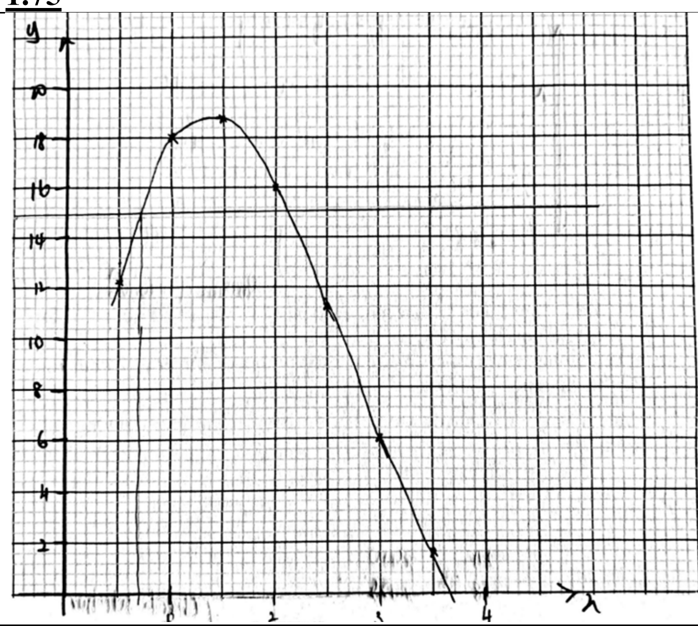
ST JOSEPH'S INSTITUTION
PRELIMINARY EXAMINATION 2025
(YEAR 4) MA4052 Paper 2 Solutions

Q/N	Solutions
1(a)(i)	$a=\sqrt{\dfrac{3n}{2n+5k}}$ $=\sqrt{\dfrac{3\left(\frac{1}{3}\right)}{2\left(\frac{1}{3}\right)+5\left(\frac{2}{3}\right)}}$ $=\underline{\dfrac{1}{2}}$
1(a)(ii)	$a=\sqrt{\dfrac{3n}{2n+5k}}$ $a^2=\dfrac{3n}{2n+5k}$ $2a^2n+5a^2k=3n$ $2a^2n-3n=-5a^2k$ $n(2a^2-3)=-5a^2k$ $\underline{n=\dfrac{-5a^2k}{2a^2-3}}$ or $\underline{n=\dfrac{5a^2k}{3-2a^2}}$

1(b)	$\frac{4-5x}{6} > 1 - \frac{x+5}{4}$ $\frac{4-5x}{6} > \frac{4-(x+5)}{4}$ $\frac{4-5x}{6} > \frac{4-x-5}{4}$ $\frac{4-5x}{6} > \frac{-x-1}{4}$ $4-5x > \frac{6(-x-1)}{4}$ $4(4-5x) > 6(-x-1)$ $16-20x > -6x-6$ $-14x > -22$ $x < \frac{22}{14}$ $\underline{x < \frac{11}{7}}$ or $\underline{x < 1\frac{4}{7}}$
1(c)	$\frac{x-2}{x+1} - \frac{x+3}{x-1} = 5$ $\frac{(x-2)(x-1)-(x+3)(x+1)}{(x+1)(x-1)} = 5$ $\frac{(x^2-x-2x+2)-(x^2+x+3x+3)}{x^2-1} = 5$ $\frac{-3x+2-4x-3}{x^2-1} = 5$ $\frac{-7x-1}{x^2-1} = 5$ $-7x-1 = 5(x^2-1)$ $5x^2-5+7x+1 = 0$ $5x^2+7x-4 = 0$ $x = \frac{-7 \pm \sqrt{(7)^2-4(5)(-4)}}{2(5)}$ $x = \frac{-7 \pm \sqrt{129}}{10}$ $x \approx 0.4357816692$ or -1.835781669 $x \approx$ $\underline{\mathbf{0.44}}$ or $\underline{\mathbf{-1.84}}$ (2dp)

2(a)(i)	114.9 million $= 114.9 \times 10^6$ $= 1.149 \times 10^2 \times 10^6$ $\approx \underline{1.15 \times 10^8}$
2(a)(ii)	Population density in 2025 = $\dfrac{116.79 \times 10^6}{300000}$ = 389.3 Since **<u>389.3 > 350</u>** The Philippines **<u>had exceeded</u>** the government's population density.
2(b)	% change = $\dfrac{(2.8x + 1.5x + 5x) - 10x}{10x} \times 100\%$ $= \dfrac{9.3x - 10x}{10x} \times 100\%$ = **<u>– 7%</u>**
2(c)	Total cost = $x \times 12 \times 1.1 \times 1.09$ $= 14.388x$ Total bill = $30 \times 11 + 29.7 = 14.388x$ $\underline{x = 25}$
3(a)	Volume of the solid $= \dfrac{1}{2}(4-x)(2x)(4-x) + (4-x)^2 x$

	$= x\left(16-8x+x^2\right)+x(16-8x+x^2)$ $\underline{= x^3-8x^2+16x+x^3-8x^2+16x}$ $= 2x^3-16x^2+32x$
3(b)	The **<u>dimensions</u>** of the figure **<u>must be greater than 0</u>**. Therefore, $4-x>0$ $x<4$ and $x>0$ $\therefore 0<x<4$
3(c)	**<u>1.75</u>**
3(d)	
3(e)	**<u>0.65</u>**
3(f)	From the graph the **<u>maximum value is 18.9</u>**, the volume cannot be greater than 18.9 cm^3 . Hence the volume of the solid **<u>cannot be 20 cm^3</u>**.

4(a)	Using similarity, $\frac{h}{h-10}=\frac{24}{18}$ $18h=24h-240$ $h=40$ Volume of frustum = $\frac{1}{3}\pi(12)^2\times 40-\frac{1}{3}\pi(9)^2\times 30$ $=1110\pi\ \text{cm}^3$ $=\underline{1.11\pi\ \text{litres}}$
4(b)	$\frac{3}{4}\pi r^2(20)=1110\pi$ $r^2=74$ $r=\sqrt{74}\approx 8.6023$ Therefore, area not in contact of water = $2\pi\left(\sqrt{74}\right)\left[\frac{1}{4}(20)\right]$ ≈ 270.250 = $\underline{\mathbf{270\ cm^2}}$ (to 3 sf)
5(a)	$3x-2y=6$ --- (1) $x+y=5$ --- (2) From (2) $x=5-y$ --- (2*) Sub (2*) into (1) $3(5-y)-2y=6$ $y=\frac{9}{5}$ Sub $y=\frac{9}{5}$ into (2*) $x=5-\left(\frac{9}{5}\right)=\frac{16}{5}$ T is $\underline{\left(3\frac{1}{5},1\frac{4}{5}\right)}$

5(b)	$x+y=5$ A is $(5,0)$ B is $(0,5)$ Length of $AB=\sqrt{(5)^2+(5)^2}$ $= 5\sqrt{2}$ (= 7.07106) [Using area of triangle OAB] $\frac{1}{2}(7.07106)h=\frac{1}{2}(5)(5)$ $\underline{h=3.54}$ units (to 3 sf)
5(c)	Let m be the distance from T $\frac{1}{2}(5+m)\times 3.2=18$ $m=6.25$ $\underline{y=-4.45}$ or $\underline{y=8.05}$
6(a)	$DF = AF$ (given F midpoint of AD) $\angle BFD = \angle BFA = 90°$ ($\perp$ bisector of chord) BF is common $\therefore$ triangle $ABF \equiv$ triangle DBF **(SAS)**
6(b)(i)	$\angle DAC \doteq \angle DBC = \underline{\mathbf{45}}°$ **<u>($\angle$ s in the same segment)</u>**
6(b)(ii)	$\angle ADC = 90°$($\angle$ in a semicircle) $\angle DCA = 180° - 90° - 45°$ **($\angle$ s sum of a triangle)** $= \underline{\mathbf{45}°}$
6(b)(iii)	$\angle DBA \doteq \angle DCA = 45°$ ($\angle$ s in the same segment) $\angle OBA = \frac{45°}{2}$ (triangle $ABF \equiv$ triangle DBF) $= 22.5°$ $\angle OAB = \angle OBA = 22.5°$ (base $\angle$ s of an isosceles triangle) $\angle AOB = 180° - 22.5° - 22.5°$ **($\angle$ s sum of a triangle)** $\underline{= \mathbf{135}°}$

6(b)(iv)	$\angle ACB = \frac{135^\circ}{2}$ ($\angle$ at centre = 2 $\angle$s at circumference) $= 67.5^\circ$ $\angle CEB = 180^\circ - 67.5^\circ - 45^\circ$ ($\angle$s sum of a triangle) $= 67.5^\circ$ $\angle AED = \angle CEB =$ **$\underline{67.5^\circ}$ (vertically opposite $\angle$s)**
7(a)	$\frac{BD}{\sin 44.2^\circ} = \frac{200}{\sin 95^\circ}$ $BD = \frac{200}{\sin 95^\circ} \times \sin 44.2^\circ$ ≈ 139.965 $\approx \underline{140}$ **m**
7(b)	$3300 = \frac{1}{2}(48)(139.965)\sin \angle CBD$ $\sin \angle CBD = \frac{3300 \times 2}{48(139.965)}$ $\angle CBD \approx 79.230$ bearing of C from B $= 360^\circ - (180^\circ - 45.8^\circ) - 95^\circ - 79.230^\circ$ ≈ 051.57 $=$ **$\underline{051.6^\circ}$** (to 1 dp)
7(c)	Let h be the height of the vertical tower. $\sin 79.230^\circ = \frac{opp}{48}$ $opp = 48 \times \sin 79.230^\circ$ ≈ 47.1544 $\tan \theta_e = \frac{80}{47.1544}$ $\underline{\theta_e = 59.5^\circ}$ (to 1 dp)

8(a)	$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \underline{3\mathbf{b} - 2\mathbf{a}}$
8(b)	$\overrightarrow{XZ} = \frac{1}{2}\overrightarrow{AZ} = \frac{1}{2}\left(\overrightarrow{AC} + \overrightarrow{CZ}\right)$ $= \frac{1}{2}(3\mathbf{b} - 2\mathbf{a} + \mathbf{a})$ $= \underline{\frac{3}{2}\mathbf{b} - \frac{1}{2}\mathbf{a}}$
8(c)	$\overrightarrow{CX} = \overrightarrow{CZ} + \overrightarrow{ZX}$ $= \mathbf{a} + \frac{1}{2}\mathbf{a} - \frac{3}{2}\mathbf{b}$ $= \underline{\frac{3}{2}\mathbf{a} - \frac{3}{2}\mathbf{b}}$
8(d)	$\overrightarrow{CY} = \overrightarrow{CA} + \overrightarrow{AY}$ $= -3\mathbf{b} + 2\mathbf{a} + \mathbf{b}$ $= \underline{2\mathbf{a} - 2\mathbf{b}}$
8(e)	Since $\underline{\overrightarrow{CY} = \frac{4}{3}\overrightarrow{CX}}$ and **<u>C is a common point,</u>** C, X and Y are **<u>collinear.</u>**
8(f)	$\frac{\text{area of } \Delta ACY}{\text{area of } \Delta ACZ} = \frac{\text{area of } \Delta ACY}{\text{area of } \Delta ABC} \times \frac{\text{area of } \Delta ABC}{\text{area of } \Delta ACZ}$ $= \frac{1}{3} \times \frac{2}{1}$ $= \underline{\frac{2}{3}}$

9(a)(i)	**56 minutes**
9(a)(ii)	IQR = $65 - 51.5 =$ **13.5 minutes**
9(a)(iii)	Number of students who took more than 70 minutes $= 600 - 490$ $= 110$ Probability that a pupil chosen at random from the school took more than 70 minutes $= \frac{110}{600} = \frac{11}{60}$
9(b)(i)	The **median time taken by students from School X** to complete the task is **56 minutes**, while for **School Y it is 52 minutes**. This means that students from **School Y, on average, complete the task faster** than those from School X.
9(b)(ii)	For School X: Upper boundary = 85.25 For School Y: Upper boundary =89.25 **88 minutes is considered an outlier for School X**, but not for School Y. Therefore, 88 minutes would be considered **more unexpected for School X**.
9(c)	Since both tasks have the same IQR, **the steepness of the curve will remain the same**, but the curve **will be shifted to the left** because of the smaller median.

10(a)	Total Stamp Duties = $1\%\times180000+2\%\times180000+3\%\times640000+4\%\times500000+5\%\times500000$ $+\quad 17\%\times2000000$ = 69 600 + 340 000 = **<u>$409 600</u>**
	If sold in April 2025 (> 2 years) Cost incurred = 2 000 000 + 409 600 = 2 409 600 Proceeds after SSD = 2 300 000 – 0.04*2 300 000 = 2 208 000 Net Profit = 2 208 000 – 2 409 600 = – 201 600 ----------------------------- if Sold in **March 2026** (> 3 years) Future price $= 2000000\times(1.07)^3$ = 2 450 086 <u>SSD (more than 3 years) dropped to 0%</u> Proceeds after SSD = 2 450 086 Net Profit = 2 450 086– 2 409 600 = **<u>$40 486</u>** If Rachel in April 2025**:** Loss = 201 600 **Wait for another year:** Net profit = **<u>$40 486</u>** The drop in seller's SSD to 0 and potential capital appreciation (property rise of 7%) mean Rachel stands to gain much more by waiting until April 2026.