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SOLUTIONS
CATHOLIC HIGH SCHOOL
2025 Preliminary Examination
Secondary 4 (O-Level Programme)



Mathematics

4052/02

Paper 2

27 Aug 2025

2 hours 15 minutes

Booklet A

Additional material: Question Booklet B

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions in the space provided.

If working is needed for any question, it must be shown with the answer.

Omission of essential working will result in loss of marks.

Calculators should be used where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to **three significant figures**.

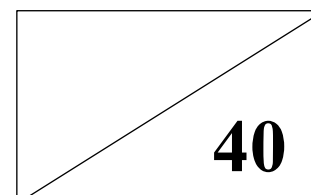
Give answers in **degrees to one decimal place**.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [] at the end of each question or part question.

The total mark for this paper is **90**.

For examiners' use



This booklet consists of **12** printed pages

Mathematical Formulae

Compound interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4 \pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} a b \sin C$$

$$\text{Arc length} = r \theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2 b c \cos A$$

Statistics

$$\text{Mean} = \frac{\sum f x}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum f x^2}{\sum f} - \left(\frac{\sum f x}{\sum f} \right)^2}$$

- 1 Every evening Juline and Evangeline play a game of tic-tac-toe. When they play against each other, Juline has a $\frac{2}{5}$ chance of winning and Evangeline has a $\frac{1}{6}$ chance of winning. Find, as a fraction in its simplest form, the probability that
- (i) Juline will win on both Monday and Tuesday next week.

$$\begin{aligned}
 &P(\text{Juline wins both days}) \\
 &= \frac{2}{5} \times \frac{2}{5} \\
 &= \frac{4}{25}
 \end{aligned}$$

Answer [1]

- (ii) Juline will win on either Wednesday or Thursday next week but not on both days.

$$P(\text{Juline wins one day only}) = P(J) \times P(\text{Not } J) + P(\text{Not } J) \times P(J)$$

$$\begin{aligned}
 &\left(\frac{2}{5} \times \frac{3}{5} \right) \times 2 \\
 &= \frac{12}{25}
 \end{aligned}$$

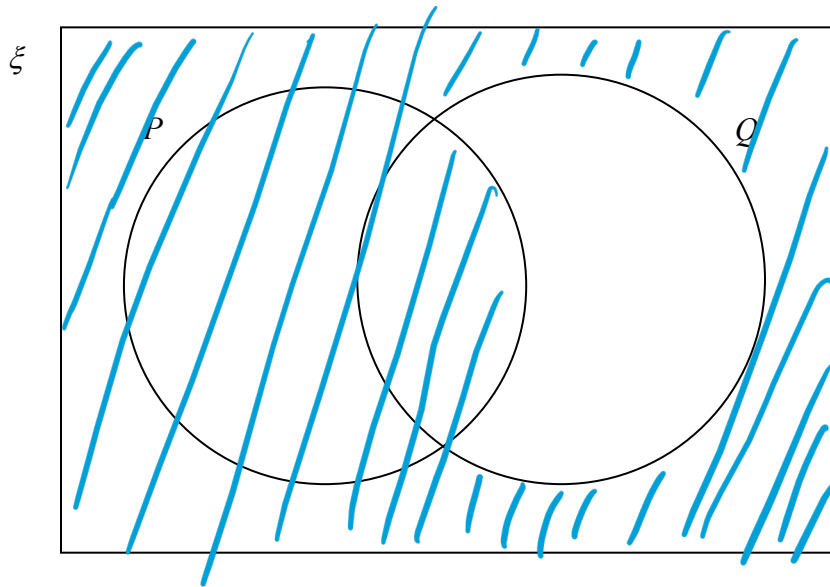
Answer[2]

- (iii) it will be a draw on Friday next week.

$$\begin{aligned}
 P(\text{Draw}) &= 1 - \frac{2}{5} - \frac{1}{6} \\
 &= \frac{13}{30}
 \end{aligned}$$

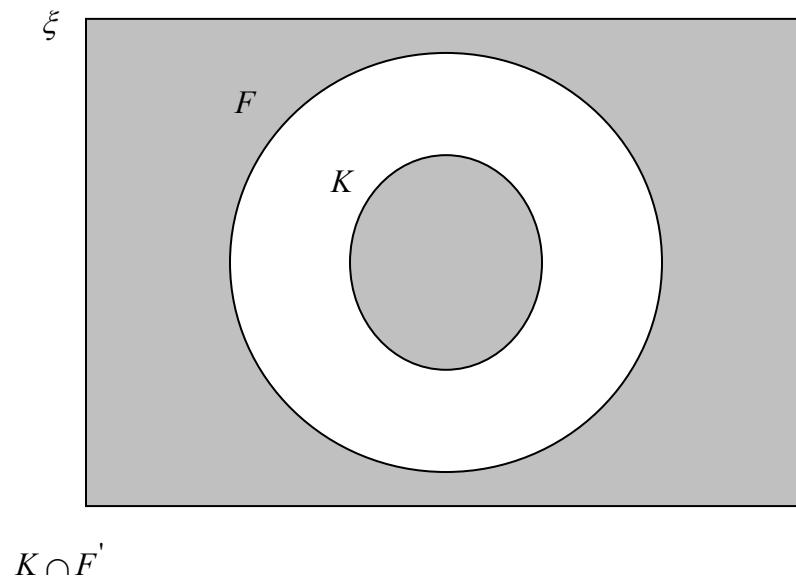
Answer[2]

2(a)(i) On the Venn diagram, shade the region which represents $(P' \cap Q)'$.



[1]

(ii) Use set notation to describe the shaded region.



Answer[1]

2(b) $\xi = \{\text{integers } x : 20 \leq x \leq 50\}$

$A = \{\text{perfect square}\}$

$B = \{\text{prime numbers}\}$

$C = \{\text{multiples of 7}\}$

- (i)** List the elements in $A \cap C'$.

25, 36

Answer[1]

- (ii)** Find the value of $n(B \cap C)$.

0

Answer[1]

- (iii)** A number, p , is chosen at random from the set $(B \cup C)$.
Find the probability that $p \in A$.

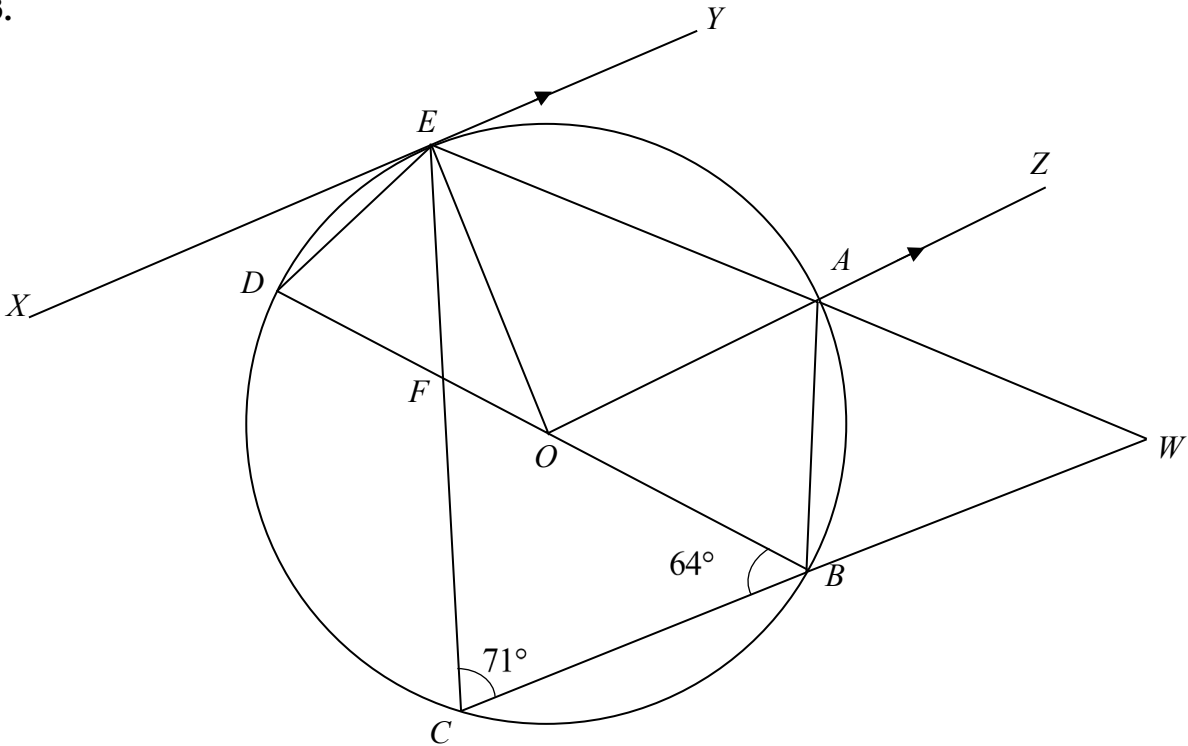
$B \cup C = \{21, 23, 28, 29, 31, 35, 37, 41, 42, 43, 47, 49\}$

$n(B \cup C) = 12$ [recognise that 49 is an element that belongs to both $B \cup C$ and A]

Required probability is $\frac{1}{12}$

Answer[2]

3.



O is the centre of the circle $ABCDE$. XY is a tangent to the circle at E and EC intersects BD at F . OZ is a straight line that passes through A and is parallel to XY . The lines CB produced and EA produced meet at W . $\angle BCE = 71^\circ$ and $\angle OBC = 64^\circ$.

(a) Complete the following statements.

(i) $\angle BDE = \dots\dots\dots 71 \dots\dots\dots^\circ$

because $\dots\dots\dots$ angles in the same segment. $\dots\dots\dots$ [2]

(ii) $\angle AOE = \dots\dots\dots 90 \dots\dots\dots^\circ$

because ...tangent is perpendicular to radius , interior angles between parallel lines $OZ // XY \dots\dots\dots$ [2]

Find

(b) $\angle BOE$,

$$\begin{aligned}\angle BOE &= 71^\circ \times 2 \\ &= 142^\circ\end{aligned}$$

Answer $\angle BOE = \dots\dots\dots^\circ$ [1]

(c) $\angle OCE$,

$$\begin{aligned}\angle EOC &= 71^\circ - 64^\circ \\ &= 7^\circ\end{aligned}$$

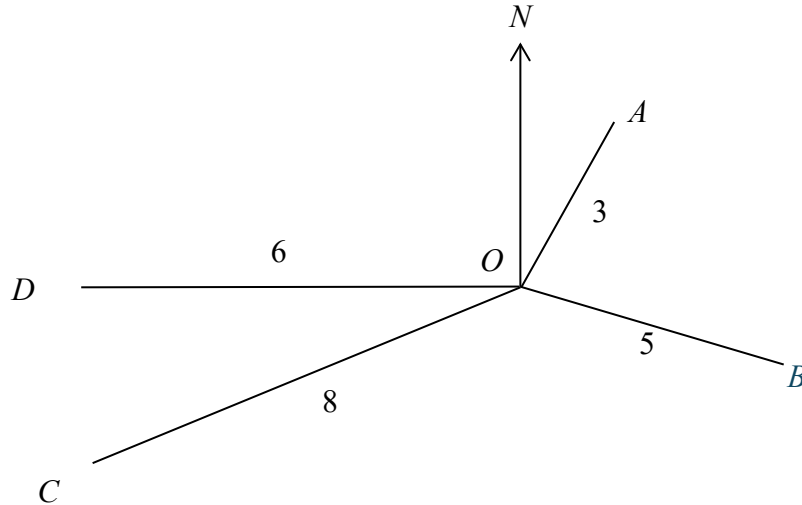
Answer $\angle OCE = \dots\dots\dots^\circ$ [1]

(d) $\angle ABW$.

$$\begin{aligned}\angle OEA &= \frac{180^\circ - 90^\circ}{2} = 45^\circ \\ \angle ABW &= \angle AEC \text{ (exterior angle of cyclic quadrilateral)} \\ &= 45^\circ + 7^\circ \\ &= 52^\circ\end{aligned}$$

Answer $\angle ABW = \dots\dots\dots^\circ$ [2]

4. In the diagram, a lighthouse is at O .
Four ships, A , B , C and D are positioned around it with $OA = 3$ km, $OB = 5$ km, $OC = 8$ km and $OD = 6$ km. The bearing of A from O is 027° and the bearing of B from O is 103° . Ship D lies due west of O and Ship C lies due south of D .



- (a) Calculate the distance between Ship A and Ship B .

Using the Cosine Rule:

$$\angle AOB = 103^\circ - 27^\circ = 76^\circ$$

$$AB^2 = 3^2 + 5^2 - 2(3)(5)\cos(76^\circ)$$

$$= 9 + 25 - 30 \times 0.241921$$

$$= 26.74235$$

$$AB = 5.1713$$

$$= 5.17 \text{ km}$$

Answerkm [3]

- (b) The keeper of the lighthouse is standing on top of the lighthouse, which is 20 m tall. Calculate the angle of depression, from where he is standing to Ship *A*.

$$\begin{aligned}\tan \theta &= \frac{20}{3000} \\ \theta &= 0.381966 \\ &= 0.381^\circ \\ &= 0.4^\circ\end{aligned}$$

Answer° [2]

- (c) Calculate the bearing of Ship *C* from *O*.

$$\begin{aligned}\cos \angle COD &= \frac{6}{8} \\ \angle COD &= 41.4096^\circ\end{aligned}$$

$$\begin{aligned}\text{Bearing of } C \text{ from } O &\text{ is } 270^\circ - 41.4096^\circ \\ &= 228.5904^\circ \\ &= 228.6^\circ\end{aligned}$$

Answer° [2]

- (d) A speedboat travels directly from Ship A to Ship B . Calculate the shortest distance between the speedboat and the lighthouse during its journey.

Let the shortest distance be x km

$$\frac{1}{2}x(5.1713) = \frac{1}{2}(5)(3)\sin 76^\circ$$

$$x = 2.81446$$

$$= 2.81$$

Answer km [2]

- (e) It is suggested that Ships A , B and the lighthouse at O form a right-angled triangle. Showing your working clearly, determine whether this is true.

$$\text{Given } OA = 3, OB = 5$$

$$\text{from (a), } AB = 5.1713$$

$$OA^2 + OB^2 = 3^2 + 5^2 = 34 \neq AB^2$$

Conclusion:

Since Pythagoras' Thm does not applied, triangle OAB is not a right-angled triangle hence Ships A , B and the lighthouse at O do not form a right-angled triangle.

“Pythagoras' theorem” must be seen, before concluding that OAB is not a right-angle triangle.

Answer

[2]

- 5. (a)** As part of a school fundraising event, Jamie buys x boxes of snacks for a total of \$180. She plans to sell each box at a profit of 80 cents.

(i) Write, in terms of x , an expression for the price, in dollars, Jamie paid for one box.

$$\text{Cost per box} = \frac{180}{x}$$

Answer \$..... [1]

(ii) Hence, write an expression for her selling price, in dollars, of one box.

$$\text{Selling price per box} = \frac{180}{x} + 0.8$$

Answer \$.....[1]

- (b)** Jamie managed to sell all except 15 of the boxes and collected \$171 from his sales. Form an equation in x , and show that it simplifies to $4x^2 - 15x - 13500 = 0$.

$$\left(\frac{180}{x} + 0.8 \right) (x - 15) = 171$$

$$180 - \frac{2700}{x} + 0.8x - 12 = 171$$

$$0.8x^2 - 2700 - 3x = 0$$

$$8x^2 - 30x - 27000 = 0$$

$$4x^2 - 15x - 13500 = 0 \text{ (Shown)}$$

Answer Shown above [3]

- (c) Showing your working clearly, solve the equation in (b) and calculate the selling price of 1 box of snacks.

$$4x^2 - 15x - 13500 = 0$$

Using general formula,

$$x = \frac{-(-15) \pm \sqrt{(-15)^2 - 4(4)(-13500)}}{2(4)}$$

$$= \frac{15 \pm \sqrt{225 + 216000}}{16}$$

$$x = \frac{15 \pm 465}{8}$$

$$x = 60 \quad \text{or} \quad x = -56.25 \text{ (reject)}$$

$$\text{Selling price} = \frac{180}{60} + 0.8$$

$$= 3.80$$

Answer $x = \dots\dots\dots$

Selling price = \$.....[3]

- (d) Jamie considers increasing her selling price by \$0.25 per box for the remaining 15 boxes. Would this allow her to make a profit overall?
Justify your conclusion with mathematical calculations.

$$\text{New selling price} = 3.80 + 0.25 = 4.05$$

$$\text{Total revenue} = 171 + 15 \times 4.05 = \$231.75 > \$180$$

Yes, Jamie's claim is correct. By increasing her selling price per box, she would raise a profit as the revenue collected is more than \$180, the amount she spent.

Answer

[2]

END OF BOOKLET A

Name:		Index Number:		Class:	
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CATHOLIC HIGH SCHOOL
2025 Preliminary Examination
Secondary 4 (O-Level Programme)

B

Mathematics

4052/02

Paper 2

27 August 2025

Booklet B

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For examiners' use

50

This booklet consists of **17** printed pages and **1** blank pages.

- 6.(a)** Complete the table of values for $y = \frac{1}{10}\left(x^2 + \frac{7}{x}\right)$.

Values are given to one decimal place where appropriate.

x	0.6	1	1.5	2.2	3.1	4	5.1	5.8
y	1.2	0.8	0.7	0.8		1.8	2.7	3.5

[1]

$$p = 1.2 \quad B1$$

- (b)** On the grid opposite, draw the graph of $y = \frac{1}{10}\left(x^2 + \frac{7}{x}\right)$ for $0.6 \leq x \leq 5.8$. [3]

- (c)** Using your graph, estimate the value(s) of x for which $x^2 + \frac{7}{x} = 20$, where $0.6 \leq x \leq 5.8$.

$$x^2 + \frac{7}{x} = 20$$

$$\div 10$$

$$\frac{1}{10}\left(x^2 + \frac{7}{x}\right) = 2$$

$$\text{Draw } y = 2$$

$$x = 4.25$$

$$[\text{Accept } 4.15 \text{ to } 4.35]$$

$$\text{Answer } x = \dots\dots\dots [2]$$

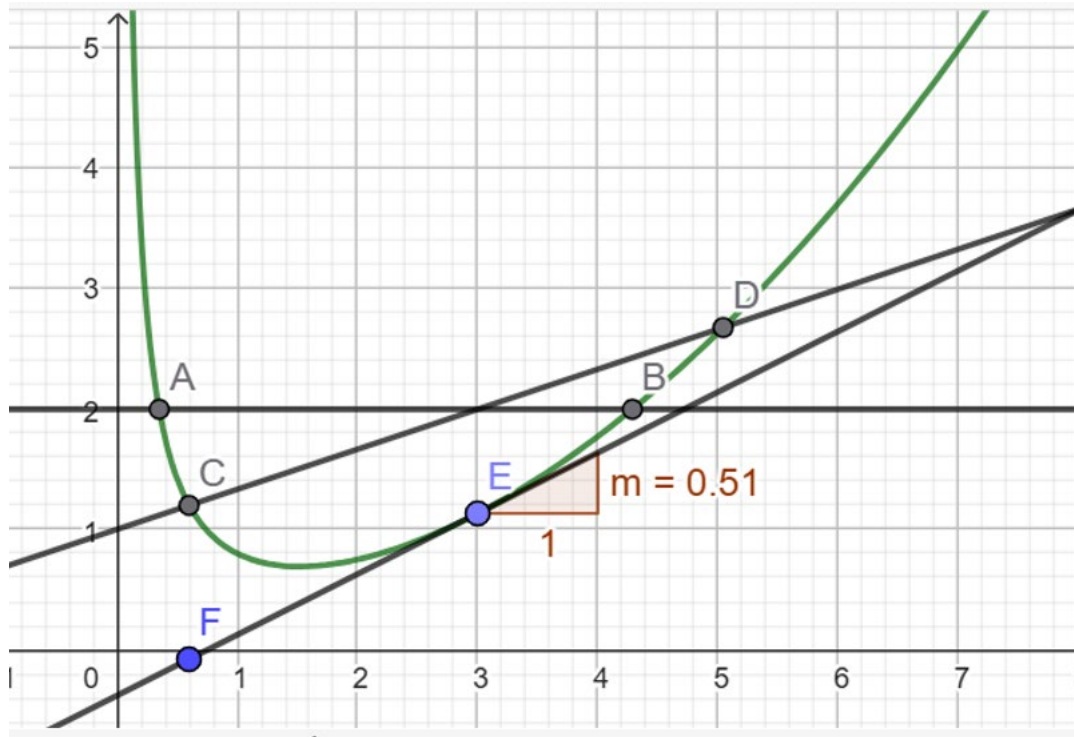
- (d) (i)** On the same grid, draw the graph of $3y - x = 3$ for $0.6 \leq x \leq 5.8$. [1]

$$\text{Draw } y = \frac{1}{3}x + 1$$

- (ii)** Write down the x -coordinates of the points where the line intersects the curve.

$$x = 0.6 \quad \text{or} \quad x = 5.05$$

$$\text{Answer } x = \dots\dots\dots \text{ and } x = \dots\dots\dots [2]$$



- (iii) These values of x are solutions of the equation $3x^3 + Ax^2 + Bx + 21 = 0$.
Find the value of A and of B .

$$\begin{aligned}\frac{1}{10}\left(x^2 + \frac{7}{x}\right) &= \frac{1}{3}x + 1 \\ x^2 + \frac{7}{x} &= \frac{10x}{3} + 10 \\ x^3 - \frac{10x}{3} - 10 + \frac{7}{x} &= 0 \\ 3x^3 - 10x - 30 + \frac{21}{x} &= 0 \\ 3x^3 - 10x^2 - 30x + 21 &= 0 \\ A &= -10 \\ B &= -30\end{aligned}$$

Answer $A = \dots\dots\dots$

$B = \dots\dots\dots$ [3]

- (e) By drawing a tangent, find the gradient of the curve where $x = 2.2$.

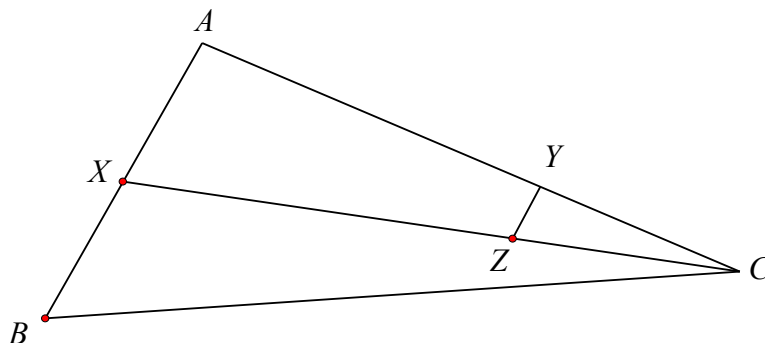
$$\begin{aligned}\text{Gradient} &= \frac{1.25 - 0.35}{3.5 - 1} \\ &= 0.36 \\ &[\text{Accept } 0.26 < m < 0.375]\end{aligned}$$

Answer[2]

7. ABC is a triangle. $\overrightarrow{AB} = \mathbf{p}$ and $\overrightarrow{AC} = \mathbf{q}$. X is the midpoint of AB . Z and Y are points on

XC and AC respectively such that $\overrightarrow{XZ} = \frac{2}{3}\overrightarrow{XC}$ and $\overrightarrow{YZ} = k\overrightarrow{AC}$.

[minus 1 mark for whole question if no vector notations]



- (a) Express the following vectors in terms of $\mathbf{p}$ and/or $\mathbf{q}$, giving your answers in the simplest terms.

- (i) $\overrightarrow{AX}$,

$$\frac{1}{2}\mathbf{p}$$

Answer $\overrightarrow{AX} = \dots\dots\dots [1]$

- (ii) $\overrightarrow{CX}$.

$$\begin{aligned}\overrightarrow{CX} &= \overrightarrow{AX} - \overrightarrow{AC} \\ &= \frac{1}{2}\mathbf{p} - \mathbf{q}\end{aligned}$$

Answer $\overrightarrow{CX} = \dots\dots\dots [2]$

- (b) Express $\overrightarrow{YZ}$ in terms of k , $\mathbf{p}$ and $\mathbf{q}$.

$$\begin{aligned}\overrightarrow{YZ} &= \overrightarrow{YC} + \overrightarrow{CZ} \\ &= k\mathbf{q} + \frac{1}{3}\overrightarrow{CX} \\ &= k\mathbf{q} + \frac{1}{3}\left(\frac{1}{2}\mathbf{p} - \mathbf{q}\right) \\ &= \frac{1}{6}\mathbf{p} + \left(k - \frac{1}{3}\right)\mathbf{q}\end{aligned}$$

Answer $\overrightarrow{YZ} = \dots\dots\dots [2]$

(c) Given that YZ is parallel to AX , find the numerical value of

(i) k ,

$$\frac{1}{3}$$

Answer $k = \dots\dots\dots [1]$

(ii) $\frac{YZ}{AB}$,

$$\frac{1}{6}$$

Answer $\frac{YZ}{AB} = \dots\dots\dots [1]$

(iii) $\frac{\text{Area of } \triangle XYZ}{\text{Area of } \triangle ABC}$.

$$\begin{aligned} \frac{\text{Area of } \triangle XYZ}{\text{Area of } \triangle AXC} &= \frac{1}{2} \\ \frac{\text{Area of } \triangle XYZ}{\text{Area of } \triangle ABC} &= \frac{1}{3+3} \\ &= \frac{1}{6} \end{aligned}$$

Answer $\frac{\text{Area of } \triangle XYZ}{\text{Area of } \triangle ABC} = \dots\dots\dots [1]$

8. A manufacturer sells lightbulbs under two different brands, **BrightLite** and **EcoGlow**. The lifespan, in days, of **500 BrightLite lightbulbs** is summarised in the table below.

Lifespan of **BrightLite Lightbulbs**

Length of life (x days)	Number of bulbs
$100 < x \leq 120$	15
$120 < x \leq 140$	35
$140 < x \leq 160$	70
$160 < x \leq 180$	130
$180 < x \leq 200$	150
$200 < x \leq 220$	80
$220 < x \leq 240$	20

- (a) (i) Calculate an estimate of the mean lifespan of the **BrighLite bulbs** in days.

$$\begin{aligned}
 \text{Mean} &= \\
 &= \frac{15 \times 110 + 35 \times 130 + 70 \times 150 + 130 \times 170 + 150 \times 190 + 80 \times 210 + 20 \times 230}{500} \\
 &= 177.4 \text{ days}
 \end{aligned}$$

Answer.....days [1]

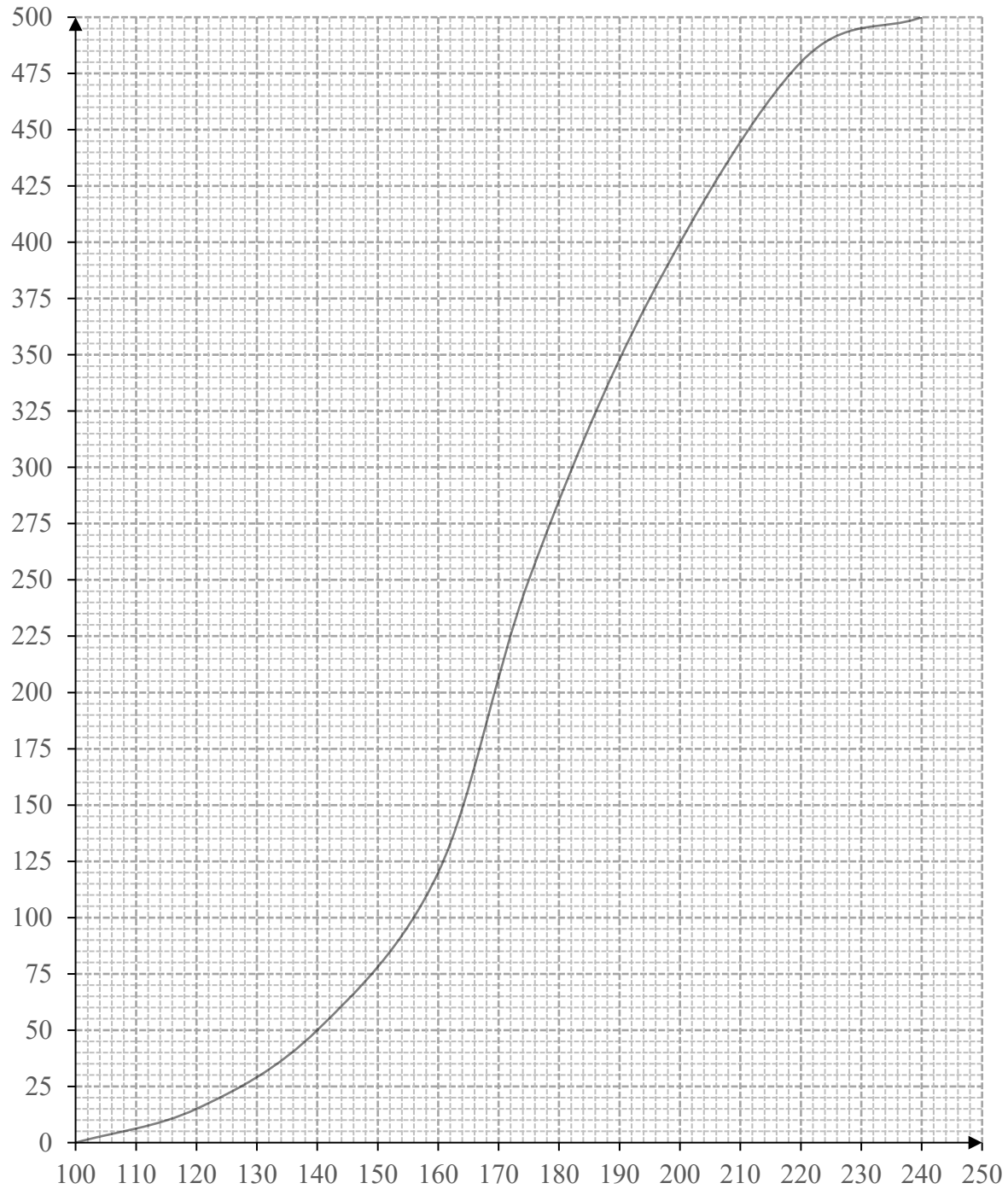
- (ii) Calculate an estimate of the standard deviation.

$$\text{Standard deviation} = 27.2 \text{ days}$$

Answerdays [1]

- (b) The cumulative frequency curve shows the same information for the 500 **BrightLite** **Bulbs**.

Cumulative Frequency



Lifespan of Lightbulbs in Days

Use the curve to estimate

- (i) the median lifespan of the bulbs,

$$\text{Median} = 175 \text{ days}$$

Answerdays[1]

- (ii) the interquartile range of the lifespans.

$$\begin{aligned}\text{Interquartile range} &= (195 - 161) \text{ days} \\ &= 34 \text{ days}\end{aligned}$$

Answerdays [2]

- (iv) If a bulb is selected, find the probability that its lifespan is more than 160 days.

$$\begin{aligned}P(\text{lifespan} > 160 \text{ days}) &= \frac{380}{500} \\ &= \frac{19}{25}\end{aligned}$$

Answer[2]

- (iv) 90% of the bulbs have lifespan more than n days. Find n .

$$n = 140 \text{ days}$$

Answer $n =$ [1]

- (v) If two bulbs are randomly chosen, find the probability that one bulb has a lifespan of more than 160 days while the other has a lifespan of less than 140 days.

$P(\text{one has lifespan} > 160 \text{ days and one has lifespan} < 140 \text{ days}) =$

$$\begin{aligned} & \frac{380}{500} \times \frac{50}{499} + \frac{50}{500} \times \frac{380}{499} \\ &= \frac{76}{499} \end{aligned}$$

Answer[2]

- (c) The manufacturer also tested 500 **EcoGlow bulbs**. These bulbs had a smaller interquartile range as the **BrightLite bulbs**, but the same median lifespan. Describe how the cumulative frequency curve for the **EcoGlow bulbs** may differ from that of the **BrightLite bulbs**.

The **EcoGlow curve** will be **steeper** than the curve of the BrightLite bulbs.

Answer

[1]

9. A company produces four types of healthy snacks — granola bars, fruit juices, protein shakes, and yoghurt cups. The quantities delivered daily to three health stores are shown in the table below.

Snack Type	Granola Bars	Fruit Juices	Protein Shakes	Yoghurt Cups
Store A	45	30	25	20
Store B	50	25	30	35
Store C	60	40	20	25

[Order of matrix must be correct]

- (a) Write down a 3×4 matrix P to show the number of each type of snack delivered to each store.

$$P = \begin{pmatrix} 45 & 30 & 25 & 20 \\ 50 & 25 & 30 & 35 \\ 60 & 40 & 20 & 25 \end{pmatrix}$$

Answer $P = \dots\dots\dots[1]$

- (b) The cost of each granola bar, fruit juice, protein shake, and yoghurt cup is \$1.50, \$2.00, \$3.20 and \$1.80 respectively. Represent these prices in a 4×1 column matrix Q .

$$Q = \begin{pmatrix} 1.50 \\ 2 \\ 3.20 \\ 1.80 \end{pmatrix}$$

Answer $Q = \dots\dots\dots[1]$

(c) Evaluate the matrix $T = PQ$.

$$\begin{aligned}
 PQ &= \begin{pmatrix} 45 & 30 & 25 & 20 \\ 50 & 25 & 30 & 35 \\ 60 & 40 & 20 & 25 \end{pmatrix} \begin{pmatrix} 1.50 \\ 2 \\ 3.20 \\ 1.80 \end{pmatrix} \\
 &= \begin{pmatrix} 45 \times 1.5 + 30 \times 2 + 25 \times 3.2 + 20 \times 1.80 \\ 50 \times 1.5 + 25 \times 2 + 30 \times 3.2 + 35 \times 1.80 \\ 60 \times 1.5 + 40 \times 2 + 20 \times 3.2 + 25 \times 1.80 \end{pmatrix} \\
 &= \begin{pmatrix} 243.50 \\ 284 \\ 279 \end{pmatrix}
 \end{aligned}$$

Answer $T = \dots\dots\dots$ [2]

(d) State what each of the elements in matrix T represents.

Each element in matrix PQ represents the **total value of items delivered** (in dollars) to each store **respectively**.

Answer

[1]

(e) (i) Find the matrix $\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}P$.

$$\begin{aligned}
 \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}P &= \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 45 & 30 & 25 & 20 \\ 50 & 25 & 30 & 35 \\ 60 & 40 & 20 & 25 \end{pmatrix} \\
 &= \begin{pmatrix} 45 + 50 + 60 & 30 + 25 + 40 & 25 + 30 + 20 & 20 + 35 + 25 \end{pmatrix} \\
 &= \begin{pmatrix} 155 & 95 & 75 & 80 \end{pmatrix}
 \end{aligned}$$

Answer $\dots\dots\dots$ [1]

(ii) Explain what each element in $(1 \ 1 \ 1)P$ represents.

Each element represents the total number of Granola Bars, Fruit Juices, Protein Shakes and Yogurt cups **respectively** delivered to all three stores.

Note: respectively must be used for the Granola Bars, Fruit Juices, Protein Shakes and Yogurt cups

Answer

[1]

10. Alex is starting a vertical farming business in Singapore, using hydroponics to grow leafy vegetables in an indoor warehouse space of floor area 50 m² with a ceiling height of 2.5 metres. The current setup consists of 8 vertical hydroponic shelves arranged within the space.

Fixed Overheads and Costs:

- Electricity: \$0.25 per kWh
- Water: \$2.50 per m³
- Nutrients: \$0.30 per kg of vegetables grown

The business rents a government-subsidised commercial unit at a special rate of **\$500/month**, under a national initiative to support sustainable urban agriculture and small business growth.

The table below gives information on the Monthly Production, Unit price, and Resource Usage for January to June 2025.

Month	Monthly production (kg)	Price (\$/kg)	Resource Usage	
			Electricity (kWh)	Water (m ³)
Jan	240	6.00	800	15
Feb	260	6.10	820	16
Mar	280	6.20	840	17
Apr	300	6.30	860	18
May	320	6.40	880	19
Jun	340	6.50	900	20

(a) Calculate the profit for the month of June.

$$\text{Revenue} = \text{Production} \times \text{Price} = 340 \text{ kg} \times \$6.50 = \$2,210.$$

Costs:

- Electricity: $900 \text{ kWh} \times \$0.25 = \225

- Water: $20 \text{ m}^3 \times \$2.50 = \50

- Nutrients: $340 \text{ kg} \times \$0.30 = \102

- Rent: \$500

$$\text{Total Cost} = \$877$$

$$\text{Profit} = \text{Revenue} - \text{Total Cost} = \$2210 - \$877 = \$1333.$$

Answer: \$1333

$$\begin{aligned} \text{June Profit} &= 340 \times 6.50 - (500 + 225 + 50 + 102) \\ &= \$1333 \end{aligned}$$

Answer \$[3]

(b) Suggest a possible reason for the increase in crop prices from January to June.

Plausible reasons:

demand grew faster than supply (e.g., new restaurant contracts or a growing subscription base), so even as output rose each month, crops still selling out—letting Alex **nudge the price up gradually** from \$6.00 to \$6.50 without losing sales.

positioning as premium/local pesticide-free produce

passing on higher operating costs like electricity or labour

inflation

to increase profits

Answer

[1]

- (c) The vertical farming system uses hydroponic shelves, each measuring 2.0 m (length) $\times$ 0.5 m (width) $\times$ 1.8 m (height), with 5 levels of planting trays per shelf. Each level grows 10 kg of vegetables per month. There are currently 8 such shelves arranged in the warehouse. Safety guidelines require a minimum clearance of 0.2 m from the ground and 0.5 m from the ceiling.

The maximum number of shelves that can be installed is limited by both walking and maintenance space requirements. Each shelf occupies **2.0 m $\times$ 0.5 m** of floor space, and minimum clear walking space of **0.5 m** must be provided around each shelf. This clearance applies to the warehouse walls and between shelves.

The warehouse has a total floor area of **50 m²** (10 m $\times$ 5 m).

Due to airflow and light distribution limits, the maximum number of shelves that can physically fit in the warehouse is limited.

- For **10 shelves or fewer**, each shelf operates at **100 %** of its production capacity.
- When **11 or more shelves** are installed, airflow saturation occurs, reducing the production of **all** shelves to **85 %** of their monthly production.
- In addition, **each shelf beyond the 10th** operates at only **70 %** of its monthly production due to poor light and air mixing at the edges.

Alex's goal is to **maximise revenue** for the month of July. Assume that the unit price and resource usage for July is the same as June.

Task:

Determine the number of shelves Alex should install. Justify your answer clearly with mathematical calculations that consider both **space constraints** and the **productivity limitations**. You may wish to draw diagrams to aid in your understanding.

[6]

Continue with solutions for 10(c)

Students to show at least 2 orientations so that they could compare

Orientation A – Long side of shelf along length

Along the 10 m length:

$$0.5 + 2.0 + 0.5 + 2.0 + 0.5 + 2.0 + 0.5 = 8.0 \text{ m (3 shelves) } \checkmark$$

$$\text{Adding a 4th shelf would be: } 0.5 + 2.0 + 0.5 + 2.0 + 0.5 + 2.0 + 0.5 + 2.0 + 0.5 = 10.5 \text{ m } \times$$

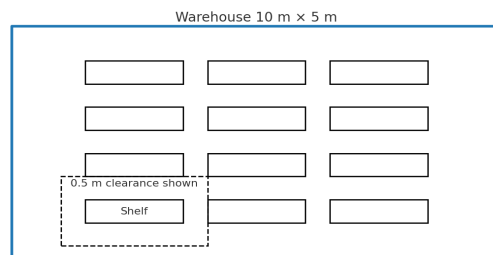
Along the 5 m width:

$$0.5 + (0.5 + 0.5 + 0.5 + 0.5) + 0.5 = 4.5 \text{ m (4 shelves) } \checkmark$$

$$\text{Adding a 5th shelf would be: } 0.5 + (5 \times 0.5) + 0.5 = 5.5 \text{ m } \times$$

Total by space = $3 \times 4 = 12$ shelves.

Orientation A (2.0 m $\times$ 0.5 m): Fits $3 \times 4 = 12$ shelves with 0.5 m gaps/margins



Orientation B – Long side of shelf along width of room

Along the 10 m length (each shelf uses 0.5 m length):

$$0.5 + 9 \times 0.5 + 8 \times 0.5 + 0.5 = 0.5 + 4.5 + 4.0 + 0.5 = 9.5 \text{ m (9 shelves) } \checkmark$$

$$\text{Trying 10 shelves: } 0.5 + 10 \times 0.5 + 9 \times 0.5 + 0.5 = 0.5 + 5.0 + 4.5 + 0.5 = 10.5 \text{ m } \times$$

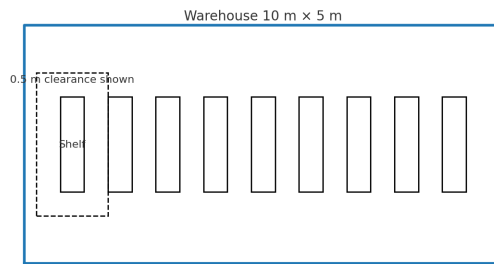
Along the 5 m width (each shelf uses 2.0 m width):

$$0.5 + 2.0 + 0.5 = 3.0 \text{ m (1 shelf across the width) } \checkmark$$

$$\text{Trying 2 across: } 0.5 + 2.0 + 0.5 + 2.0 + 0.5 = 5.5 \text{ m } \times$$

Total by space = $9 \times 1 = 9$ shelves.

Orientation B (0.5 m × 2.0 m): Fits $9 \times 1 = 9$ shelves with 0.5 m gaps/margins



Orientation C (combination of vertical and horizontal)

15 Shelves

Output and productivity per-shelf (Accept if students worked out only output to compare)

Per shelf baseline: 5 levels × 10 kg = 50 kg.

Congestion penalty (given)

- Up to 10 shelves: 50 kg each (100%).
- For 11 or more shelves: first 10 at 42.5 kg each (85%); each shelf beyond 10 at 35 kg (70%).

Revenue Comparison (Price = \$6.50/kg)

Orientation A (can reach 12 shelves):

Number of shelves	Output	Revenue	Best
8	400	2600	
9	450	2925	
10	500	3250	😊
11	460	2990	
12	495	3217	

Best with A: 10 shelves

Orientation B (space cap 9 shelves):

Number of shelves	Output	Revenue	Best
8	400	2600	
9	450	2925	😊

Best with B: 9 shelves

Orientation C (space cap of 15 shelves)

Number of shelves	Output	Revenue	Best
13	530		
14	565		
15	600	3900	😊

Conclusion (by output or revenue)

Orientation A allows up to 12 shelves by space but **10 shelves** gives the highest revenue: $500 \text{ kg} \times \$6.50 = \$3,250$

Orientation B fits at most **9 shelves**, giving at most \$2,925

Final Answer: Install 10 shelves (Orientation A).

Orientation C allows up to **15 shelves** by space and 15 shelves gives the highest revenue

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END OF BOOKLET B