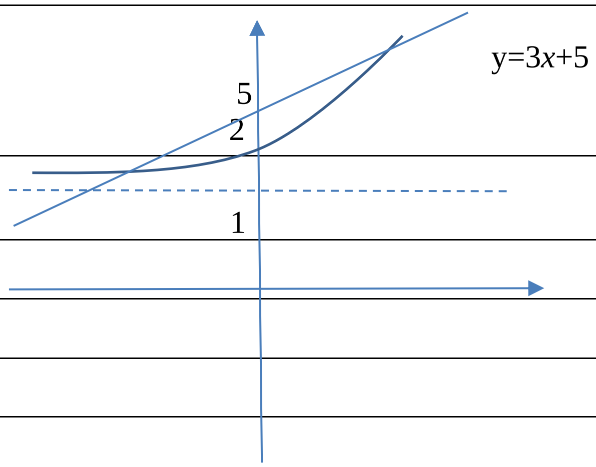
Marking Scheme

Q	Working	Marks
1	(i) Remainder = f(1)	
	$= -4$	[B1]
	(ii) Remainder = f(3) = 0	[B1]
	Since <u>remainder = 0</u>, $(x-3)$ is a factor of $f(x)$.	
	(iii) by long division, $f(x) = (x-3)(2x^2 + x - 1)$	[M2] for performing long division/ comparison method
	$= (x-3)(2x-1)(x+1)$	[A1]
	for $\boldsymbol{f(x) = 0}$, $(x-3)(2x-1)(x+1) = 0$	
	$x = 3$, $x = 0.5$, , $x = -1$	[A1] [- 1 for not writing $f(x) = 0$]
		[B1 for giving final answers only]
2	**(a)** $\sin A = \dfrac{\sqrt{55}}{8}$	[B1]
	(b) $\cos(-B) = \cos B$ $= -\dfrac{\sqrt{19}}{10}$	[B1]
	(c) $\tan A \tan B = \left(-\dfrac{\sqrt{55}}{3}\right)\left(-\dfrac{9}{\sqrt{19}}\right) = \dfrac{3\sqrt{1045}}{19}$	[M1, A1]
3	$2^{2+2x} + 2^{4+x} = 2^x + 4$	
	$2^2 \times 2^{2x} + 2^4 \times 2^x = 2^x + 4$	[M1]
	Let $2^x = u$	
	$4u^2 + 15u - 4 = 0$	[M1]
	$(4u-1)(u+4) = 0$	
	$u = \frac{1}{4}$ or $u = -4$	[M1] / [M1]
	$2^x = \frac{1}{4}$ [NO solution]	[- 1 for not writing no solution]
	$2^x = 2^{-2}$	
	$x = -2$	[A1]
4(a)		B1 (correct shape) B1 (indicate asymptote & y-intercept)

4	**(b)**	$x = 3\ln(3x+4)$	[M1]
		$\frac{x}{3} = \ln(3x+4)$ $e^{\frac{x}{3}} = 3x+4$ $e^{\frac{x}{3}} + 1 = 3x+5$	[M1]
		Therefore the equation of the line is $y = 3x+5$	[A1]
	(c)	Line drawn and labelled on graph showing y-intercept of 5	[M1]
		No of solutions = 2	[A1]
5	**(i)**	$b^2 - 4ac = 0$	[M1]
		$(-2)^2 - 4\,(3)\,(1-p) = 0$	
		$p = \frac{2}{3}$	[A1]
5	**(ii)**	$x^2 + y^2 = 10 \qquad y = 3x + k$	
		$x^2 + (3x + k)^2 = 10$	[M1]
		$10x^2 + 6kx + k^2 - 10 = 0$	[M1]
		$b^2 - 4ac < 0$	[M1]
		$-4k^2 + 400 < 0$	[M1]
		$4k^2 - 400 > 0$	
		$4(k+10)(k-10) > 0$	
		$k < -10$ or $k > 10$	[A1]
6	(a)	$T_{r+1} = \binom{7}{r}(x^2)^{7-r}\left(-\frac{3}{x}\right)^r$	[B1] – **no need to simplify** **
		$= \binom{7}{r}(x)^{14-2r}\,(-3)^r\left(\frac{1}{x}\right)^r$	
		power of $x = 14 - 3r$	[B1]
	(b)	Let $x^0 = x^{14-3r}$	
		$14 - 3r = 0$	
		$r = 4\frac{2}{3}$ r is not an integer hence the constant term is not valid in the expansion.	[M1]/FT [A1]

7	$\frac{1}{3} \times \left(\sqrt{2} + \sqrt{3}\right)^2 \times h =$	$22 + 9\sqrt{6}$	[M1]
	$\left(2 + 2\sqrt{6} + 3\right) h =$	$3\,(22 + 9\sqrt{6})$	[M1] for correct expansion
	h =	$\dfrac{3\,(22 + 9\sqrt{6})}{5 + 2\sqrt{6}}$	
	=	$\dfrac{3\,(22 + 9\sqrt{6})}{5 + 2\sqrt{6}} \times \dfrac{5 - 2\sqrt{6}}{5 - 2\sqrt{6}}$	[M1] / for rationalizing
	=	$6 + 3\sqrt{6}$------[A1]	[A1]
8	$\sqrt{3^{4x+8}} = 20^{2-2x}$.		
	$(3)^{2x+4} = 20^2 \times 20^{-2x}$		[M1] for changing square root to half /
	$(3)^{2x}\, 3^4 = \frac{20^2}{20^{2x}}$		[M1]
	$(3)^{2x} \times (20)^{2x} = \frac{20^2}{3^4}$		
	$(60)^{2x} = \frac{20^2}{3^4}$		[M1]
	$(60)^{x} = \frac{20}{9}$		[A1]
	<u>method 2</u>		
	$\sqrt{3^{4x+8}} = 20^{2-2x}$		
	$3^{4x+8} = \left(20^{2-2x}\right)^2$		[M1] for squaring 20^{2-2x}
	$(3)^{4x} \times (20)^{4x} = \frac{20^4}{3^8}$		[M1]
	$(60)^{4x} = \frac{20^4}{3^8}$		[M1]
	$(60)^{x} = \frac{20}{9}$		[A1]
9	**(i)**	$(x+4)^2 + (y-5)^2 = 81$	[M1] for standard form/formula mtd
		Centre (-4,5)	[A1]
		Radius = 9 units	[A1]
	(ii)	$m_{\tan} = \dfrac{3}{4}$	
		$m_{PQ} = -\dfrac{4}{3}$	[M1] either one of the gradient
		Equation of PQ: $y = -\dfrac{4}{3}x + 6$	[A1] equation of line

		$-\frac{4}{3}x+6=\frac{3}{4}x-\frac{26}{4}$ $\frac{25}{12}x=\frac{25}{2}$ $x=6$		[M1] substitution [A1] coordinates of Q
		Q (6,-2)		
	(iii)	$M_{PQ}=(3,2)$ Radius = $\sqrt{(0-3)^2+(6-2)^2}=5$ units		[B1] centre [M1, A1]
	(iv)	Distance between point and $C_1=\sqrt{(2+4)^2+(6-5)^2}=6.082$ Since < 9 units therefore lie in C_1 Distance between point and $C_2=\sqrt{(2-3)^2+(6-2)^2}=4.123$ Since < 5 units therefore lie in C_2		[B1] [B1]
10	**(a)**	$\log_2 2x-\log_4(x+2)=1$		
		$\log_2 2x-\frac{log_2(x+2)}{log_2 4}$	$=1$	[M1] for changing base
		$2\log_2 2x-\log_2(x+2)$	$=2$	[M1] for simplifying
			$log_2\left(\frac{4x^2}{x+2}\right)=2$	[M1] / No FT if student write $log_2\left(\frac{2x^2}{x+2}\right)=2$
			$4x^2=4(x+2)$	
			$x^2-x-2=0$	[M1]
			$(x-2)(x+1)=0$	
			$x=2$ or $x=-1$(rejected)	[A1] / No A1 if x = -1 is not rejected
	(b)	$\log_z 1000=\lg z$ $\frac{\lg 1000}{\lg z}=\lg z$ $(\lg z)^2=3$ $\lg z=\pm\sqrt{3}$ $\lg z=\sqrt{3}$ $z=10^{\sqrt{3}}=54.0$	$\lg z=-\sqrt{3}$ $z=10^{-\sqrt{3}}=0.0185$	[M1] use of change of base [M1] for simplifying [M1] solve for lgz [A1,A1] for answers
	(c)	(i)	*P = 15000*	*[B1]*
		(ii)	$28000=15000e^{bt}$	[M1]

			$\frac{28}{15} = e^{3(b)}$	[M1]
			$3b = \ln\frac{28}{15}$ $(= 0.624154)$	
			$b=\frac{1}{3}\ln\frac{28}{15} = 0.208051$	
			$P = 15000e^{bt}$	
			$P = 15000e^{(7.75\,x\ \times\ \frac{1}{3}\ln\frac{28}{15})}$	[M1] for substituting b and t = 7.75/ FT
			≈ 75200 (3sf)	[A1]/ FT

11	$\dfrac{x^2+10}{(2x^2+1)(3-x)} = \dfrac{Ax+B}{(2x^2+1)} + \dfrac{C}{(3+x)}$		
	$x^2 + 10 = (Ax + B)(3 + x) + C(2x^2 + 1)$		[M1]
	when $x = -3$,		
	$9 + 10 =$	$C(18 + 1)$	
	$C =$	1	[M1]/FT
	when $x = 0$,		
	$8 =$	$(B)(3) + 1(-1)$	
	$9 =$	$3B$	
	$B =$	3	[M1]//FT
	When $x = 1$,		
	$11 =$	$(A + 3)(4) + 1(3)$	
	$8=$	$4A + 12$	
	$A =$	-1	[M1]/FT
	$\dfrac{x^2+8}{(2x^2+1)(3+x)} = \dfrac{3-x}{(2x^2+1)} + \dfrac{1}{(3+x)}$		[A1]

12	**(i)**	Scale [B1], Points [B1], Best fit line[B1]	
	For (ii) to (iv), answers obtained by calculation are not accepted.		
	(ii)	$\ln N = 3.9$ $N = e^{3.9} = 49.4 = 49$ Accept $44 \le N \le 60$	[M1] vertical intercept value [A1] N value rounded to whole number
		$\ln N = 3.8$ $N = e^{3.8} = 44.7 = 45$ | $\ln N = 4.1$ $N = e^{4.1} = 60.34 = 60$	
	(iii)	$\ln N = \ln N_0 + kt$	[M1] form equation
		$k = \dfrac{16-8}{22-7.5} = 0.552$ Accept $0.500 \le k \le 0.600$	[A1]
	(iv)	Accepted a range of answers for $\ln N$: $\ln((44.7)(3)) = 4.898$ $\ln((49.4)(3)) = 4.998$ $\ln((54.5.)(3)) = 5.096$ $t = 2h$	[M1] find $\ln N$ values [A1] find time from the graph

13	**(a)**	grad of AC = -1	[M1]
		mid point = $\left(\frac{-2+4}{2}\ ,\ \frac{10+4}{2}\right)$ = (1, 7)	[M1]
		grad of P.B = -1	[M1]/
		eqn y = x + 6	[A1] no FT if last step is wrong such as student forgetting to find gradient of perp bisector /r mid point
	(b)	y = 0, B = (-6, 0)	[B1]
		Let D = (x,y) mid point M = $\left(\frac{-6+x}{2}\ ,\ \frac{0+y}{2}\right)$	[M1]/ FT from answer for B
		D = (8, 14)	[A1]/ FT
	(c)	Area of rhombus = $\frac{1}{2}\begin{vmatrix} -2 & -6 & 4 & 8 & -2 \\ 10 & 0 & 4 & 14 & 10 \end{vmatrix}$	[M1]/ FT
		= 84 units^2	[A1]/ FT