NORTH VISTA SECONDARY SCHOOL
End-of-Year Examination 2022
Secondary 3 Express

90

CANDIDATE NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS **4049**

5 OCTOBER 2022

2 hour 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your centre number, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use	
Category	**Question No.**
Accuracy	
Brackets	
Fractions	
Units	
Others	
Marks Deducted	

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\text{Area of } \Delta = \frac{1}{2}bc \sin A.$$

Answer **all** questions.

1 A function f(x) is given by the expression $2x^3 - 5x^2 - 4x + 3$.

(i) Find the remainder when f(x) is divided by $x - 1$. [1]

(ii) Show that $x - 3$ is a factor of f(x). [1]

(iii) Hence, solve f(x) = 0. [4]

2 Given that $\cos A = -\frac{3}{8}$, $\sin B = \frac{9}{10}$ and both A and B are in the same quadrant, find the **exact** value of

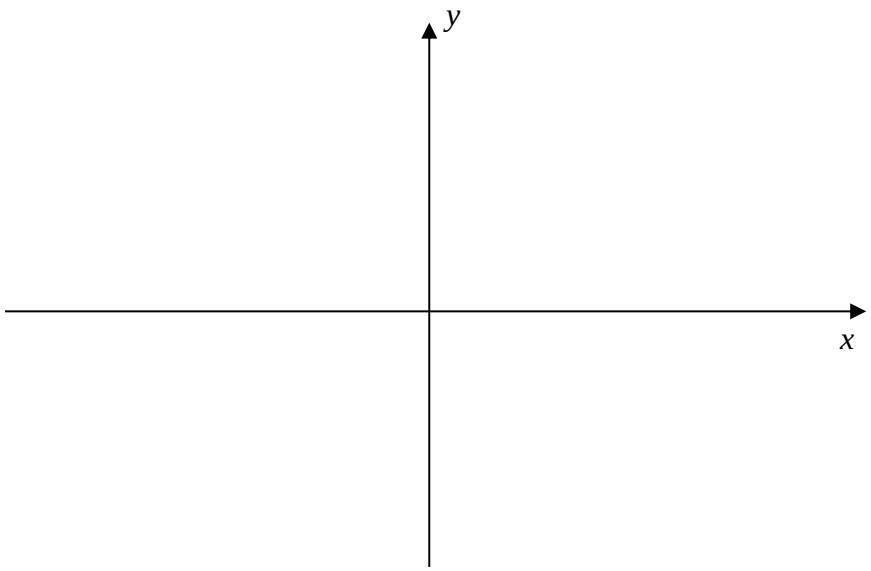
(i) $\sin A$, [1]

(ii) $\cos(-B)$, [1]

(iii) $\tan A \tan B$. [2]

3 By using a suitable substitution, solve the equation $2^{2+2x} + 2^{4+x} = 2^x + 4$. [5]

4 **(a)** Sketch the graph of $y = e^{\frac{x}{3}} + 1$. [2]



(b) To solve the equation $x = \ln(3x + 4)^3$ graphically, a suitable straight line can be drawn on the same set of axes in **(a)**. Find the equation of the straight line. [2]

(c) Hence, draw the line on the same diagram in **(a)** and state the number of solutions of the equation $x = \ln(3x + 4)^3$. [2]

5 **(a)** Find the value of p for which the x-axis is the tangent to the curve $y = 3x^2 - 2x + 1 - p$. [2]

(b) Find the range of values of k for which the line $y = 3x + k$ and the curve $x^2 + y^2 = 10$ do not intersect. [5]

6 **(i)** Find the general term for the expansion of $\left(x^2 - \frac{3}{x}\right)^7$ and write down, in terms of r, the power of x in this general term. [2]

(ii) Hence, explain whether the term independent of x in the expansion of $\left(x^2 - \frac{3}{x}\right)^7$ exist. [2]

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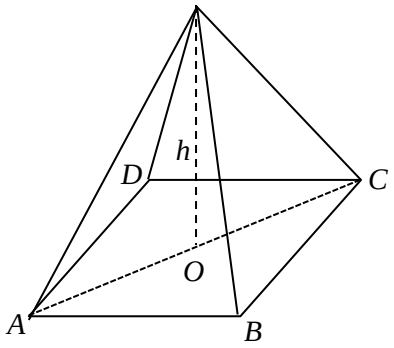


The right square pyramid has a square base *ABCD* with sides $\left(\sqrt{2}+\sqrt{3}\right)$ cm.
Given that the volume of the pyramid is $\left(22+9\sqrt{6}\right)$ cm^3, find the vertical height, *h* cm, of the pyramid. Leave your answer in the form $a + b\sqrt{3}$ where *a* and *b* are integers. [4]

8 Find the exact value of 60^x, given that $\sqrt{3^{4x+8}} = 20^{2-2x}$. [4]

9 A circle, C_1, has equation $x^2 + y^2 + 8x - 10y = 40$.

(i) Find the radius and the coordinates of the centre of C_1. [3]

A second circle, C_2, has a diameter *PQ*. The point *P* has coordinates $(0, 6)$ and the equation of the tangent to C_2 at *Q* is $4y = 3x - 26$.

(ii) Find the equation of the line *PQ* and hence coordinates of *Q*. [4]

9 **(iii)** Find the radius and the coordinates of the centre of C_2. [3]

(iv) Explain why the point $(2, 6)$ lies within both the circles C_1 and C_2. [2]

10 **(a)** Solve the equation $\log_2 2x - \log_4(x + 2) = 1$. [5]

(b) Solve the equation $\log_z 1000 = \lg z$. [5]

10 **(c)** An experimental population of bacteria increases exponentially and is given by the equation $P = 15000e^{bt}$, where t is the time in hours after the start of the experiment and P is the number of bacteria.

(i) Find the number of bacteria at the start of the experiment. [1]

Given that there are 28000 bacteria after 3 hours,

(ii) find the number of bacteria 7 hours and 45 minutes after the start of the experiment. [4]

11 Express $\frac{x^2+10}{(2x^2+1)(3+x)}$ in partial fractions. [5]

12 Recorded values of the number of bacteria, N, t hours after observations began, are shown in the table below.

N	782	12234	198789	2993707	46829457
t (hours)	5	10	15	20	25

It is known that N and t are related by the equation $N = N_0 e^{kt}$ where N_0 and k are constants.

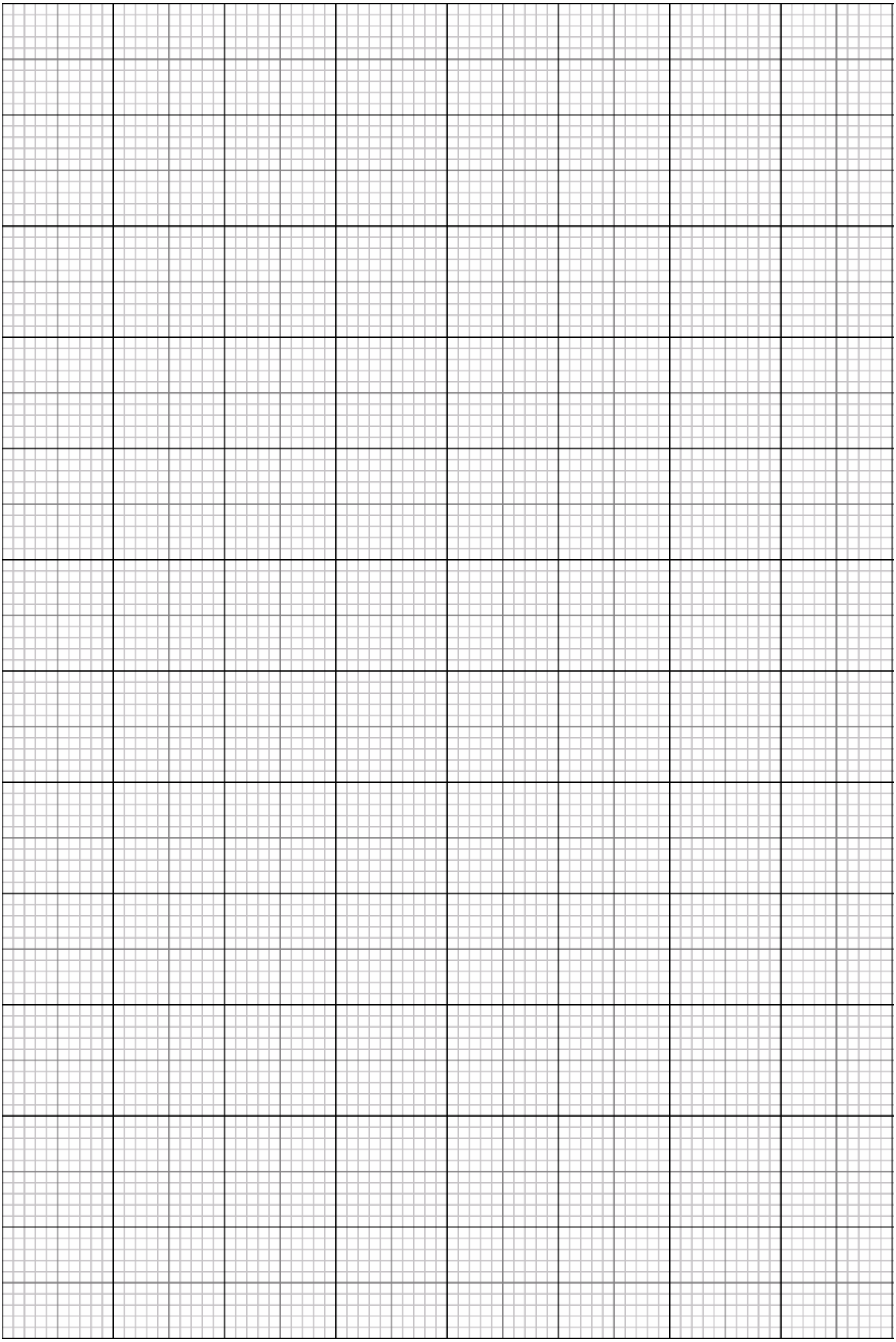
(i) Plot the graph of $\ln N$ against t. [3]

Use your graph to estimate

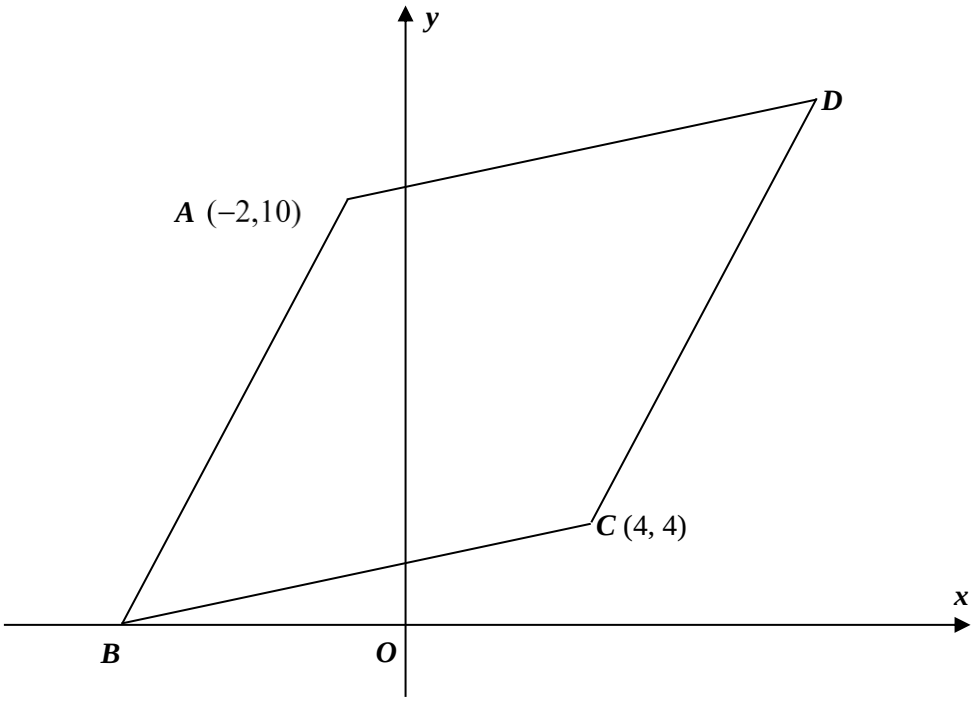
(ii) the number of bacteria when the observations began, [2]

(iii) the value of k, [2]

(iv) the time taken for the number of bacteria to be tripled since the start of observation. [2]



13 Solutions to this question by accurate drawing will not be accepted.



The figure shows a rhombus $ABCD$, where A is $(-2, 10)$ and C is $(4, 4)$ where the perpendicular bisector of AC passes though point B and D. Point B lies on the x-axis.

(i) Find the equation of the perpendicular bisector of AC. [4]

13 **(ii)** Find the coordinates of B and of D. [3]

(iii) Calculate the area of rhombus $ABCD$. [2]

END OF PAPER