



**ST JOSEPH'S INSTITUTION
PRELIMINARY EXAMINATION 2025
(YEAR 4)**

CANDIDATE
NAME

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CLASS

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INDEX
NUMBER

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MATHEMATICS

4052/02

Paper 2

21 August 2025

Candidates answer on the Question Paper.

**2 hours 15 minutes
(08:05 – 10:20)**

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use an HB pencil for any diagrams or graphs.

Do not use paper clips, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [] at the end of each question or part question.

The total marks for this paper is 90.

This document consists of **25** printed pages and **3** blank pages.

[Turn over]

Mathematical Formulae*Compound Interest*

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

TURN OVER FOR QUESTION 1

1 **(a)** It is given that $a = \sqrt{\frac{3n}{2n+5k}}$.

(i) Find the value of a when $n = \frac{1}{3}$ and $k = \frac{2}{3}$.

Answer $a =$ [1]

(ii) Express n in terms of a and k .

Answer $n =$ [3]

- (b) Solve the inequality $\frac{4-5x}{6} > 1 - \frac{x+5}{4}$.

Answer [2]

- (c) Solve the equation $\frac{x-2}{x+1} - \frac{x+3}{x-1} = 5$.

Give your solutions correct to 2 decimal places.

Answer $x =$ or [5]

- 2 (a) The total population of the Philippines was estimated at 114.9 million in June 2023 and 116.79 million in June 2025.
- (i) Write 114.9 million in standard form correct to 3 significant figures.

Answer [1]

- (ii) The land area of the Philippines is about 300 000 square kilometres.
The government wants to keep the population density below 350 people per square kilometre.

Based on the June 2025 population figure, determine whether the Philippines has exceeded this density limit. Show your calculations.

Answer

.....

..... [2]

- (b) The cost of making a bicycle was made up of three components: materials, wages, and factory rent.

These costs were split in the ratio 2 : 3 : 5, respectively.

In the following year, the cost of materials increased by 40%, the cost of wages was halved, and the cost of factory rent remained unchanged.

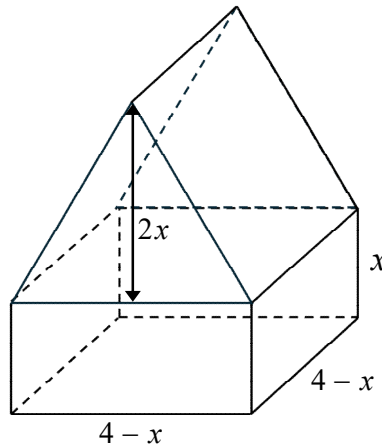
Based on these changes, calculate the percentage change in the total cost of making the bicycle.

Answer% [2]

- (c) Jia Min and 11 friends went for dinner.
The set meal cost \$ x per person excluding service charge and Goods and Services Tax (GST).
The restaurant applied a 10% service charge to the total meal cost, followed by a 9% GST on the amount including the service charge.
She collected \$30 from each of her 11 friends, then topped up the remaining \$29.70 herself to cover the full bill.
Find the value of x .

Answer $x =$ [4]

3



The diagram shows a solid made up of a cuboid with dimensions $(4-x)$ cm by $(4-x)$ cm by x cm and a triangular prism stacked on top of it. The height of the triangular prism is $2x$ cm.

- (a) Show that the volume of the solid, y cm³, is given by $y = 2x^3 - 16x^2 + 32x$.

Answer

[3]

- (b) Explain why $0 < x < 4$.

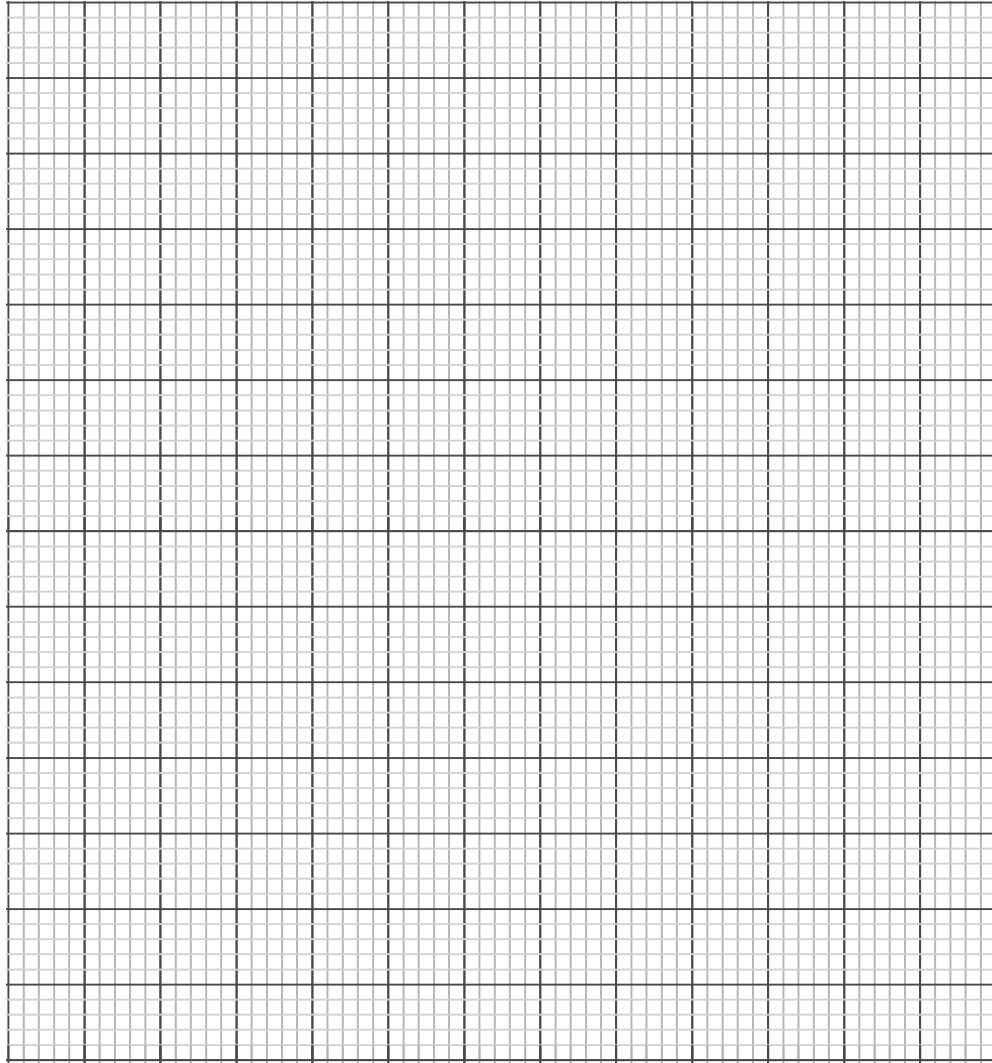
..... [1]

- (c) Complete the table of values for $y = 2x^3 - 16x^2 + 32x$.

x	0.5	1	1.5	2	2.5	3	3.5
y	12.25	18	18.75	16	11.25	6	

[1]

- (d) On the grid, draw the graph of $y = 2x^3 - 16x^2 + 32x$ for $0.5 \leq x \leq 3.5$.



[3]

- (e) Use your graph to find the smallest value of x when the volume of the solid is equal to 15 cm^3 .

Answer $x = \dots\dots\dots$ [1]

- (f) Explain how the graph shows that the volume of the solid cannot be equal to 20 cm^3 .

.....

..... [1]

4

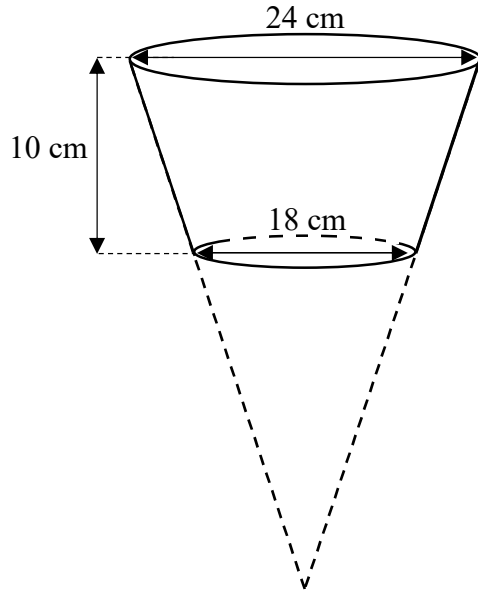


Figure 1

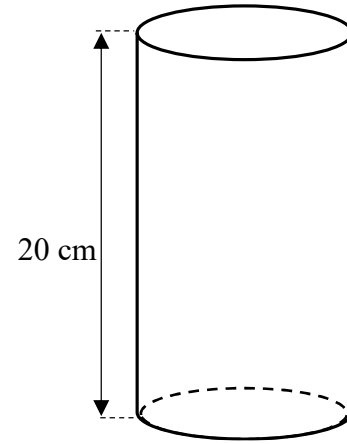


Figure 2

- (a) Figure 1 shows a container in the shape of a frustum of a cone. The container has an open circular top with a diameter of 24 cm and a circular base with a diameter of 18 cm. It is completely filled with water to a height of 10 cm. Calculate the volume of water in the container, in litres, leaving your answer in terms of π .

Answer litres [4]

- (b) All the water is poured into an open cylindrical container of height 20 cm, as shown in Figure 2.

The water fills the cylinder to $\frac{3}{4}$ of its height.

Calculate the surface area of the cylindrical container that is not in contact with the water.

Answer cm² [4]

- 5** The equation of line p is $3x - 2y = 6$.
The equation of line q is $x + y = 5$.
The two lines intersect at point T .
(a) Find the coordinates of point T .

Answer (..... ,) [3]

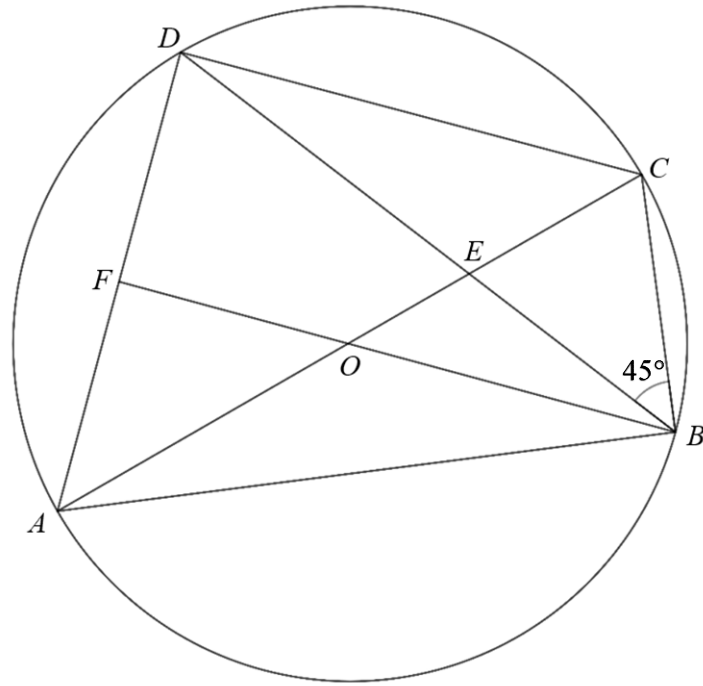
- (b) Line q intersects the x -axis at point A and the y -axis at point B .
Find the shortest distance from the origin O to the line segment AB .

Answer units [4]

- (c) Points O , B , R and T are vertices of a trapezium, where OB is parallel to RT .
The area of trapezium is 18 square units.
Write down the two possible y -coordinates of point R .

Answer y - coordinates of R = or [3]

6



The diagram shows a circle $ABCD$, centre O .

F is the midpoint of the chord AD .

BF passes through O .

E is the point of intersection of BD and diameter AC .

Angle $DBC = 45^\circ$.

- (a) Prove that triangle ABF is congruent to triangle DBF .
Give a reason for each statement you make.

Answer

[2]

- (b) Find, giving reasons for each answer,
(i) angle DAC ,

Answer Angle $DAC = \dots\dots\dots$ [1]

(ii) angle DCA ,

Answer Angle $DCA = \dots\dots\dots$ [1]

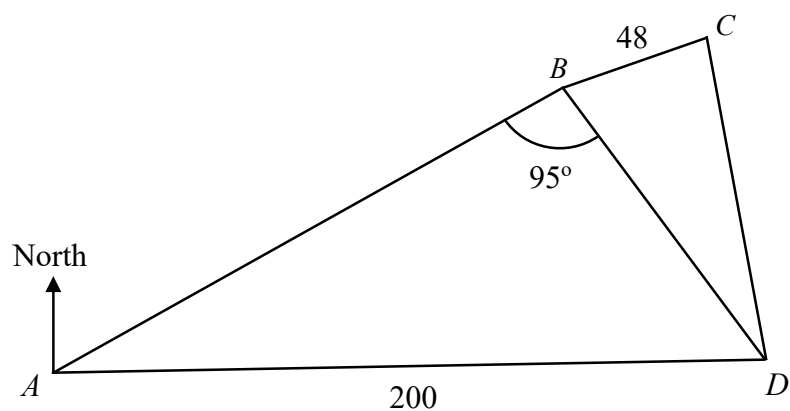
(iii) angle AOB ,

Answer Angle $AOB = \dots\dots\dots$ [2]

(iv) angle AED .

Answer Angle $AED = \dots\dots\dots$ [2]

7



The diagram shows a field $ABCD$ on horizontal ground crossed by a path BD .

D is due east of A .

$AD = 200$ m, $BC = 48$ m and angle $ABD = 95^\circ$.

The bearing of B from A is 045.8° .

(a) Calculate BD .

Answer $BD = \dots\dots\dots$ m [2]

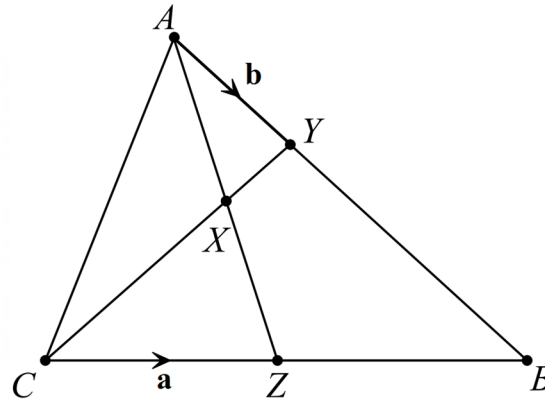
- (b) The area of triangle BCD is 3300 m^2 .
Calculate the bearing of C from B .

Answer [3]

- (c) A vertical tower of height 80 m stands at C .
Sally walks from B to D and reaches a point P where the angle of elevation of the top of the tower from P is the greatest.
Find the angle of elevation of the top of the tower from P .

Answer [4]

8



ABC is a triangle.

Y is the point on AB such that $AY : AB = 1 : 3$.

X is the midpoint of AZ .

Z is the midpoint of CB .

$\overrightarrow{CZ} = \mathbf{a}$ and $\overrightarrow{AY} = \mathbf{b}$.

(a) Express $\overrightarrow{AC}$, as simply as possible, in terms of $\mathbf{a}$ and $\mathbf{b}$.

Answer $\overrightarrow{AC} =$ [1]

(b) Express $\overrightarrow{XZ}$, as simply as possible, in terms of $\mathbf{a}$ and $\mathbf{b}$.

Answer $\overrightarrow{XZ} =$ [1]

(c) Express $\overrightarrow{CX}$, as simply as possible, in terms of $\mathbf{a}$ and $\mathbf{b}$.

Answer $\overrightarrow{CX} =$ [1]

- (d) Express $\overrightarrow{CY}$, as simply as possible, in terms of **a** and **b**.

Answer $\overrightarrow{CY} =$ [1]

- (e) Hence, use your result to (c) and (d), to explain why *C*, *X* and *Y* are collinear.

Answer

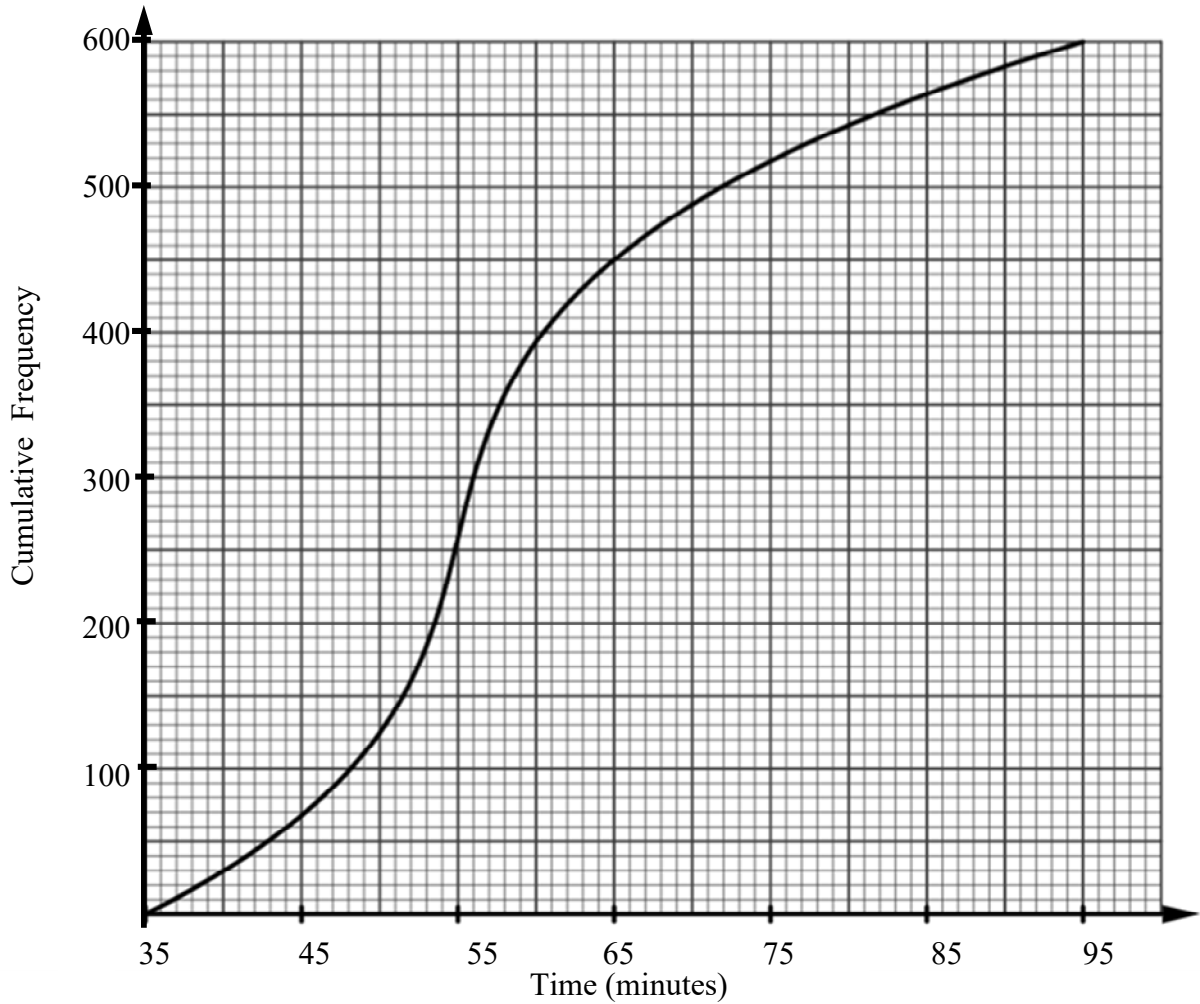
.....

..... [2]

- (f) Calculate the value of $\frac{\text{area of triangle } ACY}{\text{area of triangle } ACZ}$.

Answer [2]

- 9 School X recorded the number of minutes each of its 600 pupils took to complete Task A. The cumulative frequency curve shows the distribution of the results.



(a) Use the curve to estimate

(i) the median time taken,

Answer minutes [1]

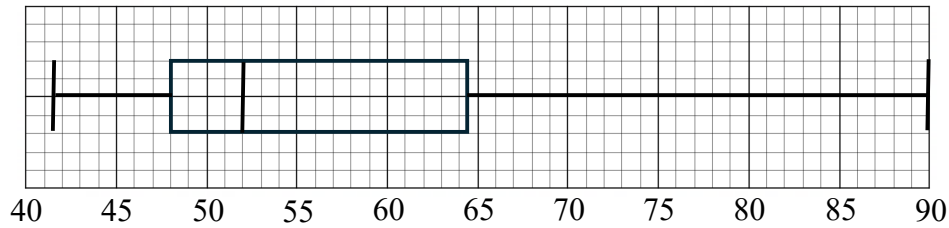
(ii) the interquartile range,

Answer minutes [1]

(iii) the probability that a pupil chosen at random from the school took more than 70 minutes.

Answer [2]

- (b) The box-and-whiskers plot shows the time taken by 600 pupils from School Y to complete Task A.



- (i) Make a comment on the performance of the students from the two schools.

.....

..... [1]

- (ii) It is given that a value is an outlier if it is below $Q1 - 1.5 \times IQR$ or above $Q3 + 1.5 \times IQR$, where $Q1$ is the lower quartile, $Q3$ is the upper quartile and $IQR = Q3 - Q1$.

If a randomly selected student from either school took 88 minutes to complete the task, explain for which school this result would be more unexpected.

Answer

.....

..... [2]

- (c) The times taken by School X to complete another task, Task B, have the same interquartile range but a smaller median.

Describe how the cumulative frequency curve will be different from the given curve in (a).

.....

..... [1]

- 10** In Singapore, stamp duties are taxes imposed on transactions related to the purchase or sale of properties. There are three main types of stamp duties:

Type 1: Buyer's Stamp Duty (BSD)

BSD is applicable for all property purchases and is computed based on the purchase price or market value, whichever is higher.

- Buyer's Stamp Duty (BSD) rates

On or after 15 Feb 2023		
Purchase price or market value of the property	BSD rates for residential properties	BSD rates for non-residential properties
First \$180,000	1%	1%
Next \$180,000	2%	2%
Next \$640,000	3%	3%
Next \$500,000	4%	4%
Next \$1,500,000	5%	5%
Remaining amount	6%	

Type 2: Additional Buyer's Stamp Duty (ABSD)

ABSD applies on top of BSD for certain property purchases, depending on the buyer's residency status and number of properties owned.

Additional Buyer's Stamp Duty (ABSD) rates is computed based on the purchase price or market value, whichever is higher.

Profile of buyer	ABSD rates from 16 Dec 2021 to 26 Apr 2023	ABSD rates on or after 27 Apr 2023
Singapore Citizens (SC) buying first residential property	Not applicable	Not applicable
SC buying second residential property	17%	20%
SC buying third and subsequent residential property	25%	30%
Singapore Permanent Residents (SPR) buying first residential property	5%	5%
SPR buying second residential property	25%	30%
SPR buying third and subsequent residential property	30%	35%
Foreigners (FR) buying any residential property	30%	60%

Type 3: Seller's Stamp Duty (SSD)

SSD applies if residential properties are sold within a specified holding period.

- Seller's Stamp Duty (SSD) rates in Singapore (2024)

How long property has been held	SSD rate (of selling price or market value, whichever is higher)
Up to one year	12%
More than one year and up to two years	8%
More than 2 years and up to 3 years	4%
More than 3 years	No SSD payable

Rachel is a Singapore citizen who owns one private residential property.

In March 2023, she decided to purchase a second private residential property, Property P, at the market price of \$2 000 000 (excluding the stamp duties) and paid the full amount without taking loans.

In April 2025, Rachel receives an offer to sell Property P for \$2 300 000.

(a) Calculate the total stamp duties Rachel paid when she purchased Property P in 2023.

Answer \$ [3]

- (b) Property prices have been rising by 7% every year. Determine whether Rachel should accept the offer to sell Property P in April 2025 or wait until April 2026 to sell it at market value. Justify your decision and show your calculations clearly.

Some other information:

- All calculations should exclude legal fees, agent commissions, loan/mortgage costs, renovation/maintenance costs, and effects of interest rate changes.
- Market environment and government housing policies are assumed stable, with no sudden changes.

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..... [6]

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ST JOSEPH'S INSTITUTION
PRELIMINARY EXAMINATION 2025
(YEAR 4) MA4052 Paper 2 Answer Key

Q/N	Solutions
1(a)(i)	$\frac{1}{2}$
1(a)(ii)	$n = \frac{-5a^2k}{2a^2 - 3}$ or $n = \frac{5a^2k}{3 - 2a^2}$
1(b)	$x < \frac{11}{7}$ or $x < 1\frac{4}{7}$
1(c)	$x = \underline{0.44}$ or $\underline{-1.84}$ (2dp)
2(a)(i)	$\underline{1.15 \times 10^8}$
2(a)(ii)	The Philippines <u>had exceeded</u> the government's population density.
2(b)	$\underline{-7\%}$
2(c)	$\underline{x = 25}$
3(a)	$\frac{1}{2}(4-x)(2x)(4-x) + (4-x)^2 x$ $= 2x^3 - 16x^2 + 32x$
3(b)	$0 < x < 4$
3(c)	$\underline{1.75}$
3(d)	

3(e)	<u>0.65</u>
3(f)	From the graph the maximum value is 18.9 , the volume cannot be greater than 18.9 cm^3 . Hence the volume of the solid cannot be <u>20 cm³</u> .
4(a)	<u>1.11π litres</u>
4(b)	<u>270 cm^2</u> (to 3 sf)
5(a)	<u>$T\left(3\frac{1}{5}, 1\frac{4}{5}\right)$</u>
5(b)	<u>$h = 3.54$</u> units (to 3 sf)
5(c)	<u>$y = -4.45$</u> or <u>$y = 8.05$</u>
6(a)	$DF = AF$ (given F midpoint of AD) $\angle BFD = \angle BFA = 90^\circ$ ($\perp$ bisector of chord) BF is common $\therefore$ triangle $ABF \equiv$ triangle DBF (SAS)
6(b)(i)	<u>45°</u>
6(b)(ii)	<u>45°</u>
6(b)(iii)	<u>135°</u>
6(b)(iv)	<u>67.5°</u>
7(a)	<u>140 m</u>
7(b)	<u>051.6°</u>
7(c)	<u>59.5°</u>
8(a)	<u>$3b - 2a$</u>
8(b)	<u>$\frac{3}{2}b - \frac{1}{2}a$</u>
8(c)	<u>$\frac{3}{2}a - \frac{3}{2}b$</u>
8(d)	<u>$2a - 2b$</u>
8(e)	<u>$\overrightarrow{CY} = \frac{4}{3}\overrightarrow{CX}$</u> <u>C is a common point</u> , C, X and Y are <u>collinear</u> .

8(f)	$\frac{2}{3}$
9(a)(i)	<u>56 minutes</u>
9(a)(ii)	<u>13.5 minutes</u>
9(a)(iii)	$\frac{11}{60}$
9(b)(i)	<u>School Y, on average, complete the task faster</u> than those from School X.
9(b)(ii)	Therefore, 88 minutes would be considered more unexpected for School X.
9(c)	Since both tasks have the <u>same IQR</u> , <u>the steepness of the curve will remain the same</u> , but the curve <u>will be shifted to the left</u> because of the <u>smaller median</u> .
10(a)	<u>\$409 600</u>
10(b)	Wait for another year: Net profit = <u>\$40 486</u>

