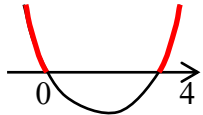
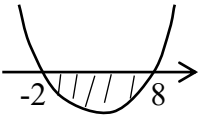
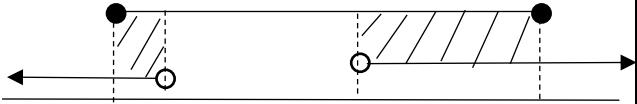
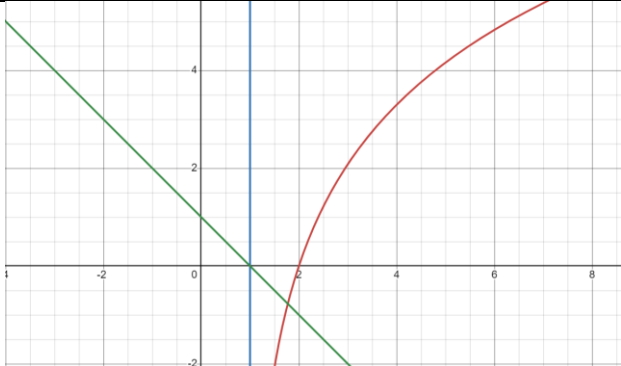


**North Vista Secondary**  
**Sec 3EXP End-of-Year Assessment 2024**  
**Additional Mathematics 4049**  
**Marking Scheme**

| Qn | Solutions                                                                                                                                                                                                                                                                                                                 | Total | Mark                                                                                                                                                                                                  | AO  |
|----|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| 1  | $3^{2x}3^{3y} = 3^0$<br>$3^{2x+3y} = 3^0$<br>Comparing Indices<br>$2x + 3y = 0 \dots\dots\dots(1)$<br><br>$\frac{a^{2x}}{a^{3y+6}} = a^{-2}$<br>$a^{2x-3y-6} = a^{-2}$<br>Comparing Indices<br>$2x - 3y - 6 = -2$<br>$2x - 3y = 4 \dots\dots\dots(2)$<br><br>$(1) + (2)$<br><br>$4x = 4$<br>$x = 1$<br>$y = -\frac{2}{3}$ | [4]   | M1 - Obtains first equation<br><br><br><br><br><br><br><br><br><br>M1 - Obtains second equation<br><br><br><br><br><br><br><br><br><br>A1 - $x$ value<br><br>A1 - $y$ value (only accept exact value) | AO2 |

|      |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |     |                                                                                  |     |
|------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|----------------------------------------------------------------------------------|-----|
| 2(a) | $y = mx^2 - 5x + 2$<br>$y$ is always positive $\Rightarrow b^2 - 4ac < 0$<br>$(-5)^2 - 4m(2) < 0$<br>$8m > 25$<br>$m > 3\frac{1}{8}$                                                                                                                                                                                                                                                                                                                                                                      | [6] | <b>M1</b> – compute D<br><br><b>A1</b>                                           | AO1 |
| (b)  | $x^2 - 4x > 0$ ----- (1)<br>$x(x - 4) > 0$<br>$x < 0$ or $x > 4$ } <br><br>$x^2 - 4x \leq 2x + 16$ ----- (2)<br>$x^2 - 6x - 16 \leq 0$<br>$(x + 2)(x - 8) \leq 0$ } <br>$-2 \leq x \leq 8$ } <br>The solutions are $-2 \leq x < 0$ or $4 < x \leq 8$ . |     | <b>M1</b><br><br><b>M1</b><br><br><b>A2</b> [only if inequality sign is correct] | AO2 |

|             |                                                                                                                                                                                                                                                           |            |                                                                                          |     |
|-------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|------------------------------------------------------------------------------------------|-----|
| <b>3(a)</b> | When $t = 0$ ,<br>$R = 300 + 6000 = 6300$                                                                                                                                                                                                                 | <b>[6]</b> | <b>B1</b>                                                                                | AO1 |
| <b>(b)</b>  | $R_0 = 6300$<br>Given $R < \frac{6300}{2}$<br>$300 + 6000(10^{-0.02t}) < 3150$<br>$(10^{-0.02t}) < \frac{2850}{6000}$<br>$10^{-0.02t} < 0.475$<br>Take lg on both sides<br>$-0.02t < \lg 0.475$<br>$t > \frac{\lg 0.475}{-0.02} = 16.165 \approx 17$ days |            | <b>M1</b> – form eqn<br><br><b>M1</b> - Take lg on both sides<br><br><b>A1</b> – in days | AO2 |
| <b>(c)</b>  | When $t \rightarrow \infty$ ,<br>$10^{-0.02t} \rightarrow 0$<br>$R = 300 + 6000(10^{-0.02t}) \rightarrow 300$<br>$\therefore$ Number of rabbits will <b>not</b> become zero.                                                                              |            | Need to explain mathematically<br><br><b>B1</b><br><br><b>B1</b> - Conclusion            | AO3 |

|         |                                                                                                               |     |                                                                                                                                        |     |
|---------|---------------------------------------------------------------------------------------------------------------|-----|----------------------------------------------------------------------------------------------------------------------------------------|-----|
| 4(a)(i) |                               | [7] | <p>B1 — correct shape and close to asymptote</p> <p>B1 - <math>x</math>-intercept: (2,0)</p> <p>B1 - asymptote: <math>x = 1</math></p> | AO2 |
| (ii)    | $x - 1 = e^{\frac{1-x}{3}}$ $\ln(x-1) = \frac{1-x}{3}$ $3\ln(x-1) = 1-x$ $y = 1-x$                            |     | <p>M1 — Take <math>\ln</math> both sides</p> <p>A1</p>                                                                                 | AO2 |
| (b)     | $100^x - e^{x \ln p} = 0$ $100^x = e^{x \ln p}$ $\ln 100^x = \ln e^{x \ln p}$ $x \ln 100 = x \ln p$ $p = 100$ |     | <p>M1 — Take <math>\ln</math> both sides</p> <p>A1</p>                                                                                 | AO1 |





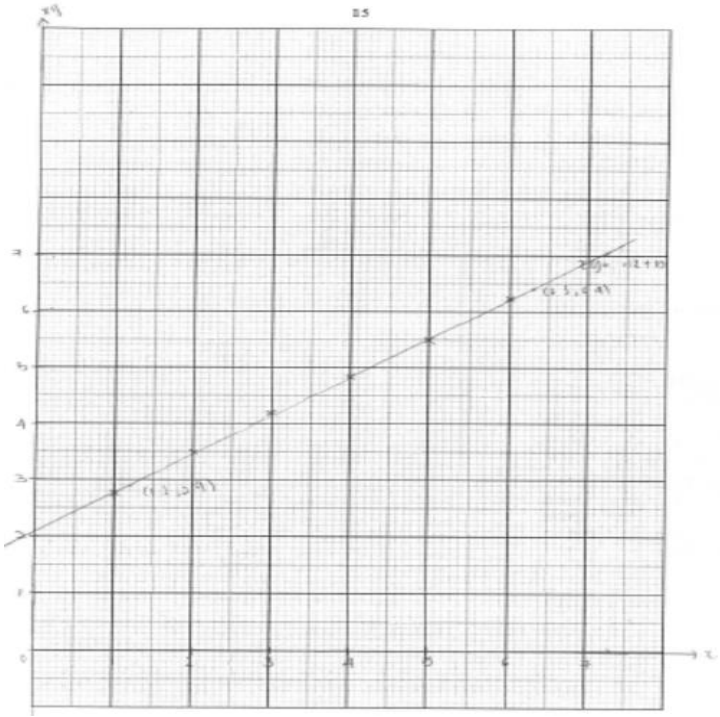
|             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |     |                                                                                                        |     |
|-------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|--------------------------------------------------------------------------------------------------------|-----|
| <b>7(a)</b> | $3x^3 - 24y^3$ $= 3 \left[ x^3 - (2y)^3 \right]$ $= 3(x-2y)(x^2 + 2xy + 4y^2)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |     | M1 – diff of cubes<br>A1                                                                               | AO1 |
| (b)         | $\frac{3-10x-x^3}{(x^2+1)(x-2)} = -1 + \frac{-2x^2-9x+1}{(x^2+1)(x-2)}$ $\frac{-2x^2-9x+1}{(x^2+1)(x-2)} = \frac{Ax+B}{(x^2+1)} + \frac{C}{(x-2)}$ <p>– <math>-2x^2-9x+1 = (Ax+B)(x-2) + C(x^2+1)</math><br/>Let <math>x = 2</math><br/><math>-25 = 5C</math><br/><math>C = -5</math></p> <p>Let <math>x = 0</math><br/><math>1 = -2B + C</math><br/><math>B = -3</math></p> <p>Comparing coefficients of <math>x^2</math><br/><math>-2 = A + C</math><br/><math>A = 3</math></p> $-\frac{x^3+10x-3}{(x^2+1)(x-2)} = -1 + \frac{3x-3}{x^2+1} - \frac{5}{x-2}$ | [8] | M1 - Long division<br>M1 - correct form<br>B3 - A, B and C values<br>A1 – conclusion with correct sign | AO2 |

|               |                                                                                                                                                                                                                                                                           |            |                                                                                                                                                       |     |
|---------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|-------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| <b>8(a)</b>   | $2g = -8 \quad \& \quad 2f = 6 \quad \& \quad c = -4$<br>$g = -4 \quad f = 3$<br>Centre $C = (4, -3)$<br>Radius $= \sqrt{16 + 9 - (-4)} = \sqrt{29}$ units<br>$CT = \sqrt{1 + 16}$<br>$= \sqrt{17}$ units $< \sqrt{29}$ units<br>Hence $T$ lies <b>inside</b> the circle. | <b>[9]</b> | <b>B1</b> – centre<br><b>B1</b> – radius<br><br><b>B1</b> f.t. for $CT$ , with conclusion<br><b>B0</b> for merely substituting $(5, 1)$ into equation | AO3 |
| <b>(b)(i)</b> | Gradient $= -\tan 45^\circ = -1$<br>( $L$ is slanting downwards)<br>Substitute $(11, 0)$<br>$\Rightarrow y = -x + 11$                                                                                                                                                     |            | <b>M1</b><br><br><b>A1</b>                                                                                                                            | AO1 |
| <b>(ii)</b>   | Midpoint $y = 5$<br>Substitute $y = 5 \Rightarrow x = 6 \Rightarrow C = (6, 5)$                                                                                                                                                                                           |            | <b>M1</b><br><b>A1</b>                                                                                                                                | AO1 |
| <b>(iii)</b>  | Radius $= \sqrt{36 + 4} = \sqrt{40}$<br>$(x - 6)^2 + (y - 5)^2 = 40$                                                                                                                                                                                                      |            | $\sqrt{\text{M1 (ecf)}}$<br><b>A1</b>                                                                                                                 | AO1 |
| <b>(iv)</b>   | Because $CR = \sqrt{5^2 + 5^2} = \sqrt{50} \neq \sqrt{40}$<br>Hence, it is <b>impossible</b> for a circle with centre $C$ to pass through all three points, $P$ , $Q$ and $R$ .                                                                                           |            | <b>B1</b> - explanation                                                                                                                               | AO3 |

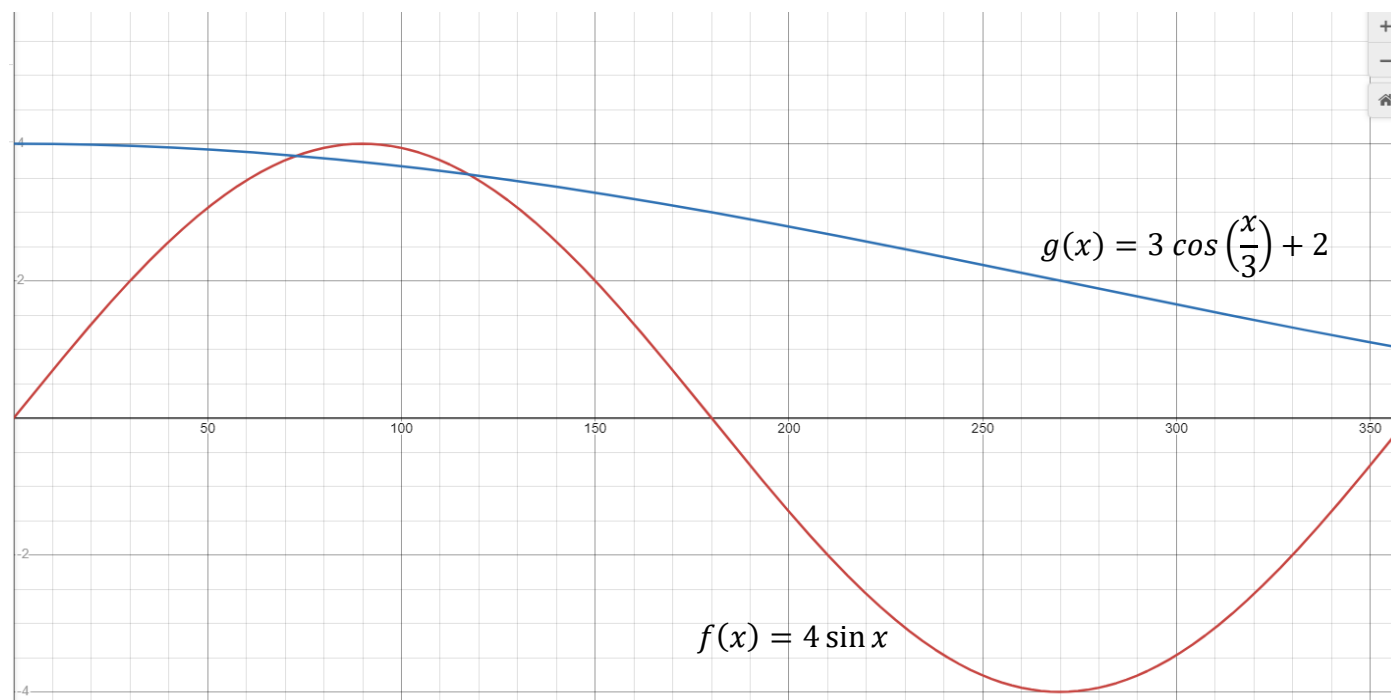


|             |                                                                                                                                                                                                                                                                                                  |             |                                                                                                                                             |     |
|-------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------|---------------------------------------------------------------------------------------------------------------------------------------------|-----|
| <b>9(a)</b> | $f(x) = 3x^3 + x^2 + 11x - 35$ <p>By Remainder Theorem,</p> $\text{Remainder} = f\left(-\frac{1}{2}\right)$ $= 3\left(-\frac{1}{2}\right)^3 + \left(-\frac{1}{2}\right)^2 + 11\left(-\frac{1}{2}\right) - 35$ $= -\frac{325}{8}$ $= -40\frac{5}{8} \text{ or } -40.625$                          | <b>[11]</b> | <p><b>M1</b> - substitution</p> <p><b>A1</b></p>                                                                                            | AO1 |
| <b>(b)</b>  | $f\left(\frac{5}{3}\right) = 3\left(\frac{5}{3}\right)^3 + \left(\frac{5}{3}\right)^2 + 11\left(\frac{5}{3}\right) - 35 = 0$ <p>By the <b>Factor Theorem</b>, <math>(3x - 5)</math> is a factor of <math>f(x)</math>. [Shown]</p>                                                                |             | <p><b>B1</b> – cite FT</p>                                                                                                                  | AO1 |
| <b>(c)</b>  | $f(x) = 3x^3 + x^2 + 11x - 35$ $= (3x - 5)(x^2 + 2x + 7)$ <p>When <math>f(x) = 0</math>,</p> $(3x - 5)(x^2 + 2x + 7) = 0$ $x = \frac{5}{3} \quad \text{or} \quad x^2 + 2x + 7 = 0$ $D = 2^2 - 4(1)(7)$ $= -24$ $< 0$ <p><math>\therefore</math> The equation has only one real root. [Shown]</p> |             | <p><b>M1</b> - By long division or by Synthetic Division</p> <p><b>M1</b></p> <p><b>M1</b> – discriminant</p> <p><b>A1</b> – conclusion</p> | AO3 |

|     |                                                                                                                                                                                                                                                                                                                                                                                       |  |                                                                                                                               |     |
|-----|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|-------------------------------------------------------------------------------------------------------------------------------|-----|
| (d) | $3^{3y+1} + 3^{2y} = 35 - 11(3^y)$ $3(3^{3y}) + 3^{2y} + 11(3^y) - 35 = 0$ $3(3^y)^3 + (3^y)^2 + 11(3^y) - 35 = 0$ <p>Let <math>u = 3^y</math></p> <p>From part (ii) and (iii)</p> $3^y = \frac{5}{3}$ <p>Take lg on both sides,</p> $y \lg 3 = \lg \left( \frac{5}{3} \right)$ $y = \frac{\lg \left( \frac{5}{3} \right)}{\lg 3}$ $= 0.46497$ $\approx 0.465 \quad (\text{to 3 sf})$ |  | <p><b>M1</b> – attempt to make the same form</p> <p><math>\sqrt{\text{M1 (ecf)}}</math></p> <p><b>M1</b></p> <p><b>A1</b></p> | AO3 |
|-----|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|-------------------------------------------------------------------------------------------------------------------------------|-----|

|         |                                                                                                                                                                                                                                        |      |      |      |      |                                                                                                                                              |                                                     |     |     |     |     |     |     |      |      |      |      |      |      |      |     |  |
|---------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|------|------|------|----------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------|-----|-----|-----|-----|-----|-----|------|------|------|------|------|------|------|-----|--|
| 10(a)   | Taking $\ln$ on both sides,<br>$\ln N(t) = \ln N_o - kt$<br>Plot $\ln N(t)$ against $t$<br>Gradient $= -k$<br>At $t = 0$ , the $y$ -intercept is $\ln N_o$ .<br>$N_o = e^{\ln(N_o)}$                                                   |      |      |      |      | [9]                                                                                                                                          | Must show<br>B1 - linearization<br><br>B1<br><br>B1 | AO3 |     |     |     |     |     |      |      |      |      |      |      |      |     |  |
| (b)(i)  | <table><tr><td><math>x</math></td><td>1.0</td><td>2.0</td><td>3.0</td><td>4.0</td><td>5.0</td><td>6.0</td></tr><tr><td><math>xy</math></td><td>2.77</td><td>3.50</td><td>4.20</td><td>4.85</td><td>5.50</td><td>6.24</td></tr></table> |      |      |      |      |                                                                                                                                              | $x$                                                 | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 6.0 | $xy$ | 2.77 | 3.50 | 4.20 | 4.85 | 5.50 | 6.24 | AO1 |  |
| $x$     | 1.0                                                                                                                                                                                                                                    | 2.0  | 3.0  | 4.0  | 5.0  | 6.0                                                                                                                                          |                                                     |     |     |     |     |     |     |      |      |      |      |      |      |      |     |  |
| $xy$    | 2.77                                                                                                                                                                                                                                   | 3.50 | 4.20 | 4.85 | 5.50 | 6.24                                                                                                                                         |                                                     |     |     |     |     |     |     |      |      |      |      |      |      |      |     |  |
|         |                                                                                                                                                      |      |      |      |      | B1 for tabulation of values<br><br>B1 for appropriate axes $Y = xy$ , $X = x$<br><br>B1 for best fit line]<br><br>B1 for vertical-intercept] |                                                     |     |     |     |     |     |     |      |      |      |      |      |      |      |     |  |
| (b)(ii) | $b = 2.1$ (accept 2.07 to 2.2)<br>Gradient $= a$<br>$= 0.694$ (accept 0.60 to 0.70)                                                                                                                                                    |      |      |      |      |                                                                                                                                              | B1<br><br>B1                                        | AO1 |     |     |     |     |     |      |      |      |      |      |      |      |     |  |

|              |                                                                                                                                                                                                                                                                                   |            |                    |     |
|--------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|--------------------|-----|
| <b>11(a)</b> | <ul style="list-style-type: none"> <li>Amplitude of <math>f(x) = 4</math></li> <li>Period of <math>f(x) = 360^\circ = 2\pi</math></li> <li>Amplitude of <math>g(x) = 3</math></li> <li>Period of <math>g(x) = \frac{360^\circ}{\frac{1}{3}} = 1080^\circ = 6\pi</math></li> </ul> | <b>[9]</b> | <b>B1</b> for both | AO1 |
| <b>(b)</b>   | Graph sketching                                                                                                                                                                                                                                                                   |            | <b>B4</b>          | A02 |
| <b>(c)</b>   | No. of solutions = 2                                                                                                                                                                                                                                                              |            | <b>B1</b>          | AO1 |



Domain:  $0^\circ \leq x \leq 360^\circ$ .

**B2** –  $f(x)$  with labelling

**B2** –  $g(x)$  with labelling

|              |                                                                                                                                                                                                                                               |            |           |     |
|--------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|-----------|-----|
| <b>12(a)</b> | 1.75 m                                                                                                                                                                                                                                        | <b>[9]</b> | <b>B1</b> | AO1 |
| <b>(b)</b>   | $h = 1.75 + 5t - 5t^2$ $= -5t^2 + 5t + 1.75$ $= -5\left(t^2 - t\right) + 1.75$ $= -5\left[\left(t - 0.5\right)^2 - 0.25\right] + 1.75$ $= 3 - 5\left(t - 0.5\right)^2$ $\therefore a = 3, \quad b = -5, \quad c = -0.5$                       |            | <b>B3</b> | AO1 |
| <b>(c)</b>   | Maximum height = 3 m<br>Time = 0.5 s                                                                                                                                                                                                          |            | <b>B2</b> | AO1 |
| <b>(d)</b>   | When $h = 0$ ,<br>$5\left(t - 0.5\right)^2 = 3$<br>$t - 0.5 = \sqrt{\frac{3}{5}}$<br>$t = \sqrt{\frac{3}{5}} + 0.5$<br>$= 1.274\text{s}$<br>$\neq 2\left(0.5\right) = 1\text{ s}$<br><br>or<br>The ball was not thrown from the ground level. |            | <b>B1</b> | AO1 |

|     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |  |                                                 |     |
|-----|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|-------------------------------------------------|-----|
| (e) | <p>Given <math>h \geq 2</math>,<br/> Let <math>3 - 5(t - 0.5)^2 = 2</math><br/> <math>5(t - 0.5)^2 = 1</math><br/> <math>(t - 0.5)^2 = 0.2</math><br/> <math>t - 0.5 = \pm\sqrt{0.2}</math></p> <p><math>-\sqrt{0.2} \leq t - 0.5 \leq \sqrt{0.2}</math><br/> <math>-\sqrt{0.2} + 0.5 \leq t \leq \sqrt{0.2} + 0.5</math></p> <p><math>\therefore</math> length of time = <math>(\sqrt{0.2} + 0.5) - (-\sqrt{0.2} + 0.5)</math><br/> (Duration) = <math>2\sqrt{0.2}</math><br/> = <math>0.894 \text{ s}</math></p> |  | <p><b>M1</b> – equate to 2</p> <p><b>A1</b></p> | A02 |
|-----|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|-------------------------------------------------|-----|