a$. There were students who used the standard form of a parabola, $(y-k)=a(x-h)^2$, but many put an additional "square" at the $(y-k)$, or were unsuccessful in finding the value of a in the equation.
(b) Many students left this blank as they were unsuccessful in part (a). Solutions of the form $\frac{1}{2}(2k)(\_\_)$ were given partial credit, as the formula for area of triangle was applied, with a base of $2k$.
(c) • Many students did not see that a is a constant, while k is the variable. Thus, $\frac{dA}{da}=0$ at the maximum area, instead of $\frac{dA}{dk}=0$. Also, after solving $\frac{dA}{dk}=0$, k should be made the subject instead of a. • There were many differentiation slips seen when finding $\frac{dA}{dk}$. For example, differentiating $4a^2$ w.r.t. k should give 0, not $8a$, since a is a constant. • A significant number of students did not show that A is maximum (with either a 1st or 2nd derivative test), or did not find the maximum value of A, as required by the question. • Answers should be sufficiently simplified. Solutions such as $\frac{8}{3}a^2\sqrt{\frac{4}{3}a^2}$ should be further simplified.

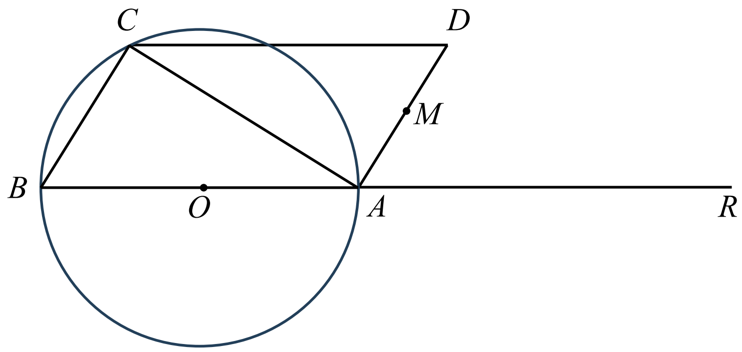
6 The diagram shows a unit circle with centre O and diameter AB. The point C lies on the circumference of the circle. The position vectors of A, B and C are $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively.



(a) By considering an appropriate scalar product, show that AC and BC are perpendicular. [4]

(b) Give a geometrical interpretation of $\left|\mathbf{a}\cdot(\mathbf{c}-\mathbf{a})\right|$. [1]

The point D is such that $ABCD$ is a parallelogram. The point M is the mid-point of AD and the point R lies on OA produced such that $OA : AR = 1:2$.

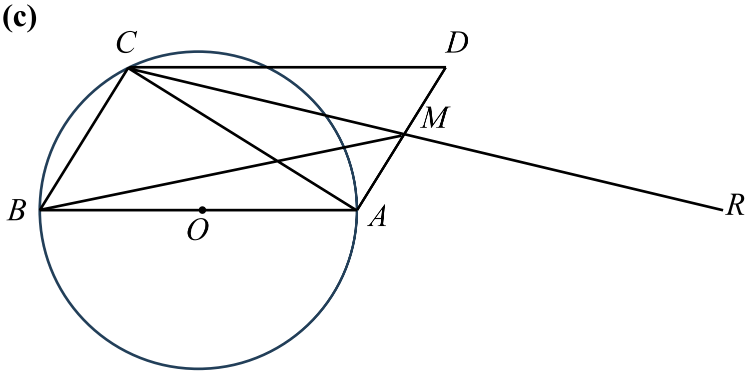


(c) By considering $\overrightarrow{CM} \times \overrightarrow{CR}$ or otherwise, show that the points C, M and R are collinear. [3]

Solution

(a)	$\overrightarrow{AC} = \mathbf{c} - \mathbf{a}$ $\overrightarrow{BC} = \mathbf{c} - \mathbf{b}$ $\overrightarrow{AC} \cdot \overrightarrow{BC} = (\mathbf{c}-\mathbf{a})\cdot(\mathbf{c}-\mathbf{b})$ $= (\mathbf{c}-\mathbf{a})\cdot(\mathbf{c}-(-\mathbf{a}))$ $= (\mathbf{c}-\mathbf{a})\cdot(\mathbf{c}+\mathbf{a})$ $= \mathbf{c}\cdot\mathbf{c} - \mathbf{a}\cdot\mathbf{a}$ $= \lvert\mathbf{c}\rvert^2 - \lvert\mathbf{a}\rvert^2$ $= 0$ ($\lvert\mathbf{c}\rvert = \lvert\mathbf{a}\rvert =$ radius of circle) Hence AC and BC are perpendicular.
(b)	$\left|\mathbf{a}.(\mathbf{c}-\mathbf{a})\right|$ is the length of projection of $\overrightarrow{AC}$ onto $\overrightarrow{OA}$.

(c)



By Ratio Theorem,

$$\overrightarrow{OM} = \frac{\overrightarrow{OD}+\overrightarrow{OA}}{2}$$

$$= \frac{1}{2}(\mathbf{c}+2\mathbf{a}+\mathbf{a})$$

$$= \frac{1}{2}\mathbf{c}+\frac{3}{2}\mathbf{a}$$

$$\overrightarrow{CM}\times\overrightarrow{CR} = \left(\overrightarrow{OM}-\overrightarrow{OC}\right)\times\left(\overrightarrow{OR}-\overrightarrow{OC}\right)$$

$$= \left[\frac{1}{2}\mathbf{c}+\frac{3}{2}\mathbf{a}-\mathbf{c}\right]\times(3\mathbf{a}-\mathbf{c})$$

$$= \left(\frac{3}{2}\mathbf{a}-\frac{1}{2}\mathbf{c}\right)\times(3\mathbf{a}-\mathbf{c})$$

$$= \frac{9}{2}\mathbf{a}\times\mathbf{a}-\frac{3}{2}\mathbf{a}\times\mathbf{c}-\frac{3}{2}\mathbf{c}\times\mathbf{a}+\frac{1}{2}\mathbf{c}\times\mathbf{c}$$

$$= \mathbf{0}-\frac{3}{2}\mathbf{a}\times\mathbf{c}+\frac{3}{2}\mathbf{a}\times\mathbf{c}+\mathbf{0}$$

$$= \mathbf{0}$$

$\Rightarrow \overrightarrow{CM}$ and $\overrightarrow{CR}$ are parallel.

Since CR and CM are parallel and C is a common point.

Hence C, M and R are collinear.

Alternatively,

$$\overrightarrow{CM} = \overrightarrow{OM}-\overrightarrow{OC}$$

$$= \frac{1}{2}\mathbf{c}+\frac{3}{2}\mathbf{a}-\mathbf{c}$$

$$= \frac{3}{2}\mathbf{a}-\frac{1}{2}\mathbf{c}$$

$$\overrightarrow{CR} = \overrightarrow{OR}-\overrightarrow{OC}$$

$$= 3\mathbf{a}-\mathbf{c}$$

$$= 2\left(\frac{3}{2}\mathbf{a}-\frac{1}{2}\mathbf{c}\right)$$

$$= 2\overrightarrow{CM}$$

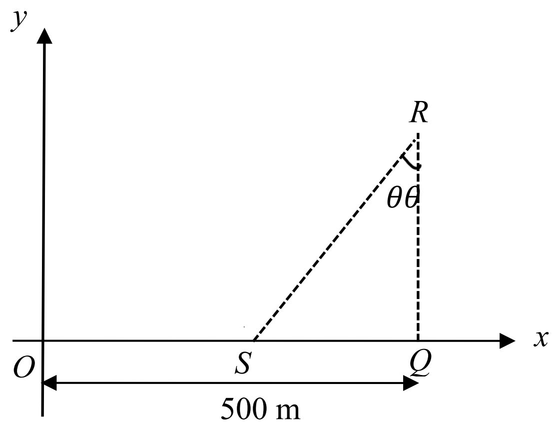
$\Rightarrow \overrightarrow{CM}$ and $\overrightarrow{CR}$ are parallel. Since *CR* and *CM* are parallel and *C* is a common point. Hence *C*, *M* and *R* are collinear.

Examiners' Report

(a) Students could generally start the question by taking the scalar product of $\overrightarrow{AC} \cdot \overrightarrow{BC}$. However, there were several poor presentations and/or misconceptions. Errors include: - $\mathbf{c} \cdot \mathbf{c} = \mathbf{c}^2$, $\mathbf{a} \cdot \mathbf{c} = \mathbf{ac}$. The use of $\cdot$ represent a scalar product of 2 vectors, it is not multiplying 2 variables. - Erroneously taking $\lvert\mathbf{c}\rvert^2 = 0$. - Erroneously assuming $\mathbf{a} = \mathbf{b} = \mathbf{c}$ when it is only $\lvert\mathbf{a}\rvert^2 = \lvert\mathbf{b}\rvert^2 = \lvert\mathbf{c}\rvert^2 = 1$. Do note that from the diagram, $\mathbf{a} = -\mathbf{b}$.
(b) This part was poorly done. Students need to identify that an application of scalar product is the length of projection, and in this case, **a** is the unit vector. Thus the geometrical interpretation of $\lvert\mathbf{a} \cdot (\mathbf{c} - \mathbf{a})\rvert$ is the length of projection of $\overrightarrow{AC}$ onto $\overrightarrow{OA}$, and not the length of projection of $\overrightarrow{OA}$ onto $\overrightarrow{AC}$. Also, $(\mathbf{c} - \mathbf{a})$ must be interpreted as $\overrightarrow{AC}$.
(c) Most students recognized the need to find $\overrightarrow{CM}$, $\overrightarrow{CR}$ or $\overrightarrow{MR}$, and attempted to use vector triangles or ratio theorem, however, varied success at arriving $\overrightarrow{CM}$, $\overrightarrow{CR}$ or $\overrightarrow{MR}$ in terms of **a** or **b**, and **c**. Thereafter, students generally knew they had to show that the vector product of $\overrightarrow{CM} \times \overrightarrow{CR} = \mathbf{0}$ or find the scalar *k* for which $\overrightarrow{CR} = k\overrightarrow{CM}$ $(k = 2)$. A significant number of students then concluded that *C*, *M* and *R* are collinear, this is insufficient. There is a need to specifically indicate that C is a common point when showing collinear.

7 **(a)** David flies a drone with a flight path C, modelled by the equation $y = 2^x$, where $x \geq 0$ and $y > 0$. He stands at the origin, O, and sees the drone at point $P(a, b)$. Assuming David's line of sight is the tangent to the curve C at point P which passes through the origin, find the exact coordinates of P. [4]

(b) In another instance, David stands at the origin O and flies the drone vertically upwards from a fixed point Q, 500 m away from O. The drone, represented by R, is ascending upwards at a constant speed of 20 m/s. At the same time, David walks along the horizontal line segment OQ at a constant speed of 5 m/s. At any time t seconds, David's position is point S on the horizontal line segment OQ.



(i) Given that the angle θ, in radians, is the angle SRQ, show that $\tan\theta = \dfrac{100 - t}{4t}$. [1]

(ii) By implicit differentiation or otherwise, find $\dfrac{\mathrm{d}\theta}{\mathrm{d}t}$ in terms of t. [4]

Solution

(a)

$y = 2^x$

$\ln y = x \ln 2$

$\dfrac{1}{y}\dfrac{\mathrm{d}y}{\mathrm{d}x} = \ln 2$

$\dfrac{\mathrm{d}y}{\mathrm{d}x} = y \ln 2 = 2^x \ln 2$

Equation of tangent (line of sight) at P:

$y - b = (2^a \ln 2)(x - a)$

Since the line of sight passes through origin

$0 - b = (2^a \ln 2)(0 - a)$

$0 - 2^a = (2^a \ln 2)(0 - a)$

$1 = (\ln 2)a$

$a = \dfrac{1}{\ln 2}$

Thus P = $\left(\dfrac{1}{\ln 2}, 2^{\frac{1}{\ln 2}}\right)$

(bi) $\tan\theta = \dfrac{SQ}{QR} = \dfrac{500-5t}{20t}$ $\tan\theta = \dfrac{100-t}{4t}$ (shown)
(bii) $\tan\theta = \dfrac{100-t}{4t}$ $\sec^2\theta\dfrac{\mathrm{d}\theta}{\mathrm{d}t} = \dfrac{(4t)(-1)-(100-t)(4)}{(4t)^2}$ $\sec^2\theta\dfrac{\mathrm{d}\theta}{\mathrm{d}t} = \dfrac{-400}{(4t)^2}$ $\left(1+\left(\dfrac{100-t}{4t}\right)^2\right)\dfrac{\mathrm{d}\theta}{\mathrm{d}t} = -\dfrac{400}{(4t)^2}$ $\left(\dfrac{(4t)^2+(100-t)^2}{(4t)^2}\right)\dfrac{\mathrm{d}\theta}{\mathrm{d}t} = -\dfrac{400}{(4t)^2}$ $\dfrac{\mathrm{d}\theta}{\mathrm{d}t} = -\dfrac{400}{(4t)^2+(100-t)^2}$
Alternative: $\theta = \tan^{-1}\left(\dfrac{100-t}{4t}\right)$ $\dfrac{\mathrm{d}\theta}{\mathrm{d}t} = \dfrac{1}{1+\left(\dfrac{100-t}{4t}\right)^2}\left[\dfrac{(4t)(-1)-(100-t)(4)}{(4t)^2}\right]$ $\dfrac{\mathrm{d}\theta}{\mathrm{d}t} = \dfrac{(4t)^2}{(4t)^2+(100-t)^2}\left[\dfrac{-400}{(4t)^2}\right]$ $\dfrac{\mathrm{d}\theta}{\mathrm{d}t} = -\dfrac{400}{(4t)^2+(100-t)^2}$

Examiners' Report

(a) While many students knew they had to differentiate 2^x to find the gradient at P, they were not successfully for

- inability to differentiate 2^x well, erroneously taking is as $\frac{\mathrm{d}}{\mathrm{d}x}2^x = x(2^{x-1})$, or
- did not recognize that gradient at P is a value $2^a \ln 2$ or $b\ln 2$, it is not in terms of x or y, or
- letting $x = 0$ and $y = 0$ even though the gradient of the curve was at P, not origin.

In instances where the gradient was successfully found, students were generally able to equate the equation of tangent to the equation of the curve to get the exact coordinates of P. There were some cases where non-exact answers were given, this wasn't accepted.

(bi) As this is a show question, students are reminded of the need for clear explanation, and a good approach would be to state clearly what is SQ, QR in terms of t. There were quite a few imprecise/inaccurate scripts stating that $SQ = \frac{500}{5} - t = 100 - t$ and $QR = 4t$. In this case, SQ was taken as time, and QR as distance, the ratio of $\tan\theta = \frac{SQ}{QR}$ is therefore inconsistent.

(bii) While there were many good responses, there were also many errors. These errors include

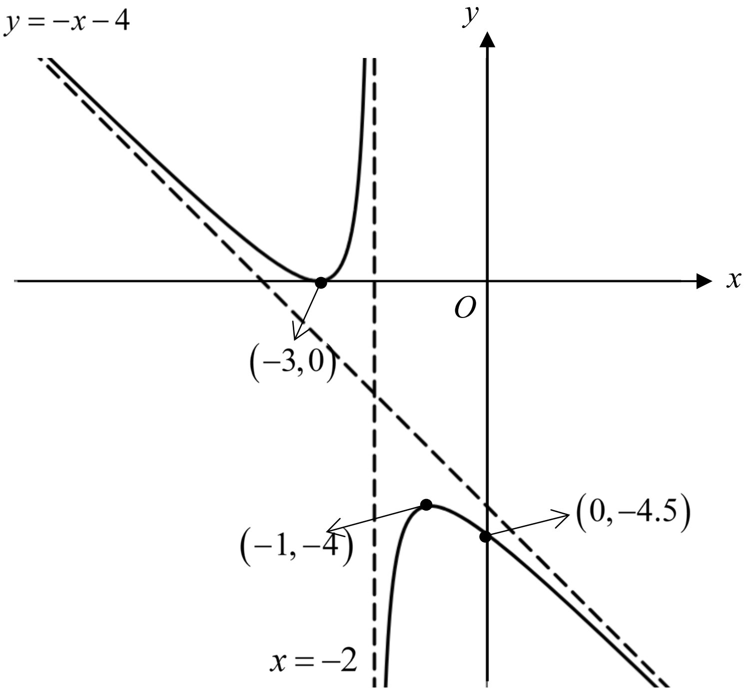
- $\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{1}{1+\left(\frac{100-t}{4t}\right)}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{100-t}{4t}\right)$
- $\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{1}{1+t^2}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{100-t}{4t}\right)$
- inaccuracies **in applying quotient rule**

In cases where implicit differentiation was used, the errors are:

- $\frac{\mathrm{d}}{\mathrm{d}t}\tan\theta = \sec\theta$ (or $\cot\theta$)
- $\tan\theta = \frac{100-t}{4t} \Rightarrow$

$\sec^2\theta = \frac{(4t)(-1)\frac{\mathrm{d}\theta}{\mathrm{d}t} - (100-t)(4)\frac{\mathrm{d}\theta}{\mathrm{d}t}}{(4t)^2}$. This is a fundamental misconception, there was no understanding of the variable are we differentiating with respect to.

8



The diagram shows the curve $y=\mathrm{f}(x)$. The curve crosses the x-axis at its minimum point $(-3,0)$. It has a maximum turning point $(-1,-4)$ and crosses the y-axis at the point $(0,-4.5)$. The asymptotes of the curve are $x=-2$ and $y=-x-4$.

On separate diagrams, sketch the graphs of

(a) $y=\mathrm{f}'(x)$, [3]

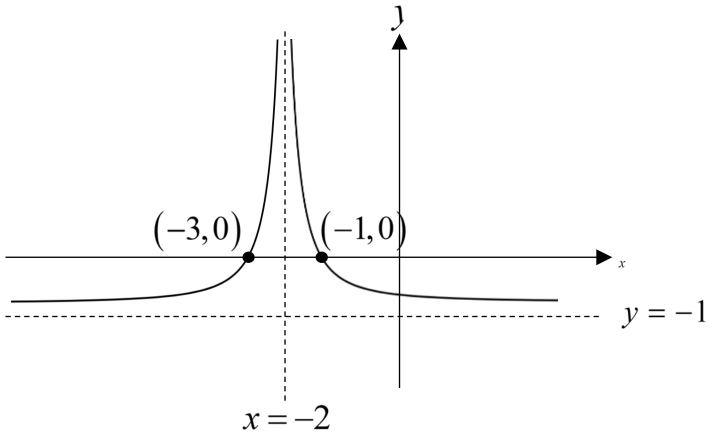
(b) $y=\dfrac{1}{\mathrm{f}(x)}$, [3]

(c) $y=|\mathrm{f}(x)|$, [3]

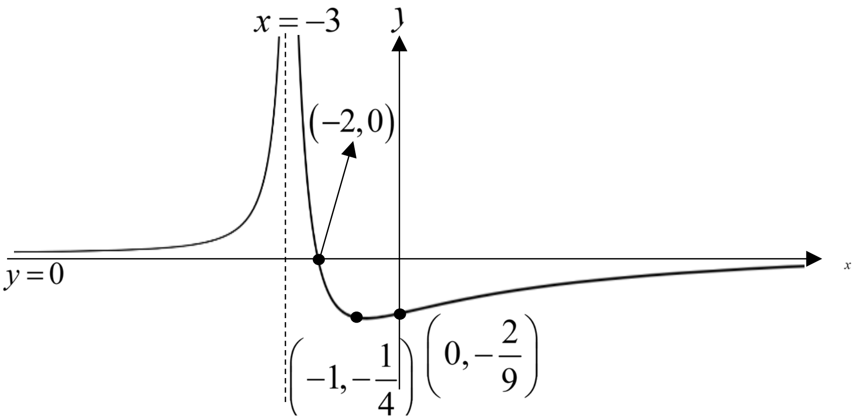
indicating the equations of any asymptotes, turning points and intersections with the axes.

Solution

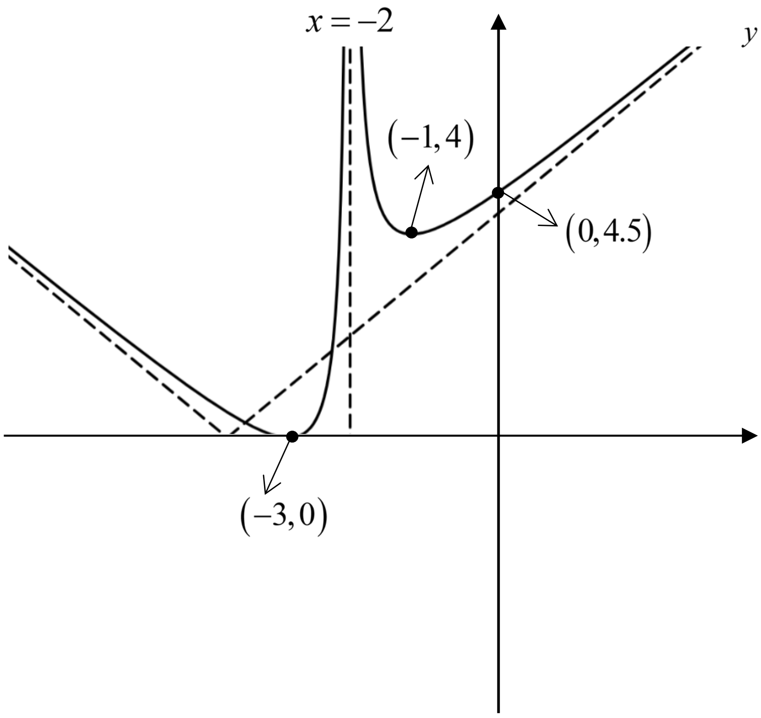
(a)



(b)



(c)



$y = |-x-4|$ or

$= |x+4|$ or

$= \pm(x+4)$

Examiners' Report

(a) Generally, students could find the correct x-intercepts and asymptotes. However, a significant number of them erroneously indicated $(0,-4.5)$ or $(0,0)$ as the y-intercept, resulting in a wrong shape of graph. Students should know that because the gradient of $(0,-4.5)$ on $y=\mathrm{f}(x)$ is unknown, the corresponding point on $y=\mathrm{f}'(x)$ need not be labelled.
(b) Students struggled to sketch a correct shape of the graph, although they were generally able to identify the correct coordinates of the axial intercepts and turning points. Students ought to revise how to obtain the shape of a reciprocal graph. For example, where $y=\mathrm{f}(x)$ is above x-axis, $y=\dfrac{1}{\mathrm{f}(x)}$ is also above x-axis. Where $y=\mathrm{f}(x)$ is increasing, $y=\dfrac{1}{\mathrm{f}(x)}$ is decreasing and vice versa.
(c) In general, students knew to reflect the bottom portion of the graph about the x-axis. They could also correctly identity the coordinates of the reflected points. However, majority of students did not know how to transform the oblique asymptote correctly. Common mistakes include indicating a wrong equation $y=x-4$ or sketching the correct equation $y=x+4$ at the wrong location. Students should know that the asymptotes $y=-x-4$ and $y=x+4$ must intersect at $(-4,0)$. A small group of students misunderstood $y=\lvert\mathrm{f}(x)\rvert$ and sketched the graph of $y=\mathrm{f}(\lvert x\rvert)$ instead.

9 **(a)** The curve C has equation

$$y=\frac{x^2+4x-5}{x-5}, \quad x\in\mathbb{R}, \quad x\neq 5.$$

(i) Sketch C, stating the equations of the asymptotes, axial intercepts and the coordinates of the turning points, if any. [4]

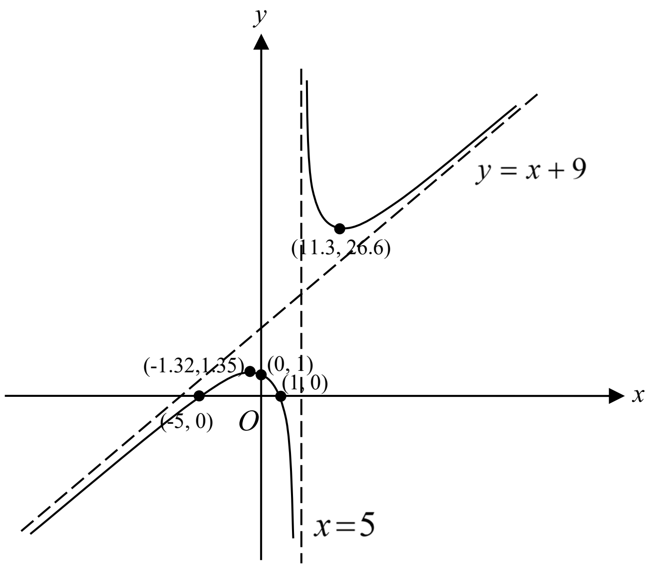
(ii) Hence, by sketching another suitable graph on the same axes as in part **(a)(i)**, solve the inequality $\dfrac{x^2+4x-5}{2x-10}\geq \ln(x+5)$. [3]

(b) Describe a pair of transformations which would transform curve C onto the graph of $y=\dfrac{4x^2+8x-5}{2x-5}-2$. [2]

Solution

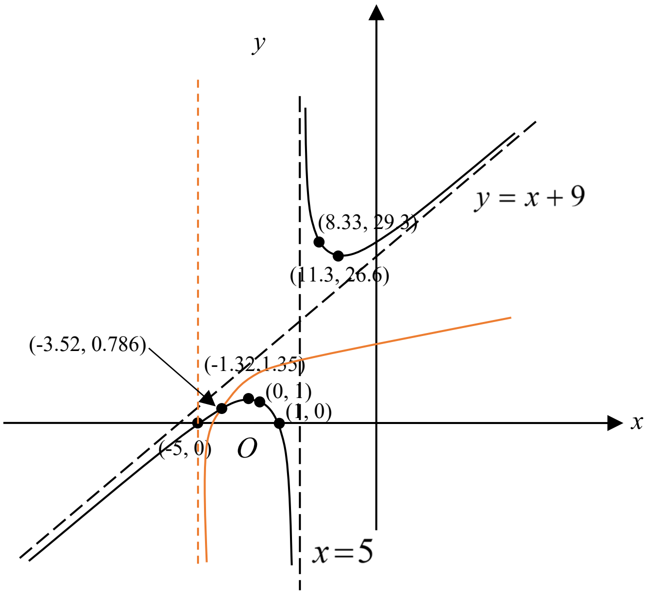
(ai) Vertical Asymptote: $x=5$

Oblique Asymptote: $y=x+9$



(aii)

$$\frac{x^2+4x-5}{x-5} \geq 2\ln(x+5)$$



$-5 < x \leq -3.52$ or $x > 5$

(b)

$$y = \frac{x^2+4x-5}{x-5}$$

$\downarrow$ Scale parallel to the $x-$axis by a factor of 0.5

$$y = \frac{(2x)^2+4(2x)-5}{(2x)-5}$$

$$y = \frac{4x^2+8x-5}{2x-5}$$

$\downarrow$ Translate 2 units in the negative $y-$direction

$$y+2 = \frac{4x^2+8x-5}{2x-5}$$

$$y = \frac{4x^2+8x-5}{2x-5} - 2$$

Examiners' Report

(ai) Many students forgot to label at least one of the axial intercepts. Students need to keep track of all the co-ordinates required, and label them clearly (using arrows if necessary). Some students did not draw the top portion of the graph. Students are advised to zoom out in their graphing calculator to ensure the entire graph is displayed.
(aii) Many were unable to achieve the full credit for this answer. Students first need to manipulate the LHS of the formula such that it will match the original equation of the curve. i.**e.** $\frac{x^2+4x-5}{2x-10} \geq \ln(x+5)$ ➔ $\frac{x^2+4x-5}{x-5} \geq 2\ln(x+5)$. Take note that only 1 additional graph should be drawn, as mentioned by the question. Many students drew $\ln(x+5)$ instead of the $2\ln(x+5)$ graph. Many students did not draw the asymptote of the Ln graph. This will affect their answer. Lastly, even when students drew the correct graphs, some fail to see that when the original curve is above or equal to the Ln graph, that will be the inequality written for their answer.
(b) Students did not use the sentence phrasing correctly. An incorrect phrasing or change of words will often change the meaning of the transformation description. For example, phrases such as "Scale the x-axis" is incorrect as it is the graph which is scaled, not the x-axis. Also, as mentioned in an A level examiner's report, "y-direction" and "in the y-axis" should not be used interchangeably.

10 The functions f and g are defined by

$$\mathrm{f}: x \mapsto \left|\frac{2x+2\lambda}{x-2}\right|, \qquad x \in \mathbb{R},\ -\lambda < x < 2$$

$$\mathrm{g}: x \mapsto x^2 - \lambda, \qquad x \in \mathbb{R},\ -\sqrt{\lambda} < x < \sqrt{\lambda}$$

where λ is a positive constant.

(a) Show that f has an inverse and find an expression for $\mathrm{f}^{-1}(x)$. [4]

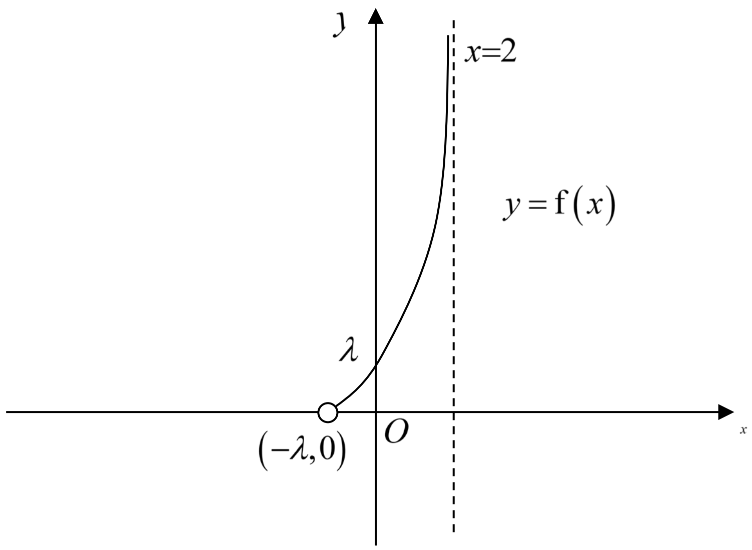
(b) Find the range of values of x such that $\mathrm{f}^{-1}\mathrm{f}(x) = \mathrm{ff}^{-1}(x)$. [1]

(c) **(i)** Explain why fg does not exist. [1]

(ii) By restricting the domain of g to $\left(0, \sqrt{\lambda}\right)$, find the rule of fg and its range. [4]

Solution

(a)



Since $y = k, k \in \mathbb{R}$ cuts the graph of $y = \mathrm{f}(x)$ at most once, f is one-to-one and hence f^{-1} exists.

Let $y = \mathrm{f}(x) \Rightarrow x = \mathrm{f}^{-1}(y)$

For $-\lambda < x < 2$,

$$y = -\frac{2x+2\lambda}{x-2}$$

$$xy - 2y = -2x - 2\lambda$$

$$x(y+2) = 2y - 2\lambda$$

$$x = \frac{2y-2\lambda}{y+2}$$

$$\therefore \mathrm{f}^{-1}(x) = \frac{2x-2\lambda}{x+2}$$

(b)

Since $\mathrm{D_f} = (-\lambda, 2)$ and $\mathrm{D_{f^{-1}}} = (0, \infty)$

For $\mathrm{f^{-1}f}(x) = \mathrm{ff^{-1}}(x)$,

$x \in \mathrm{D_f} \cap \mathrm{D_{f^{-1}}}$

$\therefore 0 < x < 2$ or The range of values of x is $(0,2)$

(ci)

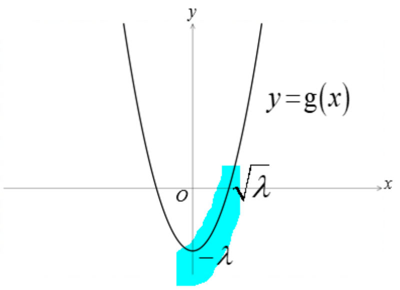
$\mathrm{R_g} = [-\lambda, 0)$ and $\mathrm{D_f} = (-\lambda, 2)$

Since $\mathrm{R_g} \not\subseteq \mathrm{D_f}$, fg does not exist.

(cii)

$$\mathrm{fg}(x) = \left| \frac{2(x^2-\lambda)+2\lambda}{(x^2-\lambda)-2} \right|$$

$$= \left| \frac{2x^2}{x^2-\lambda-2} \right| \text{ or } -\frac{2x^2}{x^2-\lambda-2}$$



$$\mathrm{D_{fg}} = \mathrm{D_g} = (0, \sqrt{\lambda}) \xrightarrow{\mathrm{g}} \mathrm{R_g} = (-\lambda, 0) \xrightarrow{\mathrm{f}} \mathrm{R_{fg}} = (0, \lambda)$$

Examiners' Report

(a) This part question was not well done.

__**Existence of Inverse**__

Despite the feedback from tutorials, CA1 and timed practice, a significant percentage of students did not draw the graph to support their explanation of a one-to-one function, did not draw the graph for the specified domain or did not draw the graph with modulus. Moreover, in the explanation of the horizontal line test, some students used one specific example which was insufficient as a proof.

A small percentage of students did not understand the one-to-one significance correctly as they wrongly wrote 'the horizontal line intersects the graph *at least* once'.

A common mistake was that a vertical asymptote was drawn as an open circle. An open circle refers to a point that is on the graph but the point is excluded from consideration. A vertical asymptote refers to the value of x when y is undefined – it is impossible to have a point on the vertical asymptote that is being excluded.

Rule of Inverse

A significant percentage of students did not know how to remove the modulus sign correctly. For $-\lambda < x < 2$, $\frac{2x+2\lambda}{x-2} < 0$ (can be seen from the graph of $y = \frac{2x+2\lambda}{x-2}$). Some attempted to square both sides but made algebraic mistakes and encountered the same problem after taking square root.

Some students found the domain of the inverse function and/or expressed the rule in similar form even though the question did not require them to do so.

(b) This part was badly done.

Despite similar questions in tutorials and timed practice, a high percentage of students had not grasped the concept of equal functions (same rule and same domain).

(ci) This part was badly done.

Despite drawing the graph to determine the range, some students did not include the minimum value. It seemed to suggest that they did not know what to look for – they should not look at end points but look for minimum and maximum values of y.

There was a small percentage of students who could not write down the condition for the existence of composite functions. Some examples of mistakes were "the domain of g is not a subset of the domain of f", "the range of f is not a subset of the domain of g" etc.

(cii) This part was not well done.

Some students did not understand the rule of a function, and gave its range as the rule.

For mapping approach, some students did not map R_g under f to find R_{fg}. A common mistake was mapping R_g under fg.

For drawing the graph of the composite function, some students did not realize that $\sqrt{\lambda} < \sqrt{\lambda+2}$ and hence gave the wrong range.

11 The plane Π_1 contains the points $A(1,4,2)$, $B(1,0,5)$ and $C(0,8,-1)$.

(a) Find a cartesian equation of Π_1. [3]

(b) Find the shortest distance between the point P (2,1,2) and the plane Π_1. [2]

A second plane Π_2 contains the point D (2,2,3) and is perpendicular to the vector $\mathbf{i}+2\mathbf{j}+2\mathbf{k}$. The point $(p,0,q)$ lies in both planes.

(c) Find p and q. [3]

(d) Hence find an equation of the line of intersection of the two planes in the form $\mathbf{r}=\mathbf{a}+t\mathbf{b}$, where t is a real constant. [2]

(e) Find the acute angle between the line AC and the plane Π_2. [2]

Solution

(a)

One possible normal vector is parallel

$$\overrightarrow{AB}=\begin{pmatrix}0\\-4\\3\end{pmatrix}\times\overrightarrow{AC}=\begin{pmatrix}-1\\4\\-3\end{pmatrix}=\begin{pmatrix}0\\-3\\-4\end{pmatrix}$$

Scalar product form of equation:

$$\mathrm{r}\cdot\begin{pmatrix}0\\-3\\-4\end{pmatrix}=\begin{pmatrix}1\\4\\2\end{pmatrix}\cdot\begin{pmatrix}0\\-3\\-4\end{pmatrix}=-20$$

$$\mathrm{r}\cdot\begin{pmatrix}0\\3\\4\end{pmatrix}=20$$

Cartesian Equation is $3y+4z=20$.

(c) Let d denote the shortest distance between P and the plane.

Then $$d=\frac{\left|\overrightarrow{AP}\cdot\begin{pmatrix}0\\3\\4\end{pmatrix}\right|}{5}$$

$$=\frac{\left|\left[\begin{pmatrix}2\\1\\2\end{pmatrix}-\begin{pmatrix}1\\4\\2\end{pmatrix}\right]\cdot\begin{pmatrix}0\\3\\4\end{pmatrix}\right|}{5}=\frac{9}{5}$$

Alternatively, we can use the points B or C or any other points lying on the plane.

(c)

Π_2 has equation given by $\mathrm{r}\cdot\begin{pmatrix}1\\2\\2\end{pmatrix}=\begin{pmatrix}2\\2\\3\end{pmatrix}\cdot\begin{pmatrix}1\\2\\2\end{pmatrix}=12$

Alternative: use dot product=0 to find p]

Since $(p, 0, q)$ lies in Π_2,

$$\begin{pmatrix} p \\ 0 \\ q \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} = 12 \Rightarrow p + 2q = 12 \text{-----}(*)$$

Since $(p, 0, q)$ lies in Π_1, $3(0) + 4(q) = 20$

$\Rightarrow q = 5$ and Subst into (*) gives $p = 2$

(d)

Direction vector of the line of intersection is parallel to $\begin{pmatrix} 0 \\ 3 \\ 4 \end{pmatrix} \times \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \\ -3 \end{pmatrix}$

Equation of line of intersection is $\mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix} + t \begin{pmatrix} -2 \\ 4 \\ -3 \end{pmatrix}, t \in \mathbb{R}$

(e)

Let the acute angle between line and plane be α.

$$\sin\alpha = \frac{\left| \begin{pmatrix} -1 \\ 4 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \right|}{\sqrt{1+16+9}\sqrt{9}} = \frac{1}{3\sqrt{26}}$$

$\alpha = 0.0654$ rads or $3.74822^\circ \approx 3.7^\circ$

Alternative

Let the acute angle between line and plane's normal vector be θ.

$$\cos\theta = \frac{\left| \begin{pmatrix} -1 \\ 4 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \right|}{\sqrt{1+16+9}\sqrt{9}} = \frac{1}{3\sqrt{26}}$$

$\theta = 1.5054$ rads or 86.252°

$\alpha = \frac{\pi}{2} - -\theta$ or $90^\circ - \theta$

$\alpha = 0.0654$ rads or $3.74822^\circ \approx 3.7^\circ$

Examiners' Report

(a)

- Instead of applying cross product to a pair of non-parallel vectors that are parallel to Π_1, many students attempted to use $\overrightarrow{OA} \times \overrightarrow{OB}$ or $\overrightarrow{OA} \times \overrightarrow{OC}$ or $\overrightarrow{OB} \times \overrightarrow{OC}$ to find the normal vector of Π_1.
- Out of the students who used the correct pair of vectors to compute the normal vector, quite a number of them computed the y-component wrongly.

- A good number of students gave the parametric form of the plane instead of the required Cartesian form.

(b) Two most common mistakes among students is to cite the formula for shortest distance between P and Π_1 as $\dfrac{\left|\overrightarrow{OP}\cdot\begin{pmatrix}0\\3\\4\end{pmatrix}\right|}{5}$ or $\dfrac{\left|\overrightarrow{AP}\times\begin{pmatrix}0\\3\\4\end{pmatrix}\right|}{5}$, instead of $\dfrac{\left|\overrightarrow{AP}\cdot\begin{pmatrix}0\\3\\4\end{pmatrix}\right|}{5}$.

(c) This part is generally well-handled, conceptually speaking. Most of the students who didn't manage to arrive at the correct values for p and q were hindered by errors that were carried over from part (a).

(d) Plenty of students did not use the p and q values found in part (c) to find the equation of the line of intersection between Π_1 and Π_2, therefore failing to fulfill the "hence" requirement. They simply used the GC to solve for the line of intersection between the two known planes.

(e)

- Many students didn't read the question carefully and tried to find the angle between line AC and Π_1 instead of the angle between line AC and Π_2.
- Quite a number of students failed to identify $\overrightarrow{AC}$ as the direction vector of line AC.
- Many students who used the formula, $\theta=\cos^{-1}\left|\dfrac{\left|\begin{pmatrix}-1\\4\\-3\end{pmatrix}\cdot\begin{pmatrix}1\\2\\2\end{pmatrix}\right|}{\sqrt{1+16+9}\sqrt{9}}\right|$, forgot or is unaware that the required angle is in fact, $\dfrac{\pi}{2}-\theta$.
- Students please be aware that angles in radians are rounded off to 3 s.f but angles in degree are rounded off to 1 d.p. as per Cambridge's requirement.

12 **(a)** Alex and Belle, together with a team of cyclists plan to ride from Singapore to Penang and back over 10 days. The distance for the route taken from Singapore to Penang and back is 1600 km.

(i) Alex suggests that the team cycle 120 km for the first day and increase the distance covered by 8 km each subsequent day. Show that the team will not be able to complete the route in 10 days. [2]

(ii) Belle suggests changing the distance covered on the first day so that the team will reach Singapore on the tenth day. Given that the increment of distance covered for each subsequent day remains at 8 km, and the distance covered on the last day is 196 km, find the distance covered on the first day. [2]

(b) On 2 January 2025, Candy saved \$8000 in a bank account which gives a compound interest of 1% per annum at the end of each year. Withdrawals of \$1200 are made at the beginning of each subsequent year.

(i) Find the amount in Candy's bank account at the beginning of the third year. [2]

(ii) Show that the amount in Candy's bank account at the beginning of the $(n+1)$th year is $\$\left[120000-112000(1.01)^{n}\right]$. [3]

(iii) In which year would Candy's bank account first have less than \$1200? [3]

Solution

(ai)

Total distance covered

$= \frac{10}{2}\left[2(120)+(10-1)(8)\right]$

$= 1560 < 1600$

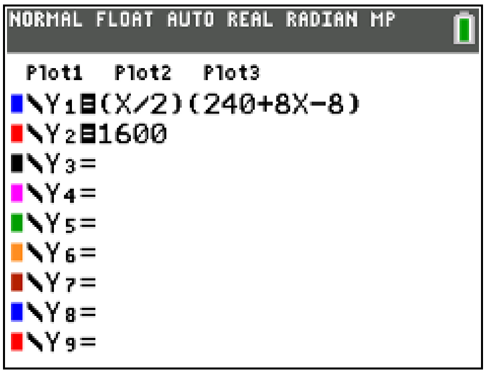
Therefore, the team will not be able to cover the route in 10 days.

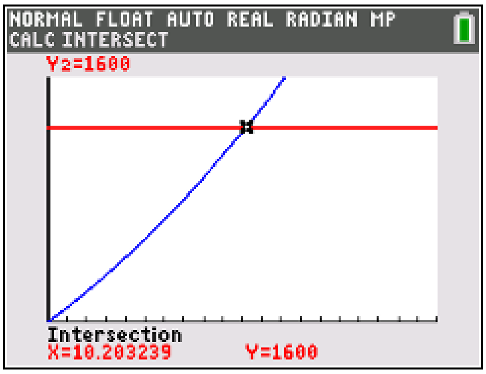
<u>Alternative:</u>

$\frac{n}{2}\left[2(120)+(n-1)(8)\right] \geq 1600$

$n \geq 10.203$

Team will take more than 10 days to complete the route.
Therefore, the team will not be able to cover the route in 10 days.





(aii)

Let a be the distance covered on the first day.
Distance covered on the 10th day = 196

$a+(10-1)(8)=196$

$a=124$

(bi)

Year	Amount in the bank account at the beginning of the Year
1	8000
2	8000 + 8000(0.01) – 1200 = 8000(1.01) – 1200
3	$\left[8000(1.01)-1200\right](1.01)-1200$ $=8000(1.01)^2-1200(1.01)-1200$
⋮	⋮
$n+1$	$8000(1.01)^n-1200(1.01)^{n-1}-\ldots-1200(1.01)-1200$ $=8000(1.01)^n-1200\left[1+(1.01)+\ldots+(1.01)^{n-1}\right]$

Amount in the bank account at the beginning of the 3rd year

$=8000(1.01)^2-1200(1.01)-1200$

$=5748.80$

(bii)

Amount in the bank account at the beginning of the (n+1)th year

$$= 8000(1.01)^n - 1200\left[1 + (1.01) + ... + (1.01)^{n-1}\right]$$

$$= 8000(1.01)^n - 1200\left[\frac{(1.01)^n - 1}{1.01 - 1}\right]$$

$$= 8000(1.01)^n - 120000\left[(1.01)^n - 1\right]$$

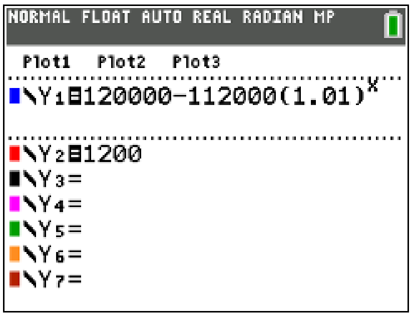
$$= 120000 - 112000(1.01)^n \quad (\text{Shown})$$

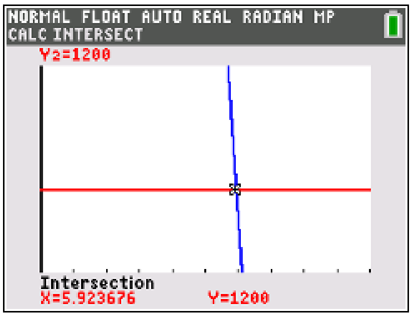
(biii)

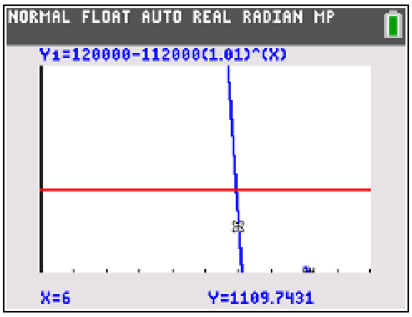
$$120000 - 112000(1.01)^k < 1200$$

$$k > \frac{\ln\left(\frac{120000-1200}{112000}\right)}{\ln(1.01)} \approx 5.9237$$

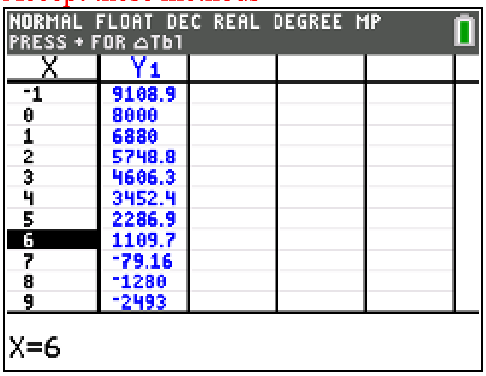
Candy's bank account first has less than \$1200 in Year 2031. (2025 + 6 years)







Accept these methods



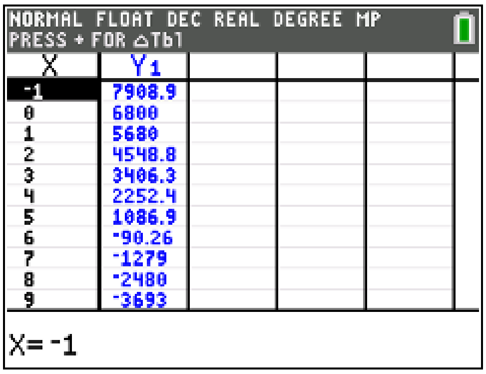
NORMAL FLOAT DEC REAL DEGREE MP
PRESS + FOR ΔTbl

X	Y1
-1	9108.9
0	8000
1	6880
2	5748.8
3	4606.3
4	3452.4
5	2286.9
6	1109.7
7	-79.16
8	-1280
9	-2493

X=6

When n = 6, the account is less than 1200 at beginning of 2031.

Or



NORMAL FLOAT DEC REAL DEGREE MP
PRESS + FOR ΔTbl

X	Y1
-1	7908.9
0	6800
1	5680
2	4548.8
3	3406.3
4	2252.4
5	1086.9
6	-90.26
7	-1279
8	-2480
9	-3693

X= -1

Examiners' Report

(ai) Most of the students could do this part, except a handful who used the formula for nth term of an AP instead of the formula for sum to first *n* terms to find total distance in 10 days. A number of students did not recall the correct formula even if the correct formula is used. Students need to know that the results of AP and GP are not in MF27 and must commit to remembering them well.
(aii) Most of the students could do this part, except a handful who used the formula for sum to first *n* terms instead of the formula for the *n*th term of an AP to equate to distance travelled for the 10^{th} day. Students need to read questions carefully to decide which formula to use. Since this is the last question in the paper, students could have rushed through reading the question. Students need to manage time well and allocate more time for long questions.
(bi) The performance of cohort is mixed in this part. Several students could not generate the correct pattern for the amount in the account at the beginning of the year. They either forgot to subtract 1200 or calculated the amount for the end of the year.
(bii) This part is poorly done. Many students did not convincingly use a suitable table to generate a pattern for the (n+1)th year. Some students skipped essential steps and went straight to the given expression and received no credit. If students could not do **b(i),** they are unlikely to do this part. Students need to practise such questions more to gain confidence in them.
(biii) Most students could get 2 out of 3 marks in this part. They lost the third mark because they did not answer the question. The question asked for "which year" and not the number of years after 2025. Another common mistake is that many students did not reverse the inequality sign when they divide both sides by a negative value. For students who used the GC method, some omitted the table of values which is an essential working.