CEDAR GIRLS' SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2025
SECONDARY FOUR

CANDIDATE NAME	**MARK SCHEME**

CLASS | | | INDEX NUMBER | | |

MATHEMATICS

4052/02

Paper 2

22 August 2025

2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Gives answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

For Examiner's Use
90

This document consists of **19** printed pages and **1** blank page.

[Turn over]

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Solve the inequality $1+4y \le \frac{2y-1}{2} < 0$.

$2+8y \le 2y-1 < 0$

$2+8y \le 2y-1$ and $2y-1<0$

$y \le -\frac{1}{2}$ $y < \frac{1}{2}$

$\therefore y \le -\frac{1}{2}$

Answer …… [2]

(b) Solve $-(1-x)\left(\frac{4}{5}x-1\right)=3$.

$(x-1)\left(\frac{4}{5}x-1\right)=3$

$\frac{4}{5}x^2-\frac{9}{5}x+1=3$

$4x^2-9x-10=0$

$x=\frac{-(-9)\pm\sqrt{(-9)^2-4(4)(-10)}}{2(4)}$

$x=-0.816$ or $x=3.07$ (3 s.f.)

Answer x = ……. and x = ……. [3]

(c) **(i)** Given that $243^p \times 2^{5p} = 6^{6-p}$, find the value of p.

$3^{5p} \times 2^{5p} = 6^{6-p}$
$6^{5p} = 6^{6-p}$
$5p = 6 - p$
$p = 1$

Answer $p =$ ……..…… [2]

(ii) Simplify $\left(\frac{16p^4}{q^8}\right)^{-\frac{3}{2}} \div pq^5$ and leave your answer in positive index notation.

$= \left(\frac{q^8}{16p^4}\right)^{\frac{3}{2}} \div pq^5$
$= \frac{q^{12}}{64p^6} \times \frac{1}{pq^5}$
$= \frac{q^7}{64p^7}$

Answer …… [3]

2 **(a)** The diameter of a red blood cell is 8.2×10^{-6} m.
Express this value as a decimal, in millimetres.

8.2×10^{-3} mm
= 0.0082 mm

Answer …… mm [1]

(b) In 2024, Changi Airport handled 67.7 million passengers.
Calculate the mean number of passengers handled by the airport each month.
Leave your answer in standard form, correct to 2 significant figures.

$$\frac{67.7\times10^{6}}{12}$$
$$= 5.6\times10^{6}$$

Answer ……... [2]

(c) LBB Bank currently charges 3% per year compound interest on all loans, compounded once every three months.
The bank manager proposes to adjust the frequency of interest compounded to once every month instead, so as to increase the earnings of the bank.

Do you agree with the bank manager's proposal?
Explain your answer with clear workings.

Answer

Compounded every 3 months:
$$P\left(1+\frac{3\%}{4}\right)^{4n} = P(1.0075)^{4n}$$
Compounded every month:
$$P\left(1+\frac{3\%}{12}\right)^{12n} = P(1.0025)^{12n}$$
$$= P(1.007518766)^{4n}$$

I agree with the bank manager's proposal, as the total earnings over the same period of ……
time is more when the frequency of interest compounded is once every month.
……
[3]
……

3 A bakery selling assorted tarts operates for 7 days a week.
The matrix, **T**, shows the number of tarts of different types that are made each day.

$$\mathbf{T} = \begin{matrix} \\ \text{Small} \\ \text{Large} \end{matrix} \begin{matrix} \begin{matrix} \text{Apple} & \text{Banana} & \text{Cheese} \end{matrix} \\ \begin{pmatrix} 55 & 35 & 70 \\ 40 & 30 & 30 \end{pmatrix} \end{matrix}$$

The cost of making a small tart is \$0.80.
The cost of making a large tart is \$1.75.

(a) Represent these costs in a 1×2 row matrix **N**.

Answer $\mathbf{N} = \begin{pmatrix} 0.8 & 1.75 \end{pmatrix}$ [1]

(b) Evaluate the matrix **C** = 7**NT**.

$$\begin{aligned} \mathbf{C} &= 7\begin{pmatrix} 0.8 & 1.75 \end{pmatrix}\begin{pmatrix} 55 & 35 & 70 \\ 40 & 30 & 30 \end{pmatrix} \\ &= 7\begin{pmatrix} 114 & 80.5 & 108.5 \end{pmatrix} \\ &= \begin{pmatrix} 798 & 563.5 & 759.5 \end{pmatrix} \end{aligned}$$

Answer **C** = [2]

(c) State what each of the elements of **C** represent.
It represents the total cost of making apple, banana and cheese tarts respectively,
……
in a week.
…… [1]

(d) The tarts are sold at 55% more than the cost to make them.
On a particular day, the number of banana tarts to cheese tarts sold is in the ratio $5:7$ while the number of apple tarts to banana tarts sold is in the ratio $5:4$. A total of 41 unsold tarts were given away.

If the number of small cheese tarts sold is twice the number of large cheese tarts sold, calculate the total amount earned by the bakery from selling cheese tarts on that day.

Apple : Banana : Cheese
$5 : 4$
$5 : 7$
$\therefore 25 : 20 : 28$

No. of cheese tarts sold $= \dfrac{260-41}{25+20+28} \times 28$
$= \dfrac{219}{73} \times 28 = 84$

Total amount earned from selling cheese tarts for the day
$= \left(\dfrac{2}{3} \times 84 \times 1.55 \times 0.80\right) + \left(\dfrac{1}{3} \times 84 \times 1.55 \times 1.75\right)$
$= \$145.39$

Answer \$ …… [3]

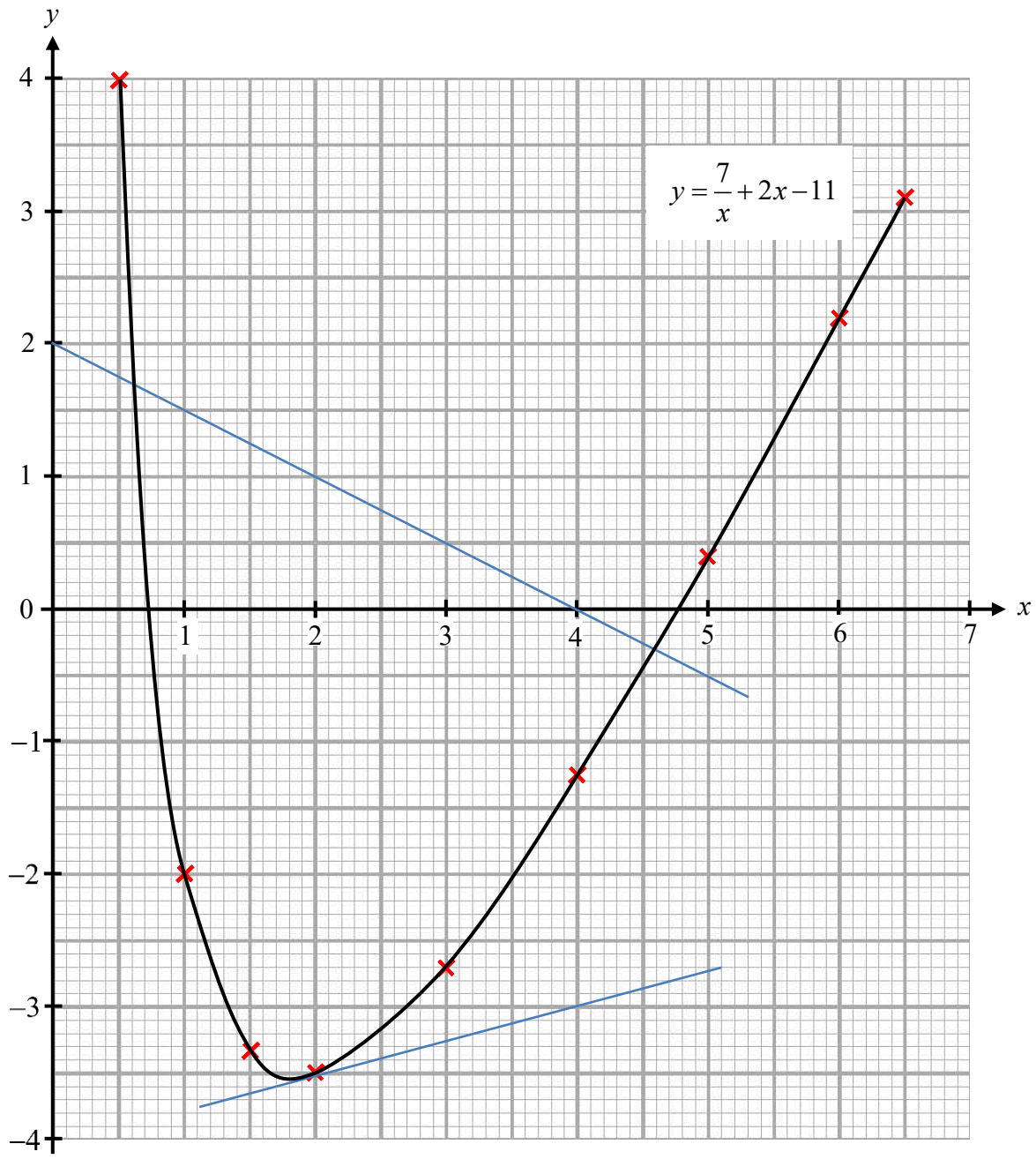
4 **(a)** The variables x and y are connected by the equation $y = \frac{7}{x} + 2x - 11$.

Some corresponding values are given in the table below.
Values are given to 1 decimal place where appropriate.

x	0.5	1	1.5	2	3	4	5	6	6.5
y	4	–2	–3.3	–3.5	–2.7	–1.25	0.4	2.2	3.1

Fill in the missing value in the table. [1]

(b) On the grid below, draw the graph of $y = \frac{7}{x} + 2x - 11$ for $0.5 \le x \le 6.5$. [2]



(c) By drawing a tangent, find the gradient of the curve at $x = 2$.

Gradient $= \dfrac{-3-(-3.5)}{4-2}$

$= 0.25\ (\pm 0.13)$

Answer Gradient = …….. [2]

(d) Using your graph,

(i) explain why the equation $2x + \dfrac{7}{x} - 7 = 0$ has no solution.

Answer

$2x + \dfrac{7}{x} - 11 = -4$

The line $y = -4$ <u>is below the minimum point of/lies entirely below</u> the curve

……

$y = 2x + \dfrac{7}{x} - 11$ and therefore, will not intersect the curve at any point.

……

[2]

(ii) find the x-coordinate of the points where the line $2y = 4 - x$ intersects the curve.

Draw $y = -\dfrac{1}{2}x + 2$

From graph, x-coordinate of points = <u>0.6</u> and <u>4.6</u> (± 0.1)

Answer x = ……. and x = ……. [2]

(iii) These values of x are the solutions of the equation $Ax^2 + Bx + 14 = 0$.

Find the value of A and of B.

$y = -\dfrac{1}{2}x + 2$ ---(1) $\qquad y = 2x + \dfrac{7}{x} - 11$ ---(2)

Sub (1) into (2): $-\dfrac{1}{2}x + 2 = 2x + \dfrac{7}{x} - 11$

$-\dfrac{1}{2}x + 2 = 2x + \dfrac{7}{x} - 11$

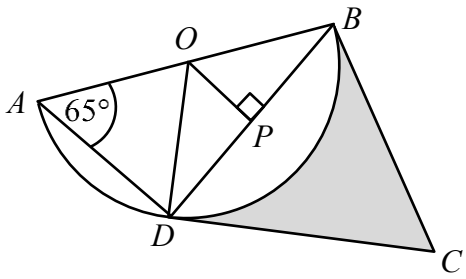
$-x^2 + 4x = 4x^2 + 14 - 22x$

$0 = 5x^2 - 26x + 14$

By comparing coefficients, $A = 5$ and $B = -26$

Answer A = …… , B = …… [3]

5 The figure shows a semi-circle $AOBD$ with centre O and OP is perpendicular to BD. BC and CD are tangents to the semi-circle at B and D.



(a) Show that triangles ABD and OBP are similar.
Give a reason for each statement you make.

Answer

$\angle ADB = 90°$ (angle in semicircle)
So, $\angle ADB = \angle OPB$
$\angle ABD$ is a common angle.
……
$\therefore \Delta ABD$ is similar to ΔOBP. (Angle-angle similarity)
……

[2]

(b) Find the ratio $\dfrac{\text{area of triangle } OBP}{\text{area of triangle } AOD}$.

Since $\dfrac{\text{area of triangle } OBP}{\text{area of triangle } OBD} = \dfrac{1}{2}$
and area of triangle OBD = area of triangle AOD,

$$\frac{\text{area of triangle } OBP}{\text{area of triangle } AOD} = \frac{1}{2}$$

Answer …… [2]

(c) Show that angle BOD = 2.2689 radians, correct to 5 significant figures.

Answer

$\angle BOP = 65°$ (similar triangles)

$$\angle BOD = (65° \times 2) \times \frac{\pi}{180°}$$
$$= 2.268928 \text{ (7 s.f.)}$$
$$= 2.2689 \text{ radians (5 s.f.)} \quad \text{(shown)}$$

[2]

(d) It is given that the radius of the semi-circle is 4 cm and $BC = 8.58$ cm.

Calculate

(i) the perimeter of the shaded part,

$BC = DC =$ 8.58 cm (tangents from ext. pt.)

Perimeter $= (8.58 \times 2) + 4(2.268928)$
$= 26.2357$ (5 s.f.)
$= 26.2$ cm (3 s.f.)

Answer …… cm [2]

(ii) the area of the shaded part.

$\angle OBC = 90°$ (tangent $\perp$ radius)

Area $= \left(2 \times \frac{1}{2} \times 4 \times 8.58\right) - \frac{1}{2}\left(4^2\right)(2.268928)$
$= 16.169$ (5 s.f.)
$= 16.2$ cm^2 (3 s.f.)

Answer …… cm^2 [3]

6 (a) (i) The position vector of point A and point B are $\begin{pmatrix} k \\ 6 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$ respectively.

Given that $\left|\overrightarrow{AB}\right| = 10$, find the negative value of k.

$$\overrightarrow{AB} = \begin{pmatrix} 2 \\ -2 \end{pmatrix} - \begin{pmatrix} k \\ 6 \end{pmatrix} = \begin{pmatrix} 2-k \\ -8 \end{pmatrix}$$

So, $(2-k)^2 + (-8)^2 = 10^2$

$4 - 4k + k^2 + 64 = 100$

$k^2 - 4k - 32 = 0$

$(k+4)(k-8) = 0$

$\therefore k = -4, \quad 8$ (rej)

Answer $k =$ …… [3]

(ii) C is the point $(5, -6)$.

Using the value of k found in part **(a)(i)**, show that A, B and C are collinear.

Answer

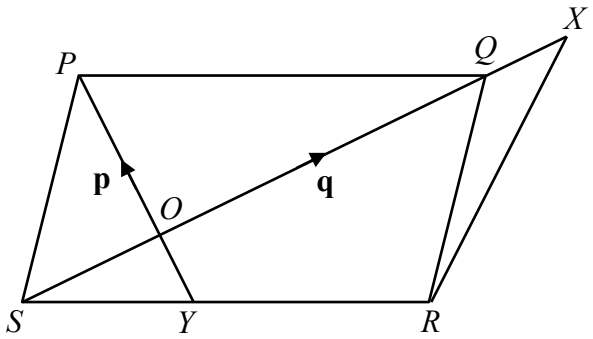
$$\overrightarrow{BC} = \begin{pmatrix} 5 \\ -6 \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$$

$$\overrightarrow{AB} = \begin{pmatrix} 6 \\ -8 \end{pmatrix} = 2\begin{pmatrix} 3 \\ -4 \end{pmatrix} = 2\overrightarrow{BC}$$

Since AB // BC and B is a common point, A, B and C must be collinear.

[3]

(b) The diagram shows a parallelogram $PQRS$ and a triangle QXR.
QS is a diagonal of the parallelogram and Y is a point on RS such that $OS : QS = 3 : 10$.



It is given that $\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OQ} = \mathbf{q}$,

(i) Express $\overrightarrow{QR}$, as simply as possible, in terms of **p** and **q**,

$$\overrightarrow{SR} = \overrightarrow{PQ} = -\mathbf{p} + \mathbf{q}$$
$$\overrightarrow{QR} = \overrightarrow{QS} + \overrightarrow{SR}$$
$$= -\frac{10}{7}\mathbf{q} + (-\mathbf{p} + \mathbf{q})$$
$$= -\mathbf{p} - \frac{3}{7}\mathbf{q}$$

Answer $\overrightarrow{QR}$ = …… [3]

(ii) If $\overrightarrow{QX} = m\overrightarrow{SQ}$ and $\overrightarrow{RX} = n\left(\frac{1}{5}\mathbf{p} + \frac{1}{7}\mathbf{q}\right)$, where m and n are rational numbers, find the value of m and of n.

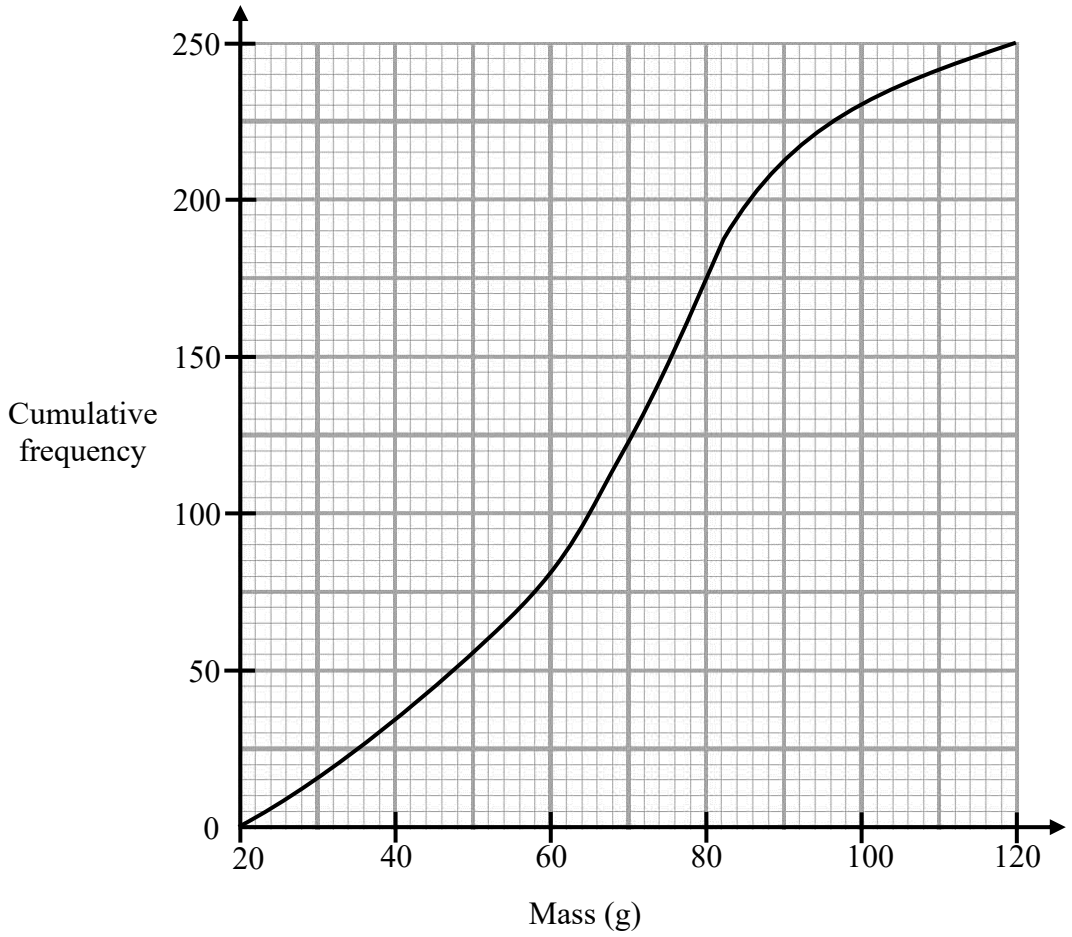
$$\overrightarrow{QX} = \frac{10}{7}m\mathbf{q} \text{ --- (1)}$$
$$\overrightarrow{QX} = \overrightarrow{QR} + \overrightarrow{RX}$$
$$= -\mathbf{p} - \frac{3}{7}\mathbf{q} + n\left(\frac{1}{5}\mathbf{p} + \frac{1}{7}\mathbf{q}\right)$$
$$= \left(\frac{1}{5}n - 1\right)\mathbf{p} + \left(\frac{1}{7}n - \frac{3}{7}\right)\mathbf{q} \text{ --- (2)}$$

Comparing coefficients, $\frac{1}{5}n - 1 = 0 \quad \rightarrow n = 5$

$$\frac{1}{7}(5) - \frac{3}{7} = \frac{10}{7}m \quad \rightarrow m = \frac{1}{5}$$

Answer m = ……, n = ……. [4]

7 A farmer harvested 250 potatoes which were grown using Fertiliser X.
The cumulative frequency curve below shows the mass distribution of the potatoes.



(a) The farmer only sells potatoes that weigh 56 g and above.

Using the curve, find the percentage of potatoes that can be sold.

$\frac{250-70}{250} \times 100\% = 72\%$

Answer …… % [1]

(b) **(i)** Complete the grouped frequency table for the mass of the potatoes.

Mass (m g)	$20 \le m < 40$	$40 \le m < 60$	$60 \le m < 80$	$80 \le m < 100$	$100 \le m < 120$
Frequency	35	45	95	55	20

[1]

(ii) Find the interval that contains the median mass.

Answer …..60…. $\le m <$ …..80.... [1]

(iii) Calculate an estimate of the mean mass.

Answer ……68.4 …… g [1]

(iv) Calculate an estimate of the standard deviation of the masses.

Answer ……22.6 …… g [1]

(c) Two of the potatoes are selected at random, without replacement.

Find the probability that

(i) none has a mass between 60 g and 100 g,

$$\text{P(none has mass between 60 g and 100 g)} = \frac{100}{250} \times \frac{99}{249}$$
$$= \frac{66}{415}$$

Answer …….... [2]

(ii) one has a mass less than 60 g and the other has a mass greater than 100 g.

P(one has mass less than 60 g and one has mass greater than 100 g)

$$= 2\left(\frac{80}{250} \times \frac{20}{249}\right)$$
$$= \frac{64}{1245}$$

Answer …….... [2]

(d) The farmer harvested another 250 potatoes that were grown using Fertiliser Y.
The estimated mean mass of this batch of potatoes is 62 g.
The estimated standard deviation of the masses is 23 g.

Make two comments comparing the mass of the potatoes grown using Fertiliser X and those grown using Fertiliser Y.

Answer
On average, potatoes grown using Fertiliser X are heavier than those grown using
……
Fertiliser Y, as they have a higher mean mass of 68.4 g, compared to 62 g.
……
The spread of the mass is comparable for both batches of potatoes, with SD being
……
22.6 g and 23 g. / Mass of potatoes grown using Fertiliser X has a smaller spread, with
…… [2]
a lower SD of 22.6 g, rather than 23 g.

8 A factory produces bowls in two sizes, small size and large size.

(a) It is found that x large bowls can be produced in a minute.
Write down an expression in terms of x, the time taken to produce 1 large bowl, **in seconds**.

$\frac{60}{x}$

Answer ……. s [1]

(b) 4 more small bowls can be produced in a minute, compared to the large bowls.
Write down an expression in terms of x, the time taken to produce 1 small bowl, **in seconds**.

$\frac{60}{x+4}$

Answer ……... s [1]

(c) Given that it takes 2.5 seconds longer to produce a large bowl than a small bowl, form an equation in x and show that it reduces to $x^2 + 4x - 96 = 0$.

Answer

$$\frac{60}{x} - \frac{60}{x+4} = 2.5$$
$$60(x+4) - 60x = 2.5x(x+4)$$
$$60x + 240 - 60x = 2.5x^2 + 10x$$
$$2.5x^2 + 10x - 240 = 0$$
$x^2 + 4x - 96 = 0$ (shown)

[2]

(d) Solve the equation $x^2 + 4x - 96 = 0$.

$$x = \frac{-4 \pm \sqrt{(-4)^2 - 4(1)(-96)}}{2(1)}$$
$$= -12 \quad \text{or} \quad 8$$

Answer x = ……. or x = ……. [2]

(e) Hence find the time taken to produce 3500 small bowls.
Give your answer in hours and minutes, correct to the nearest minute.

$$\text{Time taken} = \frac{3500}{8+4}$$
$$= 291\frac{6}{7} \text{ min}$$
$$= 4 \text{ h } 52 \text{ min}$$

Answer ……..…… h ……..…… min [2]

(f) The factory sells each small bowl at \$0.35 and each large bowl at \$0.50.

Explain, with clear workings, why it is more profitable for the factory to produce only small bowls than only large bowls.

Answer

In t seconds,

Amount earned from selling large bowls = $\$0.50 \times \frac{t}{60/8} = \$0.067t$

Amount earned from selling small bowls = $\$0.35 \times \frac{t}{60/12} = \$0.07t$

The factory earns more from selling small bowls, given the same production time.

[3]

9 Ryan has set up a home theatre system in one of his rooms, which is in the shape of a cuboid with dimensions 4 m by 3.6 m by 3 m.

Figure 1 shows the three-dimensional layout of the room, where a television is mounted on the front wall *QRVU* and two speakers are mounted on the back wall *PSWT*, at *P* and *T* respectively.

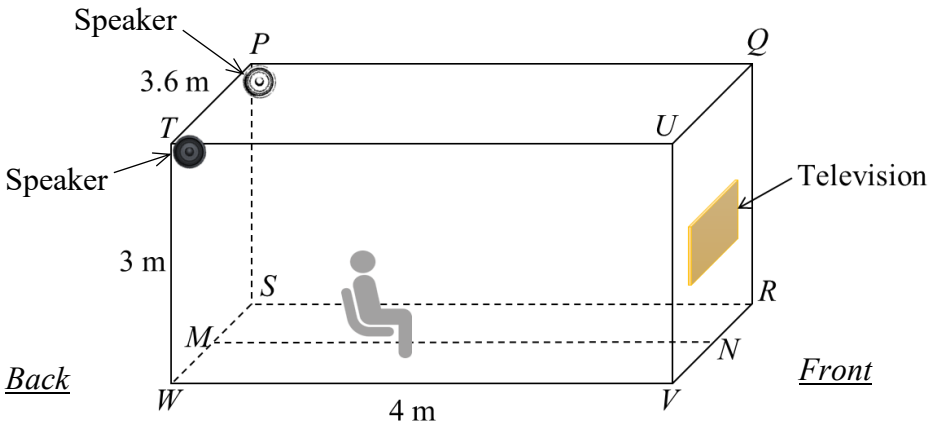


Figure 1

Ryan is planning to put an armchair along *MN*, which is a straight path that cuts across the centre of the room from the back to the front.

Figure 2 shows the cross-sectional view of him sitting in the armchair, where his eye level at *X*, is 1.24 m directly above the ground and ***d*** m away from the front wall. The centre of the television at *Y*, is 1.6 m directly above *N*.

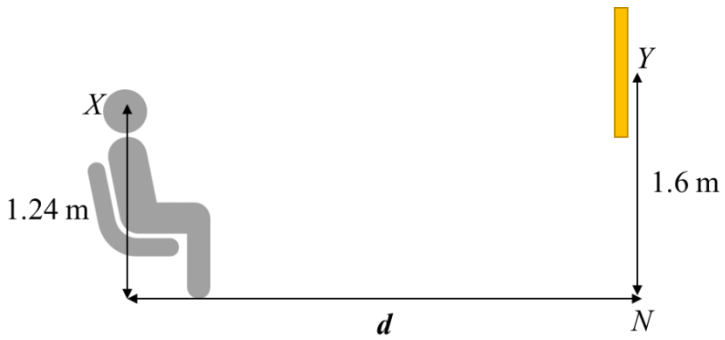


Figure 2

(a) Find the angle of elevation from *X* to *Y* when ***d*** = 1.2 m.

Let the angle of elevation be θ .

$$\tan\theta = \frac{1.6-1.24}{1.2}$$

$\theta = 16.699°$ (5 s.f.)

$= 16.7°$ (3 s.f.)

Answer ……..° [2]

(b) Ryan found out that:

- To have an optimal view of the television, the angle of elevation from X to Y should be less than $15°$.
- To have an optimal audio experience, the position of the two speakers and X should form an equilateral triangle.

Suggest a suitable value for ***d*** such that the position of the armchair will provide Ryan with best possible user experience.
Justify the decisions you make and show your calculations clearly.

Answer

1) When $\theta = 15°$, $\tan 15° = \dfrac{1.6 - 1.24}{d}$ → $\mathbf{d} = 1.3435$ m

To have optimal view of television:
For $\theta < 15°$, $\mathbf{d} > 1.3435$ m

2) To have optimal audio experience:
ΔPXT needs to be equilateral → $\angle TPX = \angle PXT = \angle XTP = 60°$

Let perpendicular bisector of PT to X be m.

So, $\sin 60° = \dfrac{m}{3.6}$ → $m = 3.1177$ m

By Pythagoras' Theorem,

$$3.1177^2 = (4 - \mathbf{d})^2 + (3 - 1.24)^2$$
$$4 - \mathbf{d} = \sqrt{3.1177^2 - 1.76^2}$$
$$\mathbf{d} = 1.4266 \text{ m } (> 1.3435)$$

∴ Ryan should position the armchair at **d** = 1.43 m [DB1] because it will provide him with the best possible user experience, with **both** optimal view of the television and audio experience. [DB1]

……

…… [7]

~ End of paper ~

BLANK PAGE