SECONDARY 4
2025 PRELIMINARY EXAMINATION

MATHEMATICS
Paper 2 **4052/02**

27 August 2025 (Wednesday) **2 hour 15 minutes**

CANDIDATE NAME	**Student's version**

CLASS	4	-			INDEX NUMBER	

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your full name, class and index number in the spaces above.
Write in dark blue or black pen in the space provided for each question.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question, it must be shown in the space below the question.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

For Examiner's Use		
Q1	10	
Q2	12	
Q3	10	
Q4	11	
Q5	10	
Q6	9	
Q7	10	
Q8	8	
Q9	10	
Total	**90**	

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer all the questions.

N1 (N1.7 & N1.8) : Numbers and Operations [10]
N2 (N2.2 & N2.4) : Ratio and Proportion
N3 (N3.4) : Percentage

1. **(a)** **(i)** The number of electric cars produced in country *Y* by company *X* in 2015 was 3 857 000. Write this number in standard form.

Answer
3.857×10^6

[1]

(ii) In 2005, 2.96 million electric cars were produced in country *Y*.
Calculate the percentage increase in the car production from 2005 to 2015.

Answer
Percentage increase

$= \frac{3.857\times10^6 - 2.96\times10^6}{2.96\times10^6} \times 100\%$

Show computation for percentage increase

$= 30.3\%$ (3 significant figures)

(iii) In 2015, 84.7% of all cars produced in country *Y* were non-electric cars.
Calculate the total number of cars produced in 2015, giving your answer to the nearest million.

Answer
Total cars in 2015

$= \frac{3.857\times10^6}{100-84.7} \times 100$

$= 2.5209 \times 10^7$

= 25 million cars (nearest million)

Or 25 000 000

(iv) In 2015, there were 420 cars per 1000 people in country *Y*.
Estimate the population of the country in 2015.
Give your answer in standard form correct to **three significant figures**.

Answer
Population

$= \frac{2.5209\times10^7}{420} \times 1000$

$= 6.00 \times 10^7$ (3 significant figures)

(b) A design team creates a scale model for a museum exhibit.
The diameter of an asteroid is 253 km.
In a scale model, the **diameter** of the asteroid is 11 m.

(i) Find the scale used for the model.
Give your answer in the form 1 : *n*.

Answer

$11\ m : 253\ km$

$1\ m : 23\ km$

$\underline{1 : 23\ 000}$

Be careful with conversion to km.

(ii) Drawn to the same scale as the asteroid, the diameter of the asteroid's moon is 1.2 m.
Find the **actual surface area of the moon**, leaving your answer in terms of $\boldsymbol{\pi}$.
Assume that the moon is spherical.

Answer

$1\ m : 23\ km$

$1.2\ m : 27.6\ km$

Actual surface area

$= 4\pi\left(\frac{27.6}{2}\right)^2$

$= 761.76\pi\ \text{km}^2$

Surface area of a sphere = $4\pi r^2$
From page 2

N7 (7.6, 7.7, 7.8) : Equations and Inequalities [12]

2. **(a)** Solve $7(x-2)-3(2x-1)=8(3x+5)-6$.

Answer

$7x-14-6x+3=24x+40-6$

$x=-\frac{45}{23}=-1\frac{22}{23}$

Based on context, Answer must be exact and shown as mixed number

(b) Solve the inequality $-3x+4<-2x-6$.

Answer

$$-x<-10$$
$$x>10$$

(c) It is given that

$$p=\frac{r^2q-2}{q-r^2}\ .$$

(i) Find p when $r=\frac{1}{2}$ and $q=-1$.

$$p=\frac{\left(\frac{1}{2}\right)^2(-1)-2}{(-1)-\left(\frac{1}{2}\right)^2}=\frac{-9/4}{-5/4}=\frac{9}{5}=1\frac{4}{5}$$

Or 1.8

Show substitution of values

(ii) Express r in terms of p and q.

$$p(q-r^2)=r^2q-2$$
$$pq-pr^2=r^2q-2$$
$$pr^2+r^2q=pq+2$$
$$r^2(p+q)=pq+2$$
$$r^2=\frac{pq+2}{p+q}$$

This is a quadratic relationship.

$$r=\pm\sqrt{\frac{pq+2}{p+q}}$$

(d) Solve the equation $\frac{x+3}{2x^2-8x+6}+\frac{5x}{3-x}=\frac{2}{x-1}$.

Give your solutions correct to **two decimal places.**

Answer

$$\frac{x+3}{2(x-3)(x-1)}+\frac{5x}{3-x}=\frac{2}{x-1}$$

$$\frac{x+3}{2(x-3)(x-1)}-\frac{5x}{x-3}=\frac{2}{x-1}$$

$$\frac{x+3}{2(x-3)(x-1)}-\frac{5x(2)(x-1)}{2(x-3)(x-1)}=\frac{2(2)(x-3)}{2(x-3)(x-1)}$$

$$x+3-10x(x-1)=4(x-3)$$

$$10x^2-7x-15=0$$

$$x=\frac{-(-7)\pm\sqrt{(-7)^2-4(10)(-15)}}{2(10)}$$

$x=1.62$ (2 d.p.). or -0.92 (2 d.p.)

Note:

$2x^2-8x+6$

$= \mathbf{2}\ (x-3)(x-1)$

Always check back when using the calculator to factorise.

$3-x=-(x-3)$

N5: finding the value of an unknown quantity in a given formula
N7: solving fractional equations that can be reduced to quadratic equations, formulating equations to solve problems [10]

3. Two water taps, X and Y, together can fill a tank in 6 hours.
Tap X takes x hours to fill the tank alone.
Tap Y takes 25 hours less than the Tap X to fill the tank alone.

(a) Write down an expression, in terms of x, for the fraction of the tank that Tap X will fill up in **one hour**.

$$\frac{1}{x}$$

(b) Write an expression, in terms of x, for the fraction of the tank that Tap Y will fill up in one hour.

$$\frac{1}{x-25}$$

(c) Write down an equation to represent the rate of filling the tank by the 2 taps and show that it reduces to $x^2 - 37x + 150 = 0$.

$$\boxed{\frac{1}{x} + \frac{1}{x-25} = \frac{1}{6}}$$

Equation that represents given context.

$$\frac{x + x - 25}{x(x-25)} = \frac{1}{6}$$

$$6(2x - 25) = x(x - 25)$$

$$12x - 150 = x^2 - 25x$$

$$x^2 - 25x - 12x + 150 = 0$$

$$x^2 - 37x + 150 = 0 \textit{ (shown)}$$

(d) Solve the equation $x^2 - 37x + 150 = 0$.

$x^2 - 37x + 150 = 0$

$$x = \frac{-(-37) \pm \sqrt{(-37)^2 - 4(1)(150)}}{2(1)}$$

Must show substitution of values.

$x = 32.365$ or $x = 4.634$

$x = 32.4$ or $x = 4.63$ (3 s.f.).

(e) Calculate how long it would take to fill the tank from empty using Tap *Y*.
Give your answer in hours and minutes, correct to the nearest minute.

Reject $x = 4.63$ as the time taken,
$x - 25$, cannot be negative.

Important to test both values and justify any rejection of solution.

Time Tap *Y* takes to fill up the tank completely
$= 32.365 - 25$
$= 7.365$

= 7 hours 22 minutes (nearest minute)

N6 (6.10) : Functions and Graphs [11]
N7 (N7.2) : Equations and Inequalities

4. **(a)** Complete the table of values for $y = \frac{x^2}{7} + \frac{2}{x} - 5$.
Values are given to one decimal place where appropriate.

x	0.2	0.5	1	2	3	4	5	6
y		-1.0	-2.9	-3.4	-3.0	-2.2	-1.0	0.5

[1]

Answer
5.0 (1 d.p.)

(b) On the grid opposite, draw the graph of $y = \frac{x^2}{7} + \frac{2}{x} - 5$ for $0 < x \leq 6$. [2]

Answer
[points plotted correctly]
[smoothness of curve **and** label]

(c) Use your graph to write down an inequality in x to describe the range of values where $y < -2$.

Answer
$0.7 < x < 4.2$ (± 0.1)

(d) **(i)** On the same grid, draw the graph of $2y + 4x = 9$ for $0 < x \leq 6$. [2]

Answer
[at least 3 plotted points]
[graph **and** label]

(ii) Write down the x-coordinates of the points where the **line intersects the curve**.

Answer
0.2, 3.55 (± 0.1)

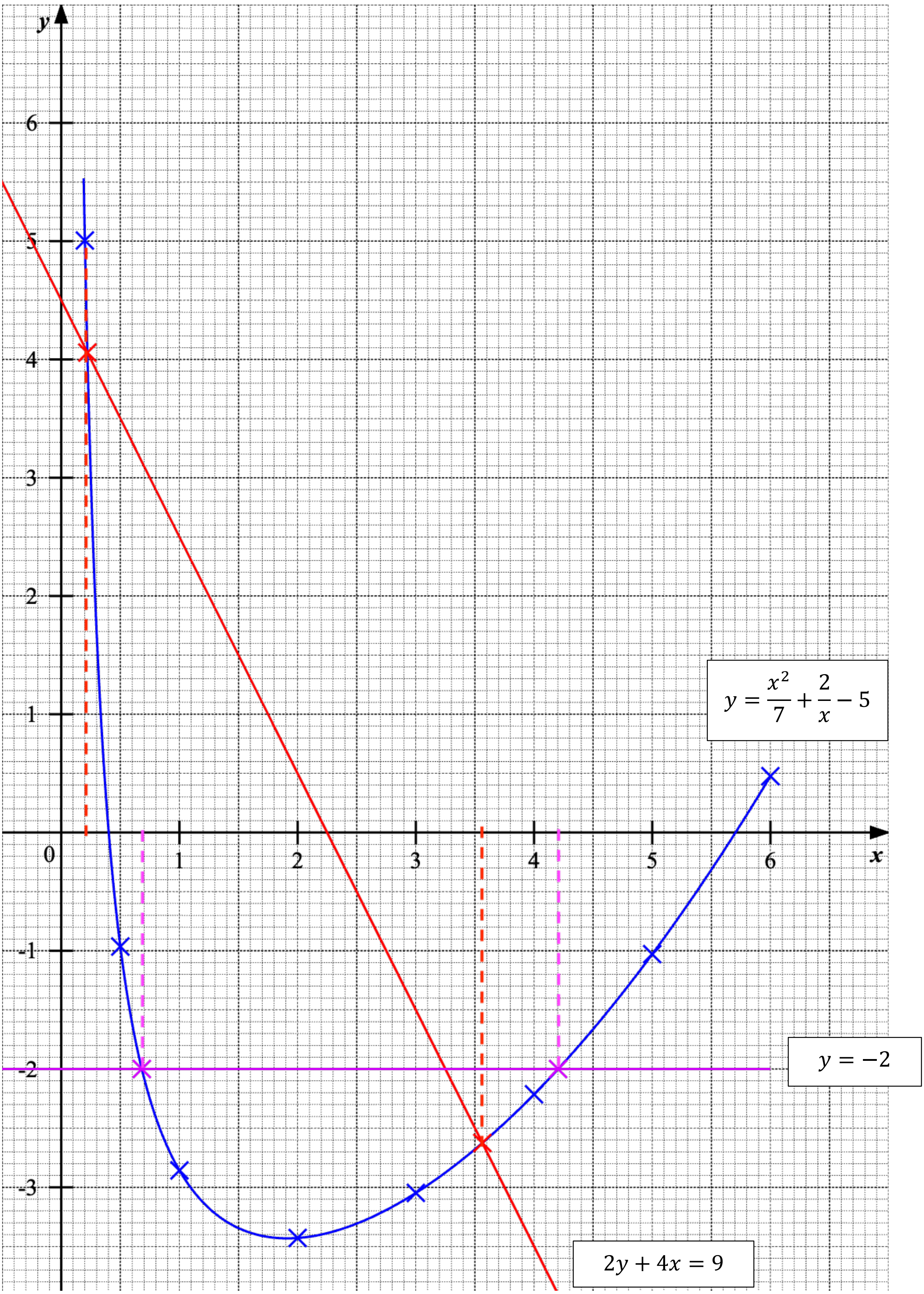
(iii) These values of x are the solutions of the equation $2x^3 + Ax^2 - Bx + 28 = 0$.
Find the value of A and the value of B.

$\frac{x^2}{7} + \frac{2}{x} - 5 = -2x + \frac{9}{2}$ [equate both functions]

$2x^3 + 28 - 70x = -28x^2 + 63x$

$2x^3 + 28x^2 - 133x + 28 = 0$

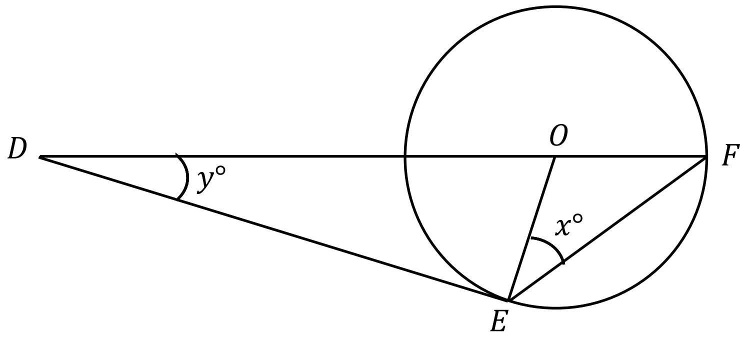
$A = 28$, $B = 133$



FYI: Creating of graphs website

G3 (3.6, 3.8, 3.9) : Properties of Circle [10]

5. **(a)**



E and F are points on the circle, centre O.
DOF is a straight line and DE is a tangent at E.
Angle $FEO = x°$ and angle $EDO = y°$.

(i) Show that $y = 90 - 2x$.
State all the reasons clearly.
Answer

$\angle OFE = x°$ (Base angles of isosceles triangle since OE and OF are radii of same circle.)

$\angle OED = 90°$ (Tangent to a circle)

$y + 90 + x + x = 180$ (Angles sum of triangle)

$\therefore y = 90 - 2x$ (Shown)

(ii) Given that $y + x = 60$, find angle EOD.

$y = 90 - 2x$ …(1)
$y + x = 60.$ …(2)
Substitute (1) into (2) :

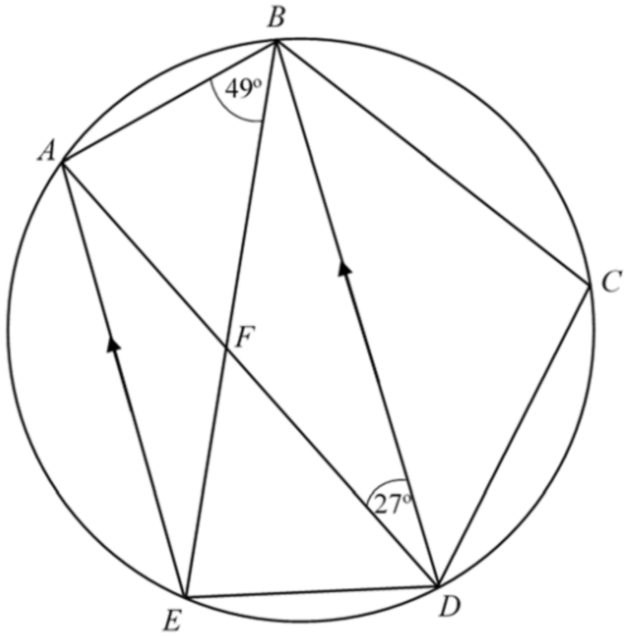
$90 - 2x + x = 60$
$x = 30, y = 30$
$\therefore \angle EOD = 90° - y°$
$\therefore \angle EOD = 90° - 30° = 60°$

\

(b) The diagram shows a circle that passes through A, B, C, D and E.

[Turn over

AFD and BFE are straight lines.
EA and DB are parallel.
Angle $ABE = 49°$ and angle $ADB = 27°$.



Find, give a reason for each step of your working,

(i) angle AFE,

$\angle AEB = \angle ADB = 27°$
(Angles in the same segment)
$\angle EAD = \angle ADB = 27°$
(Alternate angles, BD parallel AE)
$\angle AFE + \angle AEB + \angle EAD = 180°$
(Angles sum of triangle)

$\angle AFE = 126°$

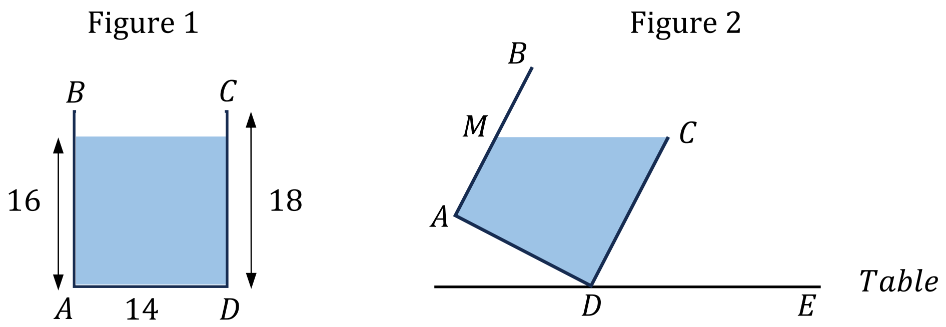
(ii) angle BCD.

$\angle AFE = \angle DAB + \angle ABE$ (Exterior angle)
$\angle DAB = 126° - 49° = 77°$

(Opposite angles of a cyclic quadrilateral)
$\angle BCD = 180° - \angle DAB$
$= 180° - 77°$
$= 103°$

G5 (5.1) : Mensuration [4]
G4 (4.3) : Trigonometry and Pythagoras Theorem
G6 (6.1) : finding the gradient of a straight line given the coordinates of two points on it [5]

6. (a)



The diagram (Figure 1) shows one side *ABCD* of a square-based prism.
Each side of the base is 14 cm. The height of the container *DC* is 18 cm.
The prism is placed on a table and filled with water until the height of the water reaches 16 cm.
Figure 2 shows the square-based prism tilted until the water level is at *MC*.
During this process of tilting, some water is spilled out.

Given that *M* is the mid-point of *AB*, find

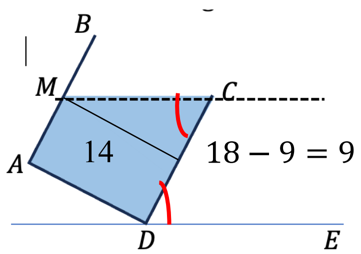
(i) the amount of water that was spilled out from the square-based prism during the tilt.

Amount of water spilled
$= 14^2 \times 16 - \frac{1}{2}(9 + 18) \times 14 \times 14$
$= 3136 - 2646$
$= 490\ cm^3$

Context: it is about volume NOT area.

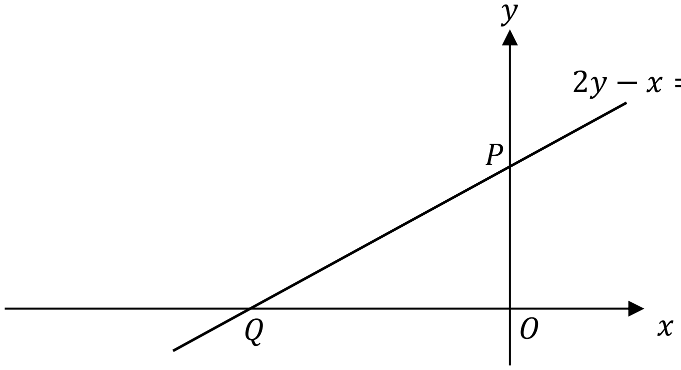
(ii) angle CDE, the angle between the square-based prism and the table.

$\angle MCD = tan^{-1}\left(\frac{14}{18-9}\right) = 57.264°.$
$\angle CDE = \angle MCD = 57.3°$ (1 d.p.)
(Alternate angles, <u>*MC* parallel *DE*</u>.)



Context: water level is parallel to the surface of the table.

(b) The equation of the line, L, is $2y - x = 12$.
It cuts the x-axis and y-axis at Q and P respectively.



(i) Write down the gradient of PQ,

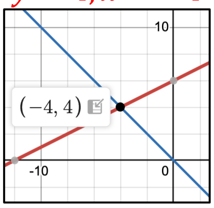
$$2y - x = 12$$
$$y = \frac{1}{2}x + 6$$

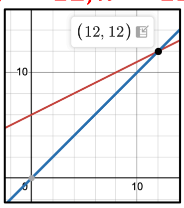
Is gradient equivalent to tangent in this context?

Gradient of $PQ = \frac{1}{2}$

(ii) A point K lies on the line L, and is equidistant from x-axis and y-axis.
Find the coordinates of K.

2 possibilities

<table>
<tr>
<td>Since point is equidistant from x-axis and y-axis, therefore $x = -y$.
From
$2y - x = 12$
$2y + y = 12$
$y = 4, x = -4$


$K = (-4\,, 4)$</td>
<td>Since point is equidistant from x-axis and y-axis, therefore $x = y$.
From
$2y - x = 12$
$2y - y = 12$
$y = 12, x = 12$


$K = (12\,, 12)$</td>
</tr>
</table>

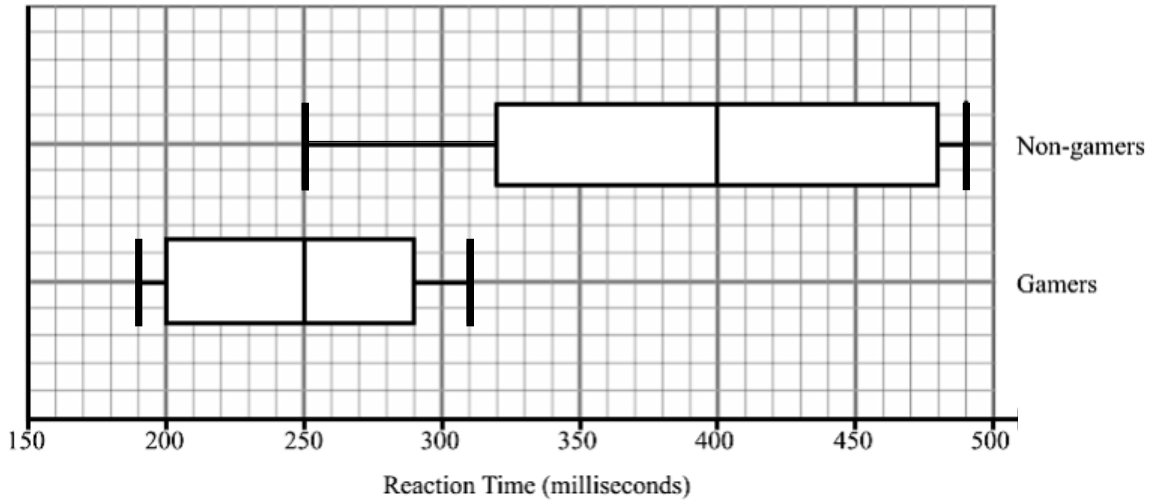
(iii) Find acute angle QPO.

$$\tan\angle QPO = \frac{OQ}{OP} = \frac{12}{6} = 2$$

$acute\ \angle QPO = 63.4°$(to 1 d.p.)

S1(S1.11) : Data Handling and Analysis [4]
S2 (S2.3) : Probability [6]

7. **(a)** A study records the reaction times of gamers and non-gamers.
The box-and-whisker plots show the distributions of the results.



(i) There are 15 non-gamers who had a reaction time slower than 480 milliseconds.

Work out the number of non-gamers in the study.

Answer
$15 \times 4/3 = 20$

(ii) Make a comment comparing the averages and a comment comparing the distributions of the times taken by the gamers and non-gamers.
Use figures to support your answers.
Answer
Gamers have a lower *median* of 250 milliseconds compared to 400 milliseconds for non-gamers. Hence, gamers generally had faster reaction times.

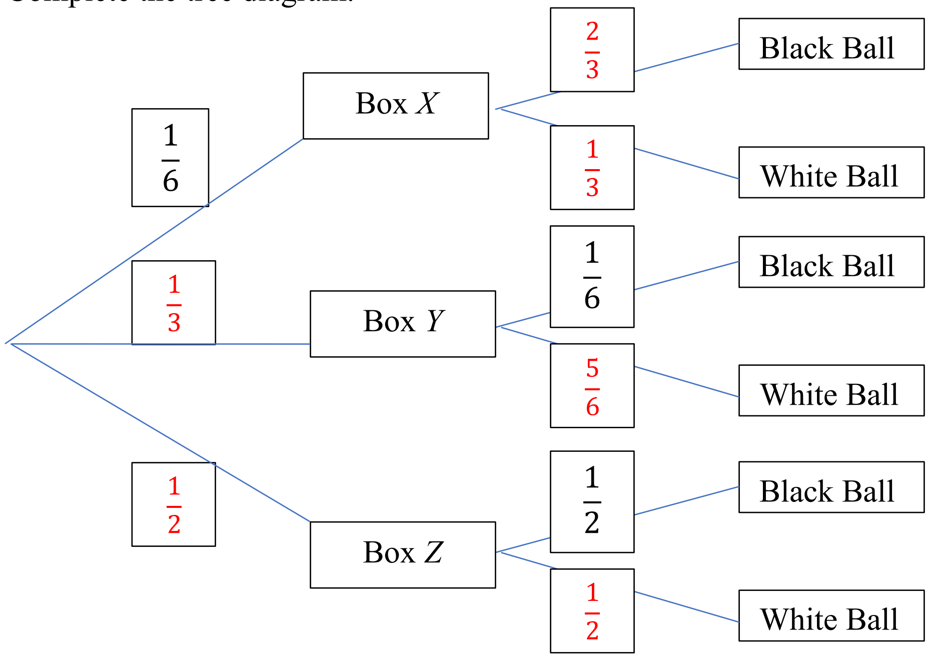
The *interquartile range* for gamers of 290 – 200 = 90 milliseconds is smaller than that for non-gamers of 480 – 320 = 160 milliseconds. Hence, non-gamers had a wider spread in reaction times, showing less consistency.

(b) Three boxes, X, Y and Z contain the following balls.
Box X : 2 black balls and 1 white ball.
Box Y : 1 black ball and 5 white balls.
Box Z : 2 black balls and 2 white balls.
A ball is picked at random from a box.
The box is chosen by throwing a die and applying the following rules.

1. If the outcome on the die is 6, box X will be chosen.
2. If the outcome on the die is a 4 or a 5, box Y will be chosen.
3. If the outcome on the die is a 1, a 2 or a 3, box Z will be chosen.

If a black ball is obtained, a prize of \$10 is awarded.
There will be a \$2 fee to play the game.

(i) Complete the tree diagram.



(ii) Find the probability of winning the \$10 prize.

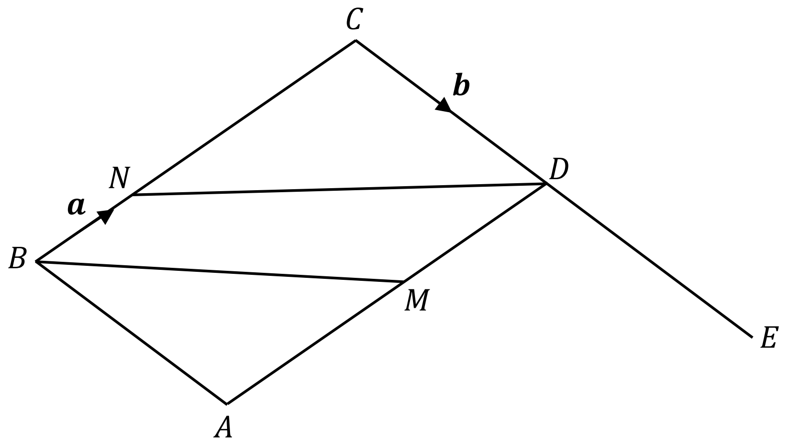
$$\frac{1}{6}\times\frac{2}{3}+\frac{1}{3}\times\frac{1}{6}+\frac{1}{2}\times\frac{1}{2}=$$
$$\frac{5}{12}$$

(iii) Find the amount gained or lost after playing 12 games.

5 wins in 12 games gains \$50
Fee incurred = \$2 (12) = \$24
Amount earned =\$50 - \$24
= *\$26 gained*

G7 (7.2, 7.4, 7.6) : Vectors in 2-dimensions [9]

8. $ABCD$ is a parallelogram, and E lies on CD produced such that $CD : CE = 1 : 2$.
M is the midpoint of AD.
N is a point on BC such that $BN : BC = 1 : 4$.
$\overrightarrow{BN} = \boldsymbol{a}$ and $\overrightarrow{CD} = \boldsymbol{b}$.



(a) Express, as simply as possible, in terms of **a** and/or **b**,

(i) $\overrightarrow{AM}$,

$$\overrightarrow{AM} = \frac{1}{2}\overrightarrow{AD} = \frac{1}{2}\overrightarrow{BC} = \frac{1}{2}(4\boldsymbol{a})$$

(ABCD is a parallelogram)

$$\overrightarrow{AM} = 2\boldsymbol{a}$$

Correct vector notation is crucial because it ensures clarity, precision, and the accurate representation of quantities with both magnitude and direction, preventing confusion between vectors and scalars and enabling correct calculations and understanding in physics, mathematics, and computer graphics.

(ii) $\overrightarrow{BM}$,

$$\overrightarrow{BM} = \overrightarrow{BA} + \overrightarrow{AM}$$

$$\overrightarrow{BM} = 2\boldsymbol{a} + \boldsymbol{b}$$

(iii) $\overrightarrow{BE}$.

$$\overrightarrow{BE} = \overrightarrow{BC} + \overrightarrow{CE}$$

$$\overrightarrow{BE} = 4\boldsymbol{a} + 2\boldsymbol{b}$$

(b) Explain why B, M and E lie on a straight line.

Answer [2]

$$\overrightarrow{BM} = 2\boldsymbol{a} + \boldsymbol{b}$$

$$\overrightarrow{BE} = 4\boldsymbol{a} + 2\boldsymbol{b} = 2(2\boldsymbol{a} + \boldsymbol{b}) = \mathbf{2}\overrightarrow{BM}$$

Since $\overrightarrow{BE} = \mathbf{2}\overrightarrow{BM}$ and B is a **common point**, B, M and E lie on a straight line. (Shown)

A proper statement and logical justification is important for such questions.

(c) Find the numerical value of

(i) $\dfrac{\text{Area of triangle } AMB}{\text{Area of triangle } DCN}$,

ABCD is a parallelogram, let h be the perpendicular height between AD and BC.

$$\frac{\text{Area of triangle } AMB}{\text{Area of triangle } DCN} = \frac{\frac{1}{2}\times h\times AM}{\frac{1}{2}\times h\times NC}$$

$$= \frac{AM}{NC} = \mathbf{\frac{2}{3}}$$

(ii) $\dfrac{\text{Area of triangle } EDM}{\text{Area of } DCBM}$.

$$\frac{\text{Area of triangle } EDM}{\text{Area of triangle } ECB} = \left(\frac{1}{2}\right)^2 = \mathbf{\frac{1}{4}}$$

$$\frac{\text{Area of triangle } EDM}{\text{Area of } DCBM} = \frac{1}{3}$$

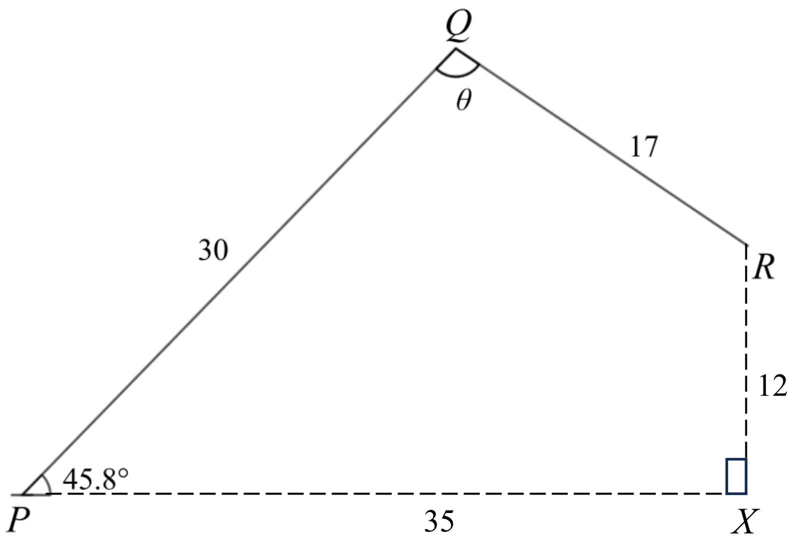
PRWC [10]

9. A packing company uses robotic arms to pick up trays from a moving conveyor belt.

A robotic arm consists of two segments connected by joints.
The length of the upper arm PQ is 30 cm.
At P, the shoulder joint is fixed at an angle of 45.8°.

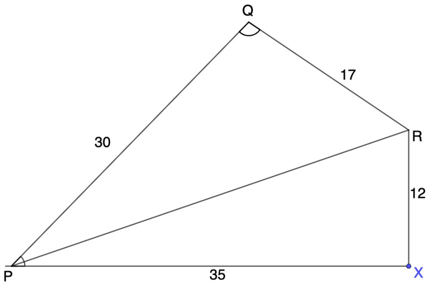
The length of the forearm QR is 17 cm.
At Q, the elbow joint rotates at an angle θ.

The robotic arm picked a tray at point R, which lies 35 cm horizontally and 12 cm vertically above X.



(a) Calculate angle θ, that allows for the arm to pick the tray at R.

Answer



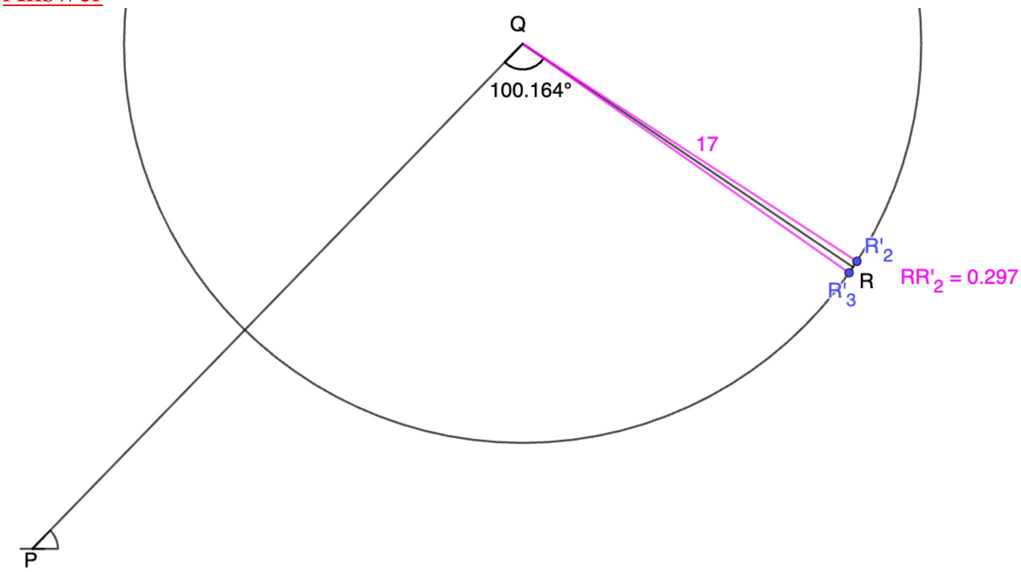
$PR = \sqrt{35^2 + 12^2} = 37$ cm

$\angle PQR = \cos^{-1}\left[\frac{30^2+17^2-37^2}{2(30)(17)}\right]$

$\theta = 100.2°$ (1 d.p.)

(b) Due to calibration errors, there may be a 1° change in θ.
Show that a 1° change in θ will lead to 0.297 cm difference from the intended position of R.

Answer [1]



Arc length

$= \frac{1°}{360°} \times 2\pi(17)$

$= 0.297$ cm (3 significant figures)

(c) The company is deciding whether to adopt a new system that uses camera feedback to calculate the joint angle θ in real time.

They are considering two new systems B and C.

System	**Description**
System A (Original)	Uses pre-programmed joint angle θ. There is a 40% chance that a 1° error in θ will occur, which causes the arm to not be able to pick up the tray.
System B (Real-time)	Uses real-time camera feedback to recalculate θ for every tray. The angle is always accurate.
System C (Hybrid)	Uses pre-programmed joint angle θ. When a 1° error in θ is detected, it uses real-time camera feedback to recalculate the angle θ.

Additional notes:

- Using the original system, the robot handles 300 trays per hour.
- Each successful tray earns the company \$0.50 revenue.
- A 1° error in θ leads to a miss, as the arm cannot pick up the tray.
- Each missed tray incurs a \$0.30 cost.
- Using the camera feedback introduces a 0.4 second delay per tray.
- There is a fixed monthly cost of \$12 000 for using the camera feedback.
- The company operates 160 hours per month.

Determine which system the company should adopt.
Justify the decision you make and show your calculations clearly.

Answer

	A (Original)	B (Real-time)	C (Hybrid)
Trays/hour	Number of missed trays $= 40\% \times 300$ $= 120$ Number of successful trays $= \frac{60}{100} \times 300$ $= 180$ [M1]	Original time taken for one tray $= \frac{3600}{300} = 12$ s New time taken for each tray (includes delay) $= 12.4\ s$ Number of successful trays $= \frac{3600}{12.4}$ $= 290$ (nearest whole number) [M1]	Similar to A Successful trays $= 180$ Total time taken for successful trays $= \frac{3600}{300} \times 180$ $= 2160$ s Includes B Number of trays picked up after recalculation. $= \frac{3600 - 2160}{12.4}$ $= 116$ (nearest whole number) Total number of trays $= 180 + 116$ $= 296$ [M1]
Revenue/hour	$180 \times \$0.50$ $= \$90$	$290 \times \$0.50$ $= \$145$	$296 \times \$0.50$ $= \$148$
Missed cost/hour	$120 \times \$0.30$ $= \$36$	\$0	\$0
Camera cost/hour	\$0	$\frac{\$12000}{160} = \$75$	$\frac{\$12000}{160} = \$75$
Net profit/hour	$\$90 - \$36 = \$54$	$\$145 - \$75$ $= \$70$	$\$148 - \$75$ $= \$73$
Monthly profit (160 hour / month)	$\$54 \times 160$ $= \$8640$ [M1]	$\$70 \times 160$ $= \$11200$ [M1]	$\$73 \times 160$ $= \$11680$ [M1]

Since System C generates the most profit, the company should adopt System C.

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