NORTH VISTA SECONDARY SCHOOL
End-of-Year Examination 2024
Secondary 3 Express

	90

CANDIDATE NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS **4049**

2 OCTOBER 2024

2 hour 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your centre number, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use	
Category	**Question No.**
Accuracy	
Brackets	
Fractions	
Units	
Others	
Marks Deducted	

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\text{Area of } \Delta = \frac{1}{2} bc \sin A.$$

1 Solve the simultaneous equations

$$\left(9^x\right)\left(27^y\right) = 1,$$

$$\frac{a^{2x}}{\left(a^{y+2}\right)^3} = \frac{1}{a^2}.$$

[4]

2 **(a)** Given that the graph of $y = mx^2 - 5x + 2$ lies entirely above the x-axis, find the range of values of m. [2]

(b) Find the range of values of x for which $0 < x^2 - 4x \leq 2x + 16$. [4]

3 In a simplified prey-predator model, fifty wolves were deliberately introduced to an island to curb the population of wild rabbits. The remaining population of rabbits, R, was given by $R = 300 + 6000\left(10^{-0.02t}\right)$ where t represented the number of days since the introduction of wolves.

(a) State the initial number of rabbits. [1]

(b) Find how long it would take before the population of rabbits dropped below half the original population. Round your answers to the nearest day. [3]

(c) Determine, with reason, whether the number of rabbits would become zero on the island in the long term. [2]

4 **(a)** **(i)** Sketch the graph of $y = 3\ln(x-1)$, clearly showing the asymptote(s) and the intercept(s) of the graph. [3]

(ii) Hence, determine the equation of the straight line needed to be drawn to obtain a graphical solution of the equation $x - 1 = e^{\frac{1-x}{3}}$. [2]

(b) Find the value of the constant p such that $100^x - e^{x\ln p} = 0$. [2]

5 **(a)** Simplify $\left(\frac{5}{2\sqrt{3}-2}+\frac{\sqrt{3}-3}{2+2\sqrt{3}}\right)\times\frac{4}{\sqrt{27}}$, giving your answer in the form $\frac{h+k\sqrt{3}}{9}$ where h and k are integers. [4]

(b) Given that $3^{2x+3}\times5^{2x-3}=27^{x}\times125^{x}\times2^{x}$, find the value of 30^{x}. [3]

6 By considering the general term in the binomial expansion of $\left(x^2+\dfrac{2}{x^3}\right)^{11}$, determine whether the ratio of the coefficient of x^7 term to that of x^3 term exists. [4]

7 **(a)** Factorise $3x^3 - 24y^3$ completely. [2]

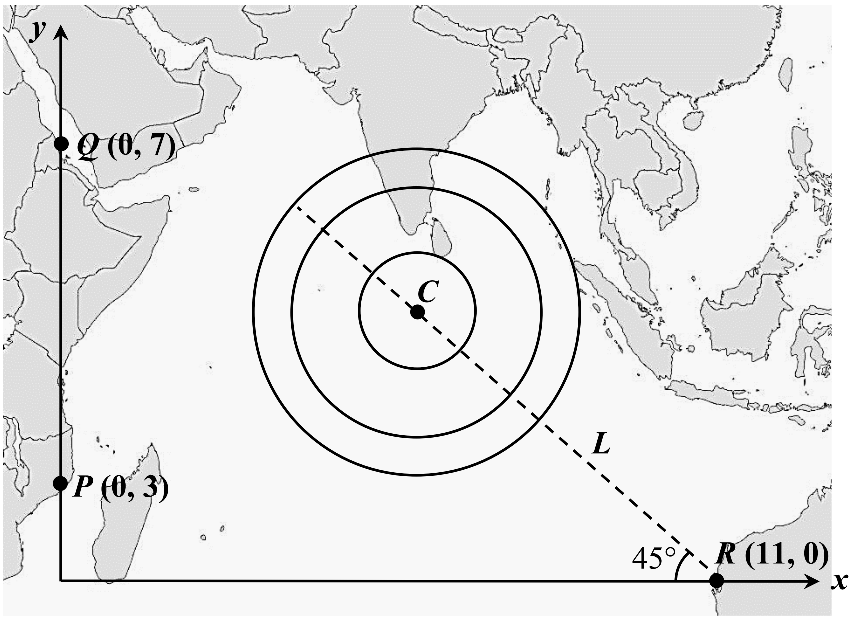
(b) Express $\dfrac{3-10x-x^3}{\left(x^2+1\right)\left(x-2\right)}$ in partial fractions. [6]

8 **(a)** A circle has equation $x^2 + y^2 - 8x + 6y - 4 = 0$.

Showing your working clearly, determine whether a point $T\,(5,\,1)$ lies inside, outside or on the circle. [3]

(b) The diagram shows a simplified map of the Indian Ocean.

Geological stations $P(0,3)$, $Q(0,7)$ and $R(11,0)$ detected an earthquake and a geologist is attempting to locate the epicentre C of the earthquake.



Instruments at P and Q detected the earthquake at the same time, indicating that the epicentre C is equidistant from P and Q. Instruments at R detected it a few seconds later in the direction as indicated by the line L in the diagram above.

The line L makes an angle of 45° with the x-axis.

8 **(i)** Show that the line L can be represented by the equation $x + y = 11$. [2]

(ii) Calculate the coordinates of C. [2]

(iii) **Hence**, find the equation of the circle passing through P and Q with centre C. [2]

(iv) Explain whether it is possible for a circle with centre C to pass through all three points, P, Q and R. [1]

9 The equation of a polynomial is given by $f(x) = 3x^3 + x^2 + 11x - 35$.

(a) Find the remainder when $f(x)$ is divided by $2x + 1$. [2]

(b) Show that $3x - 5$ is a factor of $f(x)$. [1]

(c) Show that the equation $f(x) = 0$ has only one real root. [4]

(d) Using your answers to part **(b)** and **(c)**, solve the equation $3^{3y+1} + 3^{2y} = 35 - 11(3^y)$. [4]

10 In a study of radioactive decay, the amount of radioactive substance N decreases over time according to the formula $N(t) = N_0 e^{-kt}$, where $N(t)$ is the amount of substance remaining at time t, N_0 is the initial amount of substance at $t = 0$, and k is the decay constant. Values of $N(t)$ for different values of t have been collected.

(a) Explain how a straight-line graph can be drawn to represent the formula, and state how the values of N_0 and k could be obtained from the line. [3]

10 **(b)** The table shows experimental values of two variables, x and y, which are connected by an equation of the form $xy = ax + b$, where a and b are constants.

x	1.0	2.0	3.0	4.0	5.0	6.0
y	2.77	1.75	1.40	1.21	1.10	1.04

(i) On the grid next page, plot xy against x and draw a straight line graph to illustrate the information. [4]

(ii) **From your graph**, estimate the value of a and of b. [2]

11 It is given that $f(x) = 4 \sin x$ and $g(x) = 2\cos\left(\frac{x}{3}\right) + 2$.

(a) Find the amplitude and period of $f(x)$ and $g(x)$. [4]

(b) Sketch, on the same diagram, the curves $f(x) = 4 \sin x$ and $g(x) = 2\cos\left(\frac{x}{3}\right) + 2$ for $0° \leq x \leq 360°$. [4]

(c) State the number of solutions of the equation $4\sin x - 2 = 2\cos\left(\frac{x}{3}\right)$ for $0° \leq x \leq 360°$. [1]

12 A ball is thrown vertically upwards. Its height h m, above the ground at time t seconds after being thrown is given by the formula $h = 1.75 + 5t - 5t^2$.

(a) State the height above the ground from which the ball is thrown. [1]

(b) Express h in the form of $a + b(t + c)^2$ where a, b and c are constants to be determined. [3]

(c) Hence state the maximum height attained by the ball and the time at which this occurs. [2]

12 **(d)** The ball hits the ground. Explain why the time taken for the ball to hit the ground is not twice the time found in part **(c)**. [1]

(e) Find the length of time for which the ball is at least 2 m above the ground. [2]

End of Paper