CEDAR GIRLS' SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2025
SECONDARY FOUR

CANDIDATE NAME	Marking Scheme

CLASS

INDEX NUMBER

MATHEMATICS
Paper 1

4052/01
21 August 2025

2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Gives answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

For Examiner's Use
90

This document consists of **20** printed pages.

[Turn over]

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta\text{, where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta\text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Given $a = \dfrac{v^2 - t^2}{2s}$, express v in terms of a, t and s.

$$a = \frac{v^2 - t^2}{2s}$$

$$2as = v^2 - t^2 \qquad \text{[transform to non-fractional equation]}$$

$$v^2 = 2as + t^2$$

$$v = \pm\sqrt{2as + t^2}$$

Answer $v =$ ………………… [2]

(b) Simplify $\dfrac{xy}{x^2 - 4xy + 4y^2} - \dfrac{2y}{4y - 2x}$.

$$= \frac{xy}{(x-2y)^2} - \frac{y}{2y - x} \quad \text{(factorise into a square)}$$

$$= \frac{xy}{(x-2y)^2} + \frac{y}{x-2y}$$

$$= \frac{xy + xy - 2y^2}{(x-2y)^2} \quad \text{(Attempt to change to common denominator)}$$

$$= \frac{2xy - 2y^2}{(x-2y)^2} \quad \left(\text{or } \frac{2y(x-y)}{(x-2y)^2} \text{ or } \frac{2xy - 2y^2}{x^2 - 4xy + 4y^2}\right)$$

Answer………………………… [3]

2 The diameter of a solid sphere is 7.12 cm, measured correct to 3 significant figures.
Its density is 2.2 g/cm^3.
Find the smallest possible mass of the sphere.

Mass

$$= \frac{4}{3}\pi\left(\frac{7.115}{2}\right)^3 \times 2.2 \quad \text{(See 7.115)}$$

$$= 415 \text{ g}$$

Answer…..…………………….. g [2]

3 An area of 512 km^2 is represented by 50 cm^2 on a map.
Find the scale of the map in the form $1 : n$.

512 km^2 : 50 cm^2
256 km^2 : 25 cm^2
16 km : 5 cm (Square root)
3.2 km : 1 cm
320 000 : 1

Answer ... 1 : 320 000 ... [2]

4 **(a)** Factorise completely $m^3 + 30 + 5m + 6m^2$.

$= m^3 + 6m^2 + 5m + 30$
$= m^2(m+6) + 5(m+6)$ (2 groups of factorisation)
$= (m+6)(m^2+5)$

.... [2]

(b) Given that m is a positive integer, explain why $m^3 + 30 + 5m + 6m^2$ is not a prime number.

Its factors $\underline{(m+6)}$ and $\underline{(m^2+5)}$ are whole numbers more than 1.
Hence, it's not a prime number.

...... [1]

5 Mary cycles 2.8 km at an average speed of 12 km/h and takes a break.
She then continues to cycle a further 1.4 km in 5 minutes.
Given that her average speed for the whole journey is $9\frac{1}{3}$ km/h, find the duration of her break.

Time taken for first part $= \frac{2.8}{12} = \frac{7}{30}$ h
Let x h be her break time.

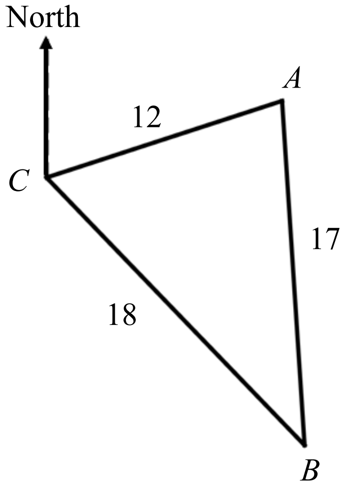
$9\frac{1}{3} = \frac{2.8+1.4}{\frac{7}{30} + x + \frac{1}{12}}$ -- M1

$\frac{19}{60} + x = \frac{4.2}{9\frac{1}{3}}$

$x = \frac{2}{15}$ (or 0.133 h)

...... h [2]

6 **(i)**



The diagram below shows 3 lighthouses *A*, *B* and *C* at sea.
AC = 12 km. *BC* = 18 km. *AB* = 17 km.
The bearing of *B* from *C* is 136°.
Find the bearing of *A* from *C*.

$$\cos \angle ACB = \frac{12^2 + 18^2 - 17^2}{2(12)(18)}$$
$$= 0.41435 \text{ (5 s.f.)}$$

$$\angle ACB = 65.522^o \text{ (3 dp)}$$

$$\text{Bearing of } A \text{ from } C = 136^o - 65.522^o$$
$$= 070.5^o \text{ (1 dp) (with zero in front)}$$

…………….. ° [2]

(ii) Find the area of the region enclosed by triangle *ABC*.

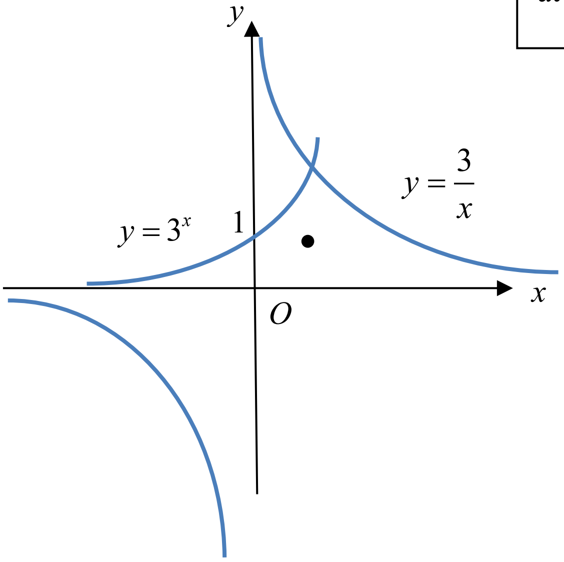
$$\text{Area of } ABC = \frac{1}{2}(12)(18)\sin 65.522^o$$
$$= 98.3 \text{ km}^2$$

Answer ……………………km^2 [2]

7 On the axes given, sketch the graphs $y = 3^x$ and $y = \frac{3}{x}$, indicating the x- and y-intercepts where relevant.
The point (1, 1) is marked in the diagram.

Answer

(shape and see intersection at above x =1)



[2]

8 **(a)** ε = {all real numbers}

A = {all perfect cubes}

B = {all rational numbers}

C = {all recurring decimals}

D = {1, 2, 3, 4, 5, 6, 7, 8, 9}

(i) List all the elements contained in the set $A \cap D$.

Answer ..... 1, 8 .... [1]

(ii) Explain why $C \cap B' = \phi$.

Answer

B' are irrational numbers which are **<u>non-recurring</u>** decimal numbers.

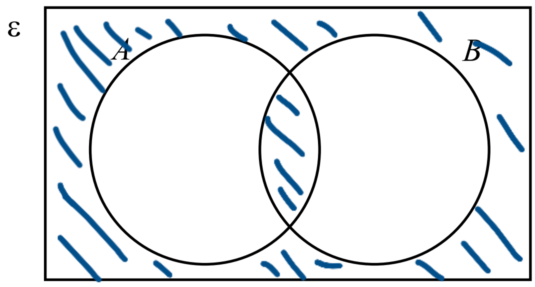
Therefore, $C \cap B' = \phi$.

(Or saying rational numbers are recurring decimals)

]

(b) On the Venn diagram, shade the region(s) which represent $(A' \cap B') \cup (A \cap B)$.

Answer



[1]

9 A quadratic curve $y = -x^2 + mx + 3.5$ has a turning point at $(-1.5, p)$, where both m and p are constants. Find the value of m and of p.

Completed square form:

$y = -(x + \frac{3}{2})^2 + p$ (Completed square form)

$= -(x^2 + 3x + \frac{9}{4}) + p$ [or see $y = -(x - \frac{m}{2})^2 + \frac{m^2}{4} + 3.5$]

$= -x^2 - 3x - \frac{9}{4} + p$

Comparing coefficients with $y = -x^2 + mx + 3.5$: (Comparing)

$m = -3$;

$-\frac{9}{4} + p = 3.5$

$\rightarrow p = \frac{23}{4}$

<u>Alternative method</u>

Completed square form:

$y = -(x - \frac{m}{2})^2 + \frac{m^2}{4} + 3.5$

Comparing coefficients with $y = -(x - \frac{m}{2})^2 + \frac{m^2}{4} + 3.5$: (Comparing)

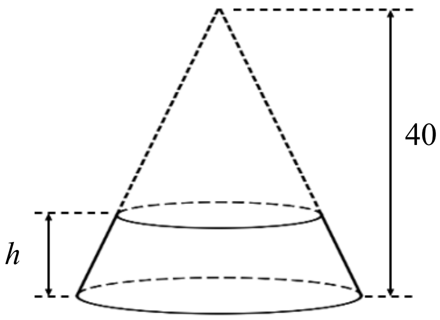
$\frac{m}{2} = -1.5 \rightarrow m = -3$;

$\frac{m^2}{4} + 3.5 = p$

$\rightarrow p = \frac{23}{4}$

10 A pet dish bowl is in the shape of a frustum, as shown in the diagram.
It is formed by removing a smaller cone from the top of a larger cone.
The smaller cone is similar to the original cone, and its volume is $\frac{3}{8}$ of the volume of the original cone. The original cone has a height of 40 cm.

(a) Find h, the height of the frustum.



$$\sqrt[3]{\frac{3}{8}} = \frac{40-h}{40} \text{--[using cube root]}$$

$$h = 40 - 40 \times \sqrt[3]{\frac{3}{8}}$$
$$h = 11.2 \ \text{(3s.f.)}$$

Answer $h =$ …………….......... [2]

(b) Given that the curved surface area of the frustum is 966 cm^2, find the curved surface area of the smaller cone.

$$\frac{A_{\text{smaller cone}}}{A_{\text{larger cone}}} = \left[\left(\frac{3}{8}\right)^{\frac{1}{3}}\right]^2 \text{--[squaring the ratio of lengths]}$$

$$\frac{A_{\text{smaller cone}}}{A_{\text{smaller cone}} + 966} = \left(\frac{3}{8}\right)^{\frac{2}{3}}$$

$$A_{\text{smaller cone}} = \left(\frac{3}{8}\right)^{\frac{2}{3}} \left(A_{\text{smaller cone}} + 966\right)$$

$$A_{\text{smaller cone}} = \frac{\left(\frac{3}{8}\right)^{\frac{2}{3}}(966)}{1 - \left(\frac{3}{8}\right)^{\frac{2}{3}}} = 1050\text{cm}^2 \text{ (3 s.f.)}$$

Answer ………………… cm^2 [2]

11 The table shows the income tax rates for different tax brackets.

Chargeable income	Income tax rate (%)
First \$20 000	0
Next \$10 000	2
Next \$10 000	3.5
Next \$40 000	7
Next \$40 000	11.5

In 2022, given that David's chargeable income is \$60,500, find his income tax payable for YA 2022.

Income tax payable

$= \$10000 \times \frac{2}{100} + \$10000 \times \frac{3.5}{100} + \$20500 \times \frac{7}{100}$

= \$1985

Answer \$ …………………... [2]

12 **(a)** Express 630 as a product of its prime factors.

$630 = 2 \times 3^2 \times 5 \times 7$

Answer ……………………… [1]

(b) Find the smallest integer value of *m* such that 630*m* is a perfect square.

$m = 2 \times 5 \times 7 = 70$

Answer *m* =……………….. [1]

(c) Given that $588 = 2^2 \times 3 \times 7^2$, find the smallest integer value of *k* such that 630*k* is a multiple of 588.

$k = 2 \times 7 = 14$

Answer *k* =………………….. [1]

(d) A rectangular piece of paper measuring 588 mm and 630 mm is being cut into identical squares with no waste.
Find the minimum number of squares that can be obtained.

$\text{HCF} = 2 \times 3 \times 7 = 42$ -- (find HCF)

$\text{No of pieces} = \frac{588 \times 630}{42^2} = 210$

Answer …………………….. [2]

13 Consider the pattern

$$L_1 : 4^2 - 1^2 = 15$$
$$L_2 : 5^2 - 2^2 = 21$$
$$L_3 : 6^2 - 3^2 = 27$$
$$L_4 : 7^2 - 4^2 = 33$$
$$\vdots$$
$$\vdots$$
L_n : ………………………...

(a) Write down L_n of the pattern.

$L_n = (n+3)^2 - n^2 = 6n + 9$

Answer L_n = ……………...…………………...…………… [1]

(b) Explain, with working, whether 3336 is a term of the sequence 15, 21, 27, 33…..

Answer

$3336 = 6n + 9$ -- [M1](Finding n)
$n = 554.5$

No, as n is not a (positive) integer / whole number.

……

……………………..…..……………………..…..……………………..…..…… [2]

(c) Find the exact value of $L_1 + L_4 + L_7 + L_{10} + L_{13} + \ldots\ldots L_{97} + L_{100}$.

$L_1 + L_4 + L_7 + L_{10} + L_{13} + \ldots\ldots L_{97} + L_{100}$
$= 4^2 - 1^2 + 7^2 - 4^2 + 10^2 - 7^2 + \ldots\ldots 100^2 - 97^2 + 103^2 - 100^2$ --(replacing the L terms with $a^2 - b^2$)
$= 103^2 - 1^2$
$= 10608$ -- [A1]

Answer ……………………...… [2]

14 **(a)** A polygon has three of its interior angles, measuring $150°$, $160°$ and $170°$.
The other interior angles are $130°$ each.
Find n, the number of sides of the polygon.

$$150^o + 160^o + 170^o + (n-3)(130^o) = (n-2)(180^o)$$
$$480^o + 130n - 390^o = 180^o n - 360^o$$
$$450^o = 50n$$
$$n = 9$$

Answer $n =$ …………………… [2]

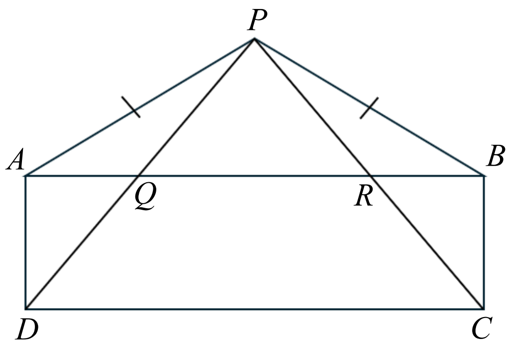
(b) Explain why it is not possible for a regular polygon to have an interior angle of 145°.

Answer

Its exterior angle is 35°. $n = \frac{360}{35} = 10\frac{2}{7}$ -- (finding n)
Since the number of sides is not a whole number / (positive) integer, it is not possible to be a regular polygon to have an interior angle of 145°.

[2]

15 The diagram shows a rectangle $ABCD$ and an isosceles triangle PAB in which $PA = PB$.
PD and PC meet AB at Q and R respectively.
Prove that triangle PDC is an isosceles triangle.



Answer

$PA = PB$ (given) (S)
$AD = BC$ (opp. sides of rectangle) (S)
$\angle PAB = \angle PBA$ (base angles of isosceles triangle)
$\angle PAD = \angle PAB + 90^o$
$= \angle PBA + 90^o$ (A) **(finding $\angle PAD = \angle PBC$)**
$= \angle PBC$
Therefore, Triangle $PAD \equiv$ Triangle PBC (SAS)
$\therefore PD = PC$,
triangle is an isosceles triangle.

[3]

16 The table below shows the number of books students read in a certain week.

Number of books	0	1	2	3	4	5
Number of students	9	13	16	21	x	17

(a) If the mode is 3, state the largest possible value of x.

Answer x = ... 20 .... [1]

(b) If the mean is 3, find the value of x.

$$\frac{0+13+32+63+4x+85}{9+13+16+21+x+17}=3$$

$$\frac{193+4x}{76+x}=3$$

$$193+4x=228+3x$$

$$x=35$$

Answer x = ... [2]

(c) Melanie claims that it is not possible for the median to be 2.
Explain whether you agree with Melanie's claim.

To find the smallest possible median is when x = 0,
(the smallest possible value of x)
– **(attempt to find smallest possible median)**
Alternative solution:
Stating that Left side has 9+13+16 = 38 students;
Right side has 21 + 17 = 38 students.
Hence the smallest median is 2.5

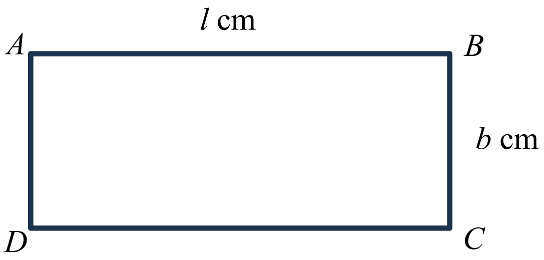
The smallest possible median = $\frac{2+3}{2}=2.5$

The smallest possible median is 2.5, and it will not be 2. Melanie is correct. -- (also accept finding median to be more than 2)

[2]

17 The diagram shows a sheet of rectangular paper $ABCD$.
The length is l cm. The breadth is b cm.
This sheet can be rolled to form an open cylinder in two different ways:

Method 1: By joining the breadths together.
Method 2: By joining the lengths together.



Justify with mathematical workings, why the volume of the open cylinder, formed using **Method 1** is larger than that formed using **Method 2**, for all real values of l and b.

Answer

	Method 1	**Method 2**	
Finding radius	$2\pi r_1 = l$ $r_1 = \dfrac{l}{2\pi}$	$2\pi r_2 = b$ $r_2 = \dfrac{b}{2\pi}$	(Correct radii for both)
Volume	$= \pi\left(\dfrac{l}{2\pi}\right)^2 \times b$ $= \dfrac{bl^2}{4\pi}$	$= \pi\left(\dfrac{b}{2\pi}\right)^2 \times l$ $= \dfrac{b^2 l}{4\pi}$	(Finding volume)

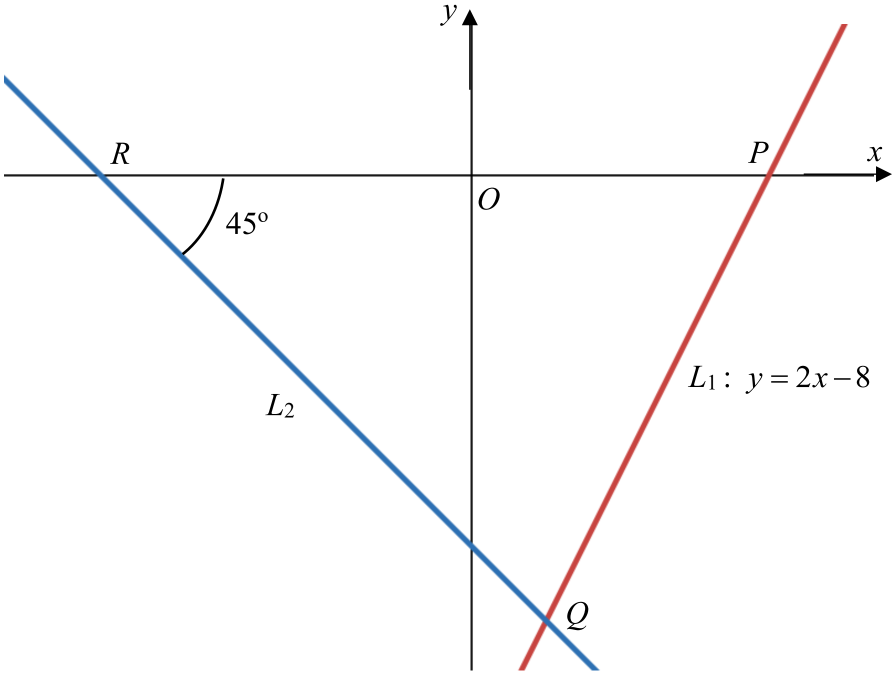
Since $l > b \rightarrow \dfrac{bl^2}{4\pi} > \dfrac{b^2 l}{4\pi}$, -- (follow through from above)

[Alternatively, $\left(\dfrac{bl}{4\pi}\right)l > \left(\dfrac{bl}{4\pi}\right)b$]

the <u>volume from method 1</u> is larger.

.... [4]

18 In the figure below, line L_1 and line L_2 cut the x-axis at P and R respectively.
The equation of L_1 is $y = 2x - 8$.
Angle PRQ is 45°. PR is 9 units.
Line L_2 meets line L_1 at Q.



(a) By finding the coordinates of R, show that the equation of Line L_2 is $y = -x - 5$.

Answer

$P = (4, 0)$
$R = (-5, 0)$

Gradient $= -\tan 45^o = -1$ --(or other similar methods to find gradient)
Therefore, $y = -x + c$

At $(-5, 0)$,

$0 = 5 + c$
$c = -5$ --(connect gradient and a point to find c)

Therefore, equation of $L_2 : y = -x - 5$ (Shown)

[4]

(b) Find the coordinates of Q.

$y = 2x - 8$ --(1)
$y = -x - 5$ --(2)

(1) = (2)

$2x - 8 = -x - 5$
$x = 1 \rightarrow y = -6$

$Q =$ $(1, -6)$

(……..… , …….......) [2]

(c) Find the shortest distance from R to line PQ.

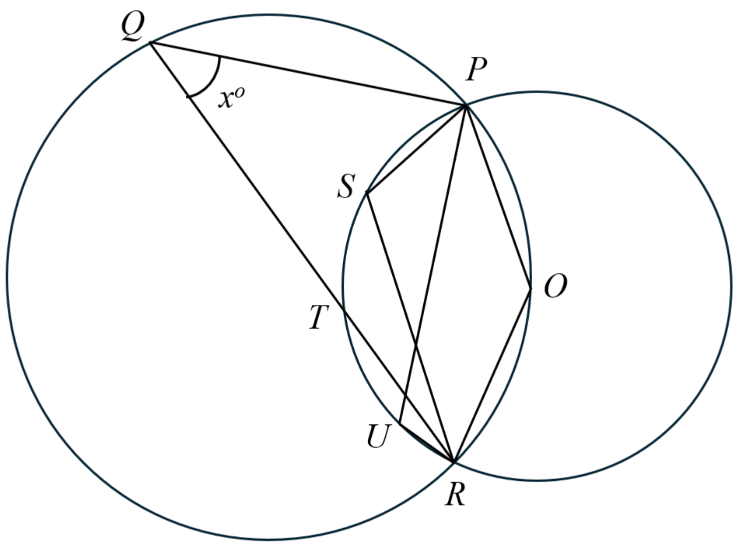
Area of triangle $= \dfrac{1}{2} \times 6 \times 9$
$= 27$ units^2

$PQ = \sqrt{(4-1)^2 + (0+6)^2}$ –(finding base of triangle)
$= \sqrt{45}$ units

Shortest distance $= \dfrac{27 \times 2}{\sqrt{45}}$ = 8.05 units (3s.f.)

Answer …………………… units [3]

19



In the diagram, O is the centre of the smaller circle passing through the points P, S, T, U and R.
QTR is a straight line.
The points P, Q, R and O lie on the larger circle. It is given that angle $PQR = x°$.

(a) Find angle PSR, in terms of x.
Give a reason for each step of your working

Answer

$\angle POR = (180 - x)^o$ (angles in opp segments)
reflex$\angle POR = 360^o - (180 - x)^o$ (angles at a pt)
$= (180 + x)^o$
$\angle PSR = \dfrac{(180 + x)^o}{2}$ (angle at centre = 2 x angle at circumference)
(also accept $\left(90 + \dfrac{x}{2}\right)^o$) [3]

(b) It is now given that angle PUR is 105°, find the value of x.

$\angle PUR = \dfrac{(180 + x)^o}{2}$ (angles in the same segment)
$105 = \dfrac{(180 + x)}{2}$
$x = 30$

Answer $x =$ …………………… [2]

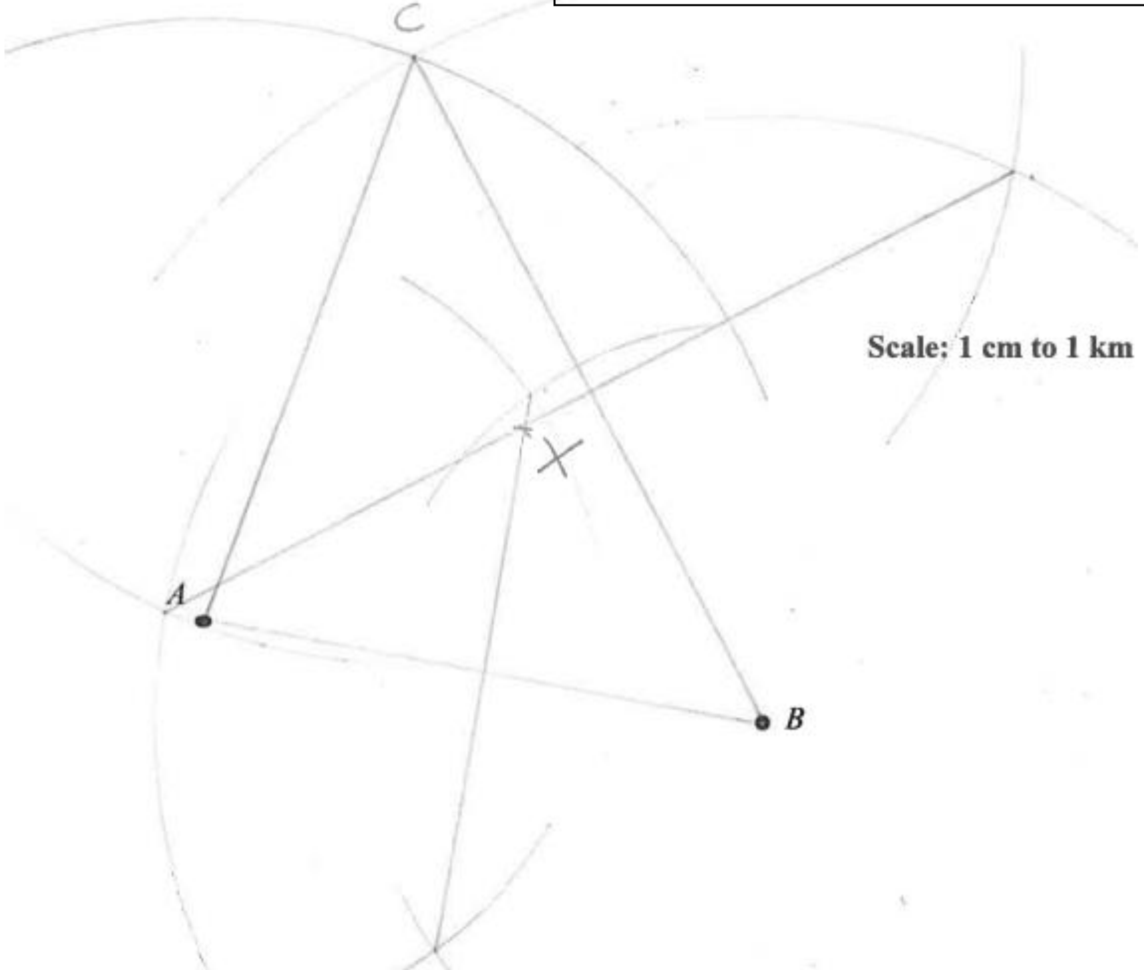
20 There are 3 statues located at points A, B and C respectively.
The diagram in the space below shows the positions of points A and B.

(a) It is given that AC is 8 km and BC is 10 km.
Using appropriate constructions, find and label the position of point C.

[1]

Answer

Draw 2 arcs Find C, the intersection of the 2 arcs



(b) There is a treasure chest buried in a location that is equidistant from A, B and C. Find and label this point X in the diagram above.

[2]

Draw at least 2 perpendicular bisectors Find X, the intersection of the 2 perpendicular bisectors

21 P is inversely proportional to the square of Q.
Given that Q increases by 10%, find the percentage decrease in P.

$P=\frac{k}{Q^2}$, $Q_{\text{new}}=1.1Q$;

$$P_{\text{new}}=\frac{k}{(1.1Q)^2}$$
$$=\frac{k}{1.21Q^2}$$

Percentage change $=\dfrac{\frac{k}{1.21Q^2}-\frac{k}{Q^2}}{\frac{k}{Q^2}}\times 100\%$ (finding % change)

$$=\left(\frac{1}{1.21}-1\right)\times 100\%=-17.4\% \text{ (3s.f.)}$$

Decrease % = 17.4%

.. % [3]

22 In Australia, a 16-oz bottle of shampoo costs A$60.
The shampoo company wants to price a 500ml bottle of the same shampoo at the same rate in France.

€ 1 = A$1.65.
1 oz = 29.5735 ml.

What should the price be in euros (€) ?
Leave your answer to the nearest 10 cents.

In Australia, a 16-oz bottle costs A$60 = €$\frac{60}{1.65}$ --(Converting to euros)

29.5735ml (also 1 oz) of shampoo costs = €$\frac{60}{1.65}\div 16$

500 ml of shampoo costs = €$\frac{60}{1.65}\div 16\times\frac{500}{29.5735}$ --(Converting to ml)
= €38.40 (to nearest 10 cents)

Answer € …………………… [3]

23 A bag contains 12 counters, n of which are green and the remainder are blue. One counter is drawn at random and not replaced.

(a) Write down, in terms of n, the probability that the counter is blue.

Answer $\boxed{\dfrac{12-n}{12}}$ ……………………………… [1]

A second counter is then drawn at random.

(b) Given that the probability that both counters drawn are blue is $\dfrac{5}{22}$, form an equation in the form $n^2+bn+c=0$, where b and c are integers.

Answer

$$\frac{12-n}{12}\times\frac{11-n}{11}=\frac{(12-n)(11-n)}{132}$$

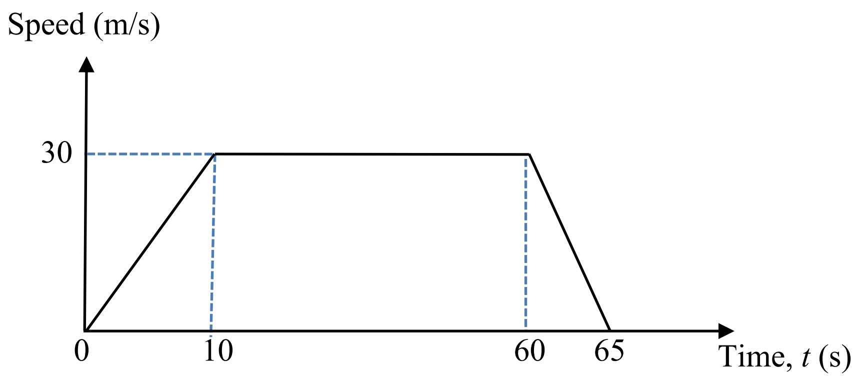
$$\frac{(12-n)(11-n)}{132}=\frac{5}{22}$$

$$(12-n)(11-n)=30$$

$$n^2-23n+102=0$$

[3]

24 The diagram shows the speed-time graph of a car as it travelled from point A to B.



(a) Find the deceleration of the car from 60s to 65s.

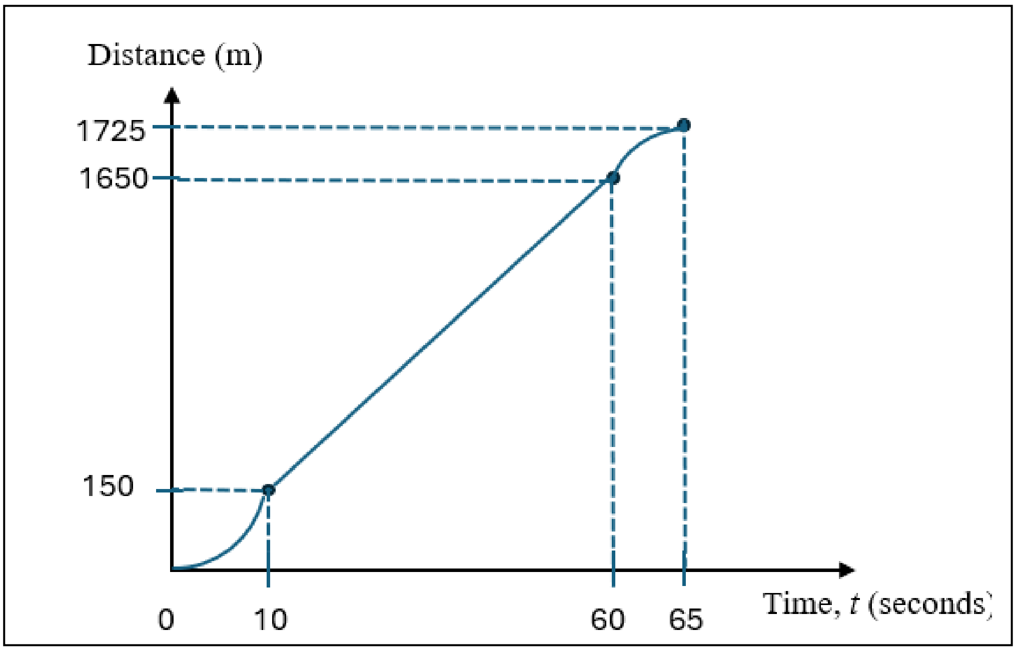
$$\boxed{\frac{30}{5} = 6 \text{ m/s}^2}$$

Answer ……………………………m/s^2 [1]

The area beneath the speed-time graph represents the distance travelled by the object.

(b) Draw the distance-time graph to represent the journey.

Answer



[3]

End of Paper