MARKING SCHEME

Qn			Marks	Remarks
1	Curved surface area = $2\pi rh$ $height=\dfrac{10\pi\sqrt{5}-18\pi}{2\pi(3+2\sqrt{5})}$ $=\dfrac{10\sqrt{5}-18}{6+4\sqrt{5}}\times\dfrac{6-4\sqrt{5}}{6-4\sqrt{5}}$ $=\dfrac{60\sqrt{5}-40(5)-18(6)+72\sqrt{5}}{-44}$ $=\dfrac{132\sqrt{5}-308}{-44}$ $=-3\sqrt{5}+7$	$height=\dfrac{2\pi\left(\sqrt{5}-9\right)}{2\pi(3+2\sqrt{5})}$ $=\dfrac{5\sqrt{5}-9}{3+2\sqrt{5}}\times\dfrac{3-2\sqrt{5}}{3-2\sqrt{5}}$ $=\dfrac{15\sqrt{5}-50-27+18\sqrt{5}}{3^2-\left(2\sqrt{5}\right)^2}$ $=\dfrac{33\sqrt{5}-77}{9-20}$ $=-3\sqrt{5}+7$	M1 (form equation / find height) M1 (rationalise) M1 (expansion) A1	Wrong formula for curved surface area – award zero. Candidates do not possess the habit of reducing the fraction to the lowest term before simplifying. Resulting in “slips”.
2(a)(i)	$tan(-B)=-\dfrac{3}{4}$		B1 (negative angles)	This question is badly done by most candidates. They are completely clueless about what the question is asking. The inability to apply CAST diagram to solve this question is very apparent. Candidates can write the exact form but unable to
(ii)	$\tan(90-A)=\dfrac{1}{\tan A}$ $=-\dfrac{12}{5}$		B1 (complementary angles)	
(iii)	$cos\,A\,sin\,B=-\dfrac{12}{13}\times\dfrac{3}{5}=-\dfrac{36}{65}$		M1, A1	

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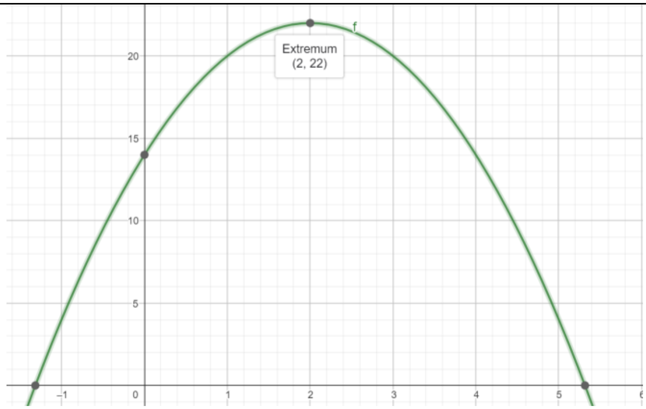
(b)	$\left(\sin\frac{\pi}{6}+\cos\frac{\pi}{4}\right)\tan\frac{\pi}{3}=\left(\frac{1}{2}+\frac{\sqrt{2}}{2}\right)\left(\sqrt{3}\right)$ $=\frac{\sqrt{3}+\sqrt{6}}{2}$	M1 (exact form of special angles) A1	perform expansion.
3(i)	$y=-2x^2+8x+14$ $=-2\left[(x-2)^2-(2)^2-7\right]$ $=-2(x-2)^2+22$	M1 (completing the square) A1	Generally well done. Weaker students are unable to complete the square.
3(ii)	 $\boxed{y=-2x^2+8x+14}$ Turning point = (2, 22) y-intercept = (0, 14)	Correct shape with y-intercept given in **coordinates form** – M1 Correct max point with label given in **coordinates form** – M1	Candidates do not label the axes and the coordinates of both the max point and y-intercept. They only gave the former in coordinates form.
4	$\frac{x^3+7x^2-5x+12}{(x-1)\left(x^2+2\right)}=1+\frac{8x^2-7x+14}{(x-1)(x^2+2)}$ $=1+\frac{A}{x-1}+\frac{Bx+C}{x^2+2}$ $A(x^2+2)+(Bx+C)(x-1)=8x^2-7x+14$ $(A+B)x^2+(-B+C)x+(2A+C)=8x^2-7x+14$ $A=5$ $B=3$ $C=-4$	M1 (long division – obtain1 $+\frac{8x^2-7x+14}{(x-1)(x^2+2)}$) M1 M1, M1, M1 (values of A, B and C)	Award zero when the form of the partial fractions is incorrect right from the start. If form is partially correct, only award the M1 for performing the long division. Candidates who fail to solve successfully didn't

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	$\dfrac{x^3+7x^2-5x+12}{(x-1)(x^2+2)}=1+\dfrac{5}{x-1}+\dfrac{3x-4}{x^2+2}$	A1 (conclusion with correct signs)	realised that the fraction improper.
5(a)	$p(x)=4x^3+ax^2+bx-12$ $p(2)=4(2)^3+a(2)^2+b(2)-12=0$ $2a+b=-10$ ---------(1) $p\left(\frac{1}{2}\right)=4\left(\frac{1}{2}\right)^3+a\left(\frac{1}{2}\right)^2+b\left(\frac{1}{2}\right)-12=-18$ $a+2b=-26$ ------- (2) Solving simultaneously, $a=2, \quad b=-14$	M1 (obtain equation (1)) M1 (substitution) M1 (obtain equation (2)) A1 (value of b)	Many candidates compute the value of b before proving that $a=2$. Formulating eqn (2) poses a challenge to some candidates.
5(b)	$4x^3+2x^2-14x-12=0$ $(x-2)(4x^2+10x+6)=0$ $2(x-2)(x+1)(2x+3)=0$ $x=2, \quad -1, \quad -\frac{3}{2}$	M1, M1 (obtain $4x^2+10x+6$ from comparing coefficients or long division) M1 (factorise) A1	Well done.
6(a)	**<u>Method1</u>** $3x+2y=5\Rightarrow x=\dfrac{5-2y}{3}$ ----(1) $\dfrac{3}{x}+\dfrac{2}{y}=10$ ----(2) Sub (1) into (2), $\dfrac{9}{5-2y}+\dfrac{2}{y}=10$ $9y+10-4y=50y-20y^2$ $20y^2-45y+10=0$ $(y-2)(4y-1)=0$ $y=2 \qquad y=\dfrac{1}{4}$ $x=\dfrac{1}{3} \qquad x=1\dfrac{1}{2}$ **<u>Method 2</u>** $3x+2y=5\Rightarrow y=\dfrac{5-3x}{2}$ ----(1) $\dfrac{3}{x}+\dfrac{2}{y}=10$ ----(2) Sub (1) into (2), $3(5-3x)+4x=10x(5-3x)$ $15-9x+4x=150x-30x^2$ $30x^2-55x+15=0$ $(3x-1)(2x-3)=0$ $x=\dfrac{1}{3} \qquad x=\dfrac{3}{2}$ $y=2 \qquad y=\dfrac{1}{4}$	M1 (substitution) M1 (factorisation) M1 (values of y)	Algebraic manipulation is a huge concern for equation 2. Common mistake seen: • $3y+2x=10$

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	$\therefore \left(\frac{1}{3}, 2\right)$ and $\left(1\frac{1}{2}, \frac{1}{4}\right)$	$\therefore \left(\frac{1}{3}, 2\right)$ and $\left(1\frac{1}{2}, \frac{1}{4}\right)$	A1 (coordinates of points of intersection)	

6(b)	$ax + 2a - 1 = 8 + 2x - x^2$ $x^2 + (a-2)x + (2a-9) = 0$ $D = (a-2)^2 - 4(1)(2a-9)$ $= a^2 - 12a + 40$ $= (a-6)^2 + 4$ Since $(a-6)^2 \geq 0$ for all values of a, $(a-6)^2 + 4 > 0 \Rightarrow D > 0$ Therefore, the line cuts the curve at two distinct points.	M1 M1 (complete the square) A1 (explain how D > 0)	Candidates are able to compute the discriminant but fail to see the need to complete the square for the mathematical explanation to prove that the curve intersects the line twice.
6(c)	Since curve lies entirely below lie, $p < 0$ $px^2 + 4x + p + 3 < 0$ $(4)^2 - 4(p)(p+3) < 0$ $16 - 4p^2 - 12p < 0$ $p^2 + 3p - 4 > 0$ $(p+4)(p-1) > 0$ $\therefore p < -4$ or $p > 1$ (rej)	M1 (substitution to D < 0) M1 (obtain quadratic equation) M1 (solve for values of p) A1 (reject $p > 1$)	Common misconception: $D < -3$ instead of $y < -3$. Common mistake seen: Students are unable to formulate the inequality. The understanding of curve lies below the line is missing. $px^2 + 4x + p - 3 < 0$
7(a)	$\left(3x - \frac{1}{2}\right)^5 = (3x)^5 + \binom{5}{1}(3x)^4\left(-\frac{1}{2}\right) + \binom{5}{2}(3x)^3\left(-\frac{1}{2}\right)^2 + \ldots$ $= 243x^5 - 202\frac{1}{2}x^4 + 67\frac{1}{2}x^3 + \ldots$	B2, B1, 0	Well done except for mistakes in sign. Failed to see that the binomial expansion will alternate in sign.

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7(b)(i)	General term $= \binom{7}{r}(x^2)^{7-r}\left(\frac{-3}{x}\right)^r$ $= \binom{7}{r}(x^{14-3r})(-3^r)$ For term to be independent of x, $14-3r=0$ $r=\frac{14}{3} \Rightarrow$ since r is not a whole number, (or positive integer) there is no term independent of x.	M1 (correct simplification) M1 (solve for r) A1 (explanation)	Generally, well done.
7(b)(ii)	For term to be independent of x, $14-3r=-1$ $r=5$ Therefore, term independent of x $= \binom{7}{5}(x^{14-15})(-3^5) \times -3x$ $= -5103x^{-1} \times -3x$ $= 15309$	M1 (value of r) M1 (evaluation of general term) A1	Badly done. Only about 4 students in the entire cohort can determine the constant term correctly.
8(a)	$\frac{125^{2+x}}{25} = 5^x \times 625^{2x-1}$ $\frac{5^{6+3x}}{5^2} = 5^x \times 5^{8x-4}$ $5^{6+3x-2} = 5^{9x-4}$ $4+3x=9x-4$ $x=\frac{4}{3}$	M1 (change to base 5) M1 (combine to base 5) M1 (compare powers) A1	Generally well done for most students.
8(b)	$log_4 x + 3\,log_x 4 = 4$ $log_4 x + \frac{log_4 4^3}{log_4 x} = 4$	M1 (change of base)	Generally okay for most students. Some students were not able to observe that they can use an

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	$log_4 x + \dfrac{3}{log_4 x} = 4$ Let $a = log_4 x$ $a + \dfrac{3}{a} = 4$ $a^2 + 3 - 4a = 0$ $(a-3)(a-1) = 0$ $a = 3$ or $a = 1$ $\log_4 x = 3$ or $\log_4 x = 1$ $x = 4^3 = 64$ $x = 4$	M1 (bring down 3) M1 (quadratic equation) M1 (solving for *a*) A1	appropriate substitution to solve for *x*. A few students forgot to solve for *x*.
9(a)	5600 stands for the <u>initial</u> population of bacterium, when <u>$t = 0$</u>.	B1	Good. A few students stated that 5600 stands for the population 'before' the start of the experiment. However, 'before' is too vague.
9(b)	$5600e^{kt} = 28000$ $e^{4k} = 5$ $k = \dfrac{\ln 5}{4}$ $= 0.40235$ $P = 5600e^{0.40235(14)}$ $= 1565039.9$ $= 1570000$ (3 s.f.)	M1 M1 (value of *k*) M1 (substitute $t = 14$) A1	Generally well done. Most students were able to perform the logarithmic laws correctly. A few students did not round off answers to 3 sf, which resulted in marks deducted for accuracy.

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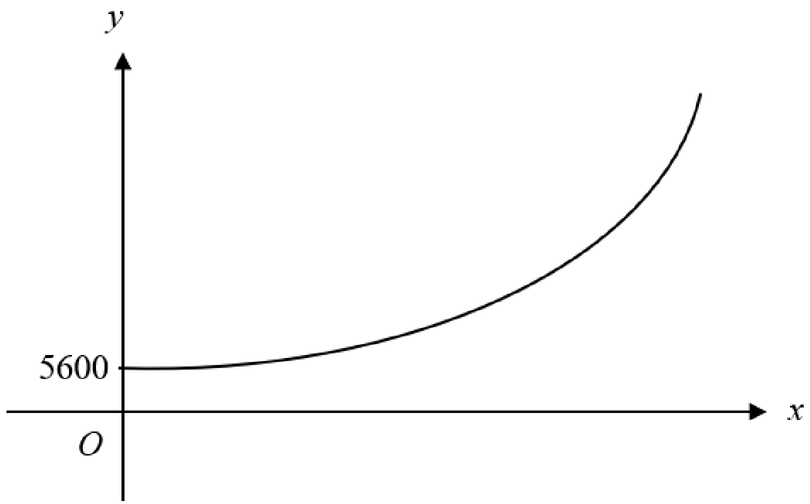
9(c)		M1 (correct shape) M1 (correct y-intercept)	Generally well done. Most students were able to get the shape of the exponential graph correct.
10(a)	$m_{BC} = \frac{4-2}{-2-2} = -\frac{1}{2}$ $m_{AB} = 2$ $y = 2x + c$ $c = -2$ $y = 2x - 2$	 M1 (gradient of AB) M1 (value of c) A1	Generally okay. Most students were able to find the correct equation of straight line. A handful of students missed out the negative sign when finding the gradient of *AB* and hence did not score for this question.
10(b)	Equation of *AD*: $y = -\frac{1}{2} - 4\frac{1}{2}$ Point *A* is the point of intersection between line *AD* and line *BC*. Equating both equations, $-\frac{1}{2} - 4\frac{1}{2} = 2x - 2$ $2.5x = -2.5$ $x = -1$ $y = -4$ (shown)	 M1 (equation of *AD*) A1	Most students were able to write down the equation of *AD*. A few students went to sub in the *x*-value -1, to obtain the *y*-value -4. This was not accepted.

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10(c)	$\frac{x+2}{2} = -2\frac{1}{2}$ $\frac{y+2}{2} = \frac{1}{2}$ $x = -7$ $y = -1$ Therefore, D is $(-7,-1)$.	B1, B1 (deduct 1 mark if did not show coordinates of *D*)	Most students were able to get this right. However, a few students used the wrong formulae to find midpoint instead.
10(d)	area $ABCD = \frac{1}{2}\left\| \begin{matrix} -1 & 2 & -2 & -7 & -1 \\ -4 & 2 & 4 & -1 & -4 \end{matrix} \right\|$ $= \frac{1}{2}\left[(36)-(-39)\right]$ $= 37.5 \text{ units}^2$	M1/DB1 for shoelace method A1	Badly done. Many students forgotten how to use the shoelace method. Many students made careless mistakes in the expansion, leaving out negative signs. A few students did not repeat the first coordinate, and some left out the absolute value and got confused.
11(a)	$m_{PQ} = \frac{1}{7}$ Gradient of perpendicular bisector = -7 Midpoint $PQ = \left(1\frac{1}{2}, -3\frac{1}{2}\right)$ Equation of perpendicular bisector of PQ: $y = -7x + 7$	M1 (gradient) M1 (midpoint) A1 (equation)	Generally okay. However, a few students went to find the gradient of the normal at *Q* instead, and therefore did not receive any marks.
11(b)	To find centre of circle, find the point of intersection between *PQ* and normal at *Q*. Find equation of normal at *Q*: $y = \frac{4}{3}x - 9\frac{2}{3}$ Solving simultaneously,	M1 (eqn of normal at *Q*)	Badly done. Most students were able to obtain the equation of normal at *Q*, but many did not perform simultaneous correctly and hence not able to get the coordinates of the centre of the circle. This might be due to carelessness in the manipulation of fractions, or even simple algebraic manipulation.

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	$\frac{4}{3}x-9\frac{2}{3}=-7x+7$ $8\frac{1}{3}x=16\frac{2}{3}$ $x=2$ $y=-7$ To find radius of circle: $\sqrt{(2+2)^2+(-7+4)^2}=5$ Hence, equation of the circle: $(x-2)^2+(y+7)^2=25$	M1/ DB1 for simultaneous M1 (centre of circle) M1 (radius) A1	
11(c)	$\sqrt{(2-2)^2+(-7-1)^2}=8>5$ Since the distance between the centre of circle and the point is > 5, the point is outside the circle.	M1 / DB1 for finding distance A1 (determine correctly)	Most students were able to get the first mark (due to DB1).
12(i)	$P=ab^t$ $lg\,P=lg(\,ab^t)$ $lg\,P=lg\,a+t\,lg\,b$	B1 Cut y-axis B1 Join 5 points with a straight line	Generally well done. However, a handful of students did not draw the line long enough to cut the y-axis.
12(ii)	Using the graph, Y-intercept = 2.20 $\lg a=2.20$ $\lg a=2.195$ $a=158.489$ or $a=156.67$ $=158$ $=157$ Gradient = 0.041 $\lg b=0.041$ $b=1.099$ $=1.10$ Range of values for Y-intercept and gradient is accepted.	M1 (value of Y-intercept) A1 (value of a) M1 (value of gradient) A1 (value of b)	Generally well done. Most students show good understanding of logarithmic laws. A few students used substitution of values to obtain a and b, which was not accepted. No marks were awarded for this method.

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12(iii)	$P = 158(1.10)^{10}$ $= 409.811$ $= 410$ (to 3 s.f.)	M1 / DB1 for correct substitution of their values A1	Most students were able to obtain DB1.