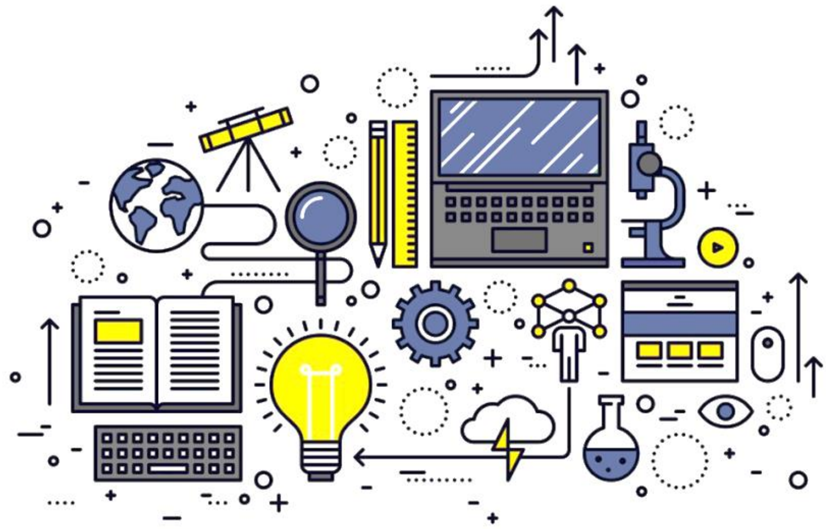
St Andrew's Junior College

PHYSICS



H2 Physics Summary

H2

Name: ____________________ Class: ___________

Data

speed of light in free space	$c = 3.00 \times 10^{8}$ m s^{-1}
gravitational constant	$G = 6.67 \times 10^{-11}$ N m^{2} kg^{-2}
acceleration of free fall	$g = 9.81$ m s^{-2}

Formulae

uniformly accelerated motion	$s = u\,t + \frac{1}{2}\,a\,t^2$
	$v^2 = u^2 + 2\,a\,s$
pressure	$p = \frac{F}{A}$
gravitational potential	$\phi = -\frac{Gm}{r}$
displacement of particle in s.h.m.	$x = x_o \sin \omega t$
velocity of particle in s.h.m.	$v = v_0 \cos \omega t = \pm \omega \sqrt{x_0^2 - x^2}$

H2 Physics Summary (Syllabus 9478)

(Definitions, Equations, Terms, Qualitative Explanations, Common Mistakes)

Key: ( ) optional but good to include; { } Tutor's comments/Common Mistakes; [ ] Alternative Term

Topic 1: Quantities and Measurements

1. **Base quantities** (with their SI base units/ symbol): length (metre/ m), mass (kilogram/ kg), time (second/ s), amount of substance (mole/ mol), temperature (kelvin/ K), current (ampere/ A).
   Base units: are (a choice of well-defined) units by which all other units are expressed.
   Derived unit: a unit expressed as a product and/or quotient of the base units. (eg. newton, pascal, joule, watt, hertz, coulomb, volt, ohm)

   Unitless quantities: all numbers, trigo functions, log functions ($\log_x$, ln), powers, certain physical constants (eg. refractive index)

2. **Homogeneous** equations**:** An equation is homogeneous if every term on both sides of the equation have the same SI base units. A physically correct equation must be homogeneous.

3. **Prefixes:** to recall certain prefixes & their decimal equivalents**:**
   - 10^{12} tera (T), 10^9 giga (G), 10^6 mega (M), 10^3 kilo (k), 10^{-1} deci (d), 10^{-2} centi (c), 10^{-3} milli (m), 10^{-6} micro (μ), 10^{-9} nano (n), 10^{-12} pico (p).
   - Sample TYS qns: decimetre = 10^{-1} m; megametre = 10^6 m; 500 cN = 500 x 10^{-2} N (2016 P1Q1)

4. Making reasonable **estimates** of certain physical quantities:
   - to give the figure to 1 significant figure. {N08P1Q2, N09P1Q2}
   - need to express a more complicated quantity in terms of other simpler quantities using a formula first.
   - 'correct to an order-of-magnitude' means value quoted is reliable to within a factor of ten or so.
   - Significant figures: rules to follow:
     - All non-zero digits are significant digits
     - Zeros that occur between significant digits are significant digits
     - Zeros to the right of the decimal point and to the right of a non-zero digit are significant

5. **Errors**
   - An error is the difference between the measured value and the 'true value'. A total error can be a combination of both systematic error and/or random error.

6. **Random error**:
   - An error {not reading} which causes measurements to be sometimes larger than the true value and sometimes smaller than the true value.
   - It is equally likely to be positive or negative with respect to true value, and can have different magnitudes.
   - Can be reduced (eg. by taking the average of repeated readings, or by plotting a graph to obtain the line or curve of best fit (Note: presence of random errors is represented by the scattering of points about the best-fit line)).
   - Eg. parallax error, non-uniform diameter of wire.

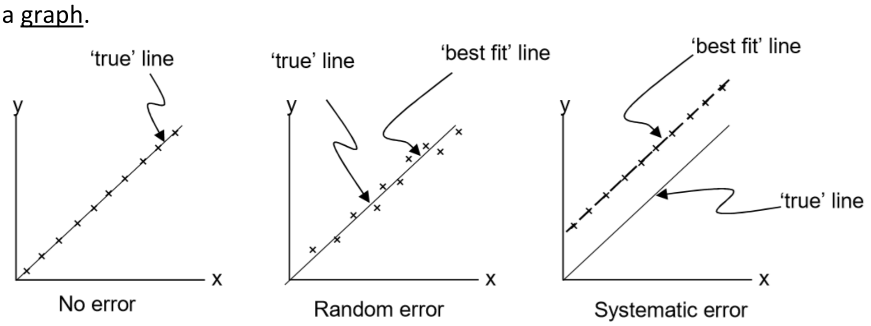
7. **Systematic error**:
   - An error {not reading} which causes measurements to be either, always larger than the true value, or always smaller than the true value.
     Hence in an expt, a systematic error is of the same magnitude & with the 'same sign'.
   - Cannot be reduced by taking the average of repeated measurements.
   - Can be eliminated {for eg, if a faulty instrument is replaced, if the experimental technique/procedure is improved, or a different experimental approach is used}.
   - Eg. zero error, incorrect calibration of the scale.

8. Accuracy*:*

- refers to the degree of agreement between the result of a measurement and the true value of the quantity.
- Note: If several readings of a quantity are taken, the "result of the measurement" refers to the mean (average) value of the readings. We take the average of the readings and compare it with the true value to see if it is accurate.
- is a measure of the magnitude of the systematic error present; high accuracy implies a small systematic error.
- check by looking at average value in a table or if gradient or y-intercept agree with the equation in a graph.

9. Precision*:*

- refers to the degree of agreement [scatter, spread] of repeated measurements of the same quantity. {Note: regardless of whether or not they are correct with respect to the true value.}
- is a measure of the magnitude of the random errors present; high precision implies a small random error.
- check by looking at how close the repeated values are in a table or if the data scattering is close to the best fit line in a graph.



10. For a quantity x = (2.0 $\pm$ 0.1) mm,

Actual [Absolute] uncertainty, Δx = 0.1 mm

Fractional uncertainty, $\dfrac{\Delta x}{x}$ = 0.05

Percentage uncertainty, $\dfrac{\Delta x}{x} \times 100\%$ = 5 %

11. To determine the overall (compound/consequential) uncertainty Y that is measured from other quantities, we follow the steps:

- Make the required quantity the subject of the equation
  (Sometimes the equation has to be deduced, if not already given by question)
- Categorise the problem (purely +/- or $\times$/$\div$ for simple equations OR complex equations)
- Decide on the method (Rules 1, 2, 3)
- Calculate the value of the unknown quantity (leave in many s.f. first)
- Calculate and express the uncertainty (e.g. ΔY) to 1 s.f.
- Write the calculated value (Y) to the same place value as the uncertainty (ΔY).
  Express answer to standard form (not always compulsory but good practice)

Here is an overview of the 3 Rules:

Rule 1	Rule 2	Rule 3
Add or subtract (for simple equations)	Multiply, divide or power (for simple equations)	(for complex equations)
Sum all contributing *absolute* uncertainties If $Y = nA \pm mB$, (n & m are constants), $\Rightarrow \Delta Y = \lvert n \rvert \Delta A + \lvert m \rvert \Delta B$ Note: Retain the coefficients	Sum all contributing *fractional* uncertainties If $Y = aX^n Z^m$, (a, n & m are constants) $\Rightarrow \frac{\Delta Y}{Y} = \lvert n \rvert \frac{\Delta X}{X} + \lvert m \rvert \frac{\Delta Z}{Z}$ Note: Coefficient *a* is ignored as it is considered error-free. Power *n* (or *m*): bring down, modulus it, and treat it as coefficient!	Calculate the maximum and minimum of the quantity *Y*. Then determine the absolute uncertainty ΔY. $\Rightarrow \Delta Y = ½ (Y_{max} - Y_{min})$

12. For **Practical Exam** (not theory papers): Absolute Uncertainty is generally estimated to:
- (a) 1 x smallest div of instrument if conditions of measurement are **'ideal'**
  (**Exceptions**: stopwatch timing where precision is 0.1 s; analogue meters, measuring cylinder, thermometer (steady temp) where ½ smallest division is required instead)
- (b) In general, Δx = (3 to 5) x smallest div if circumstances of measurement are '**challenging'**.

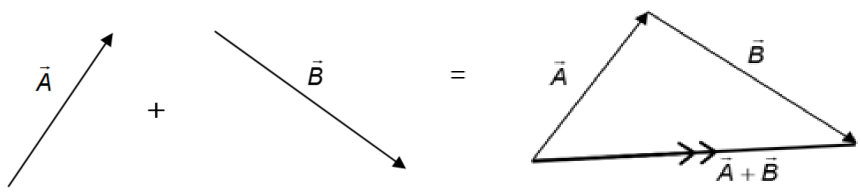
13. Actual uncertainty must be recorded to only 1 significant figure (1 sf).
The number of sig fig to be recorded for a calculated quantity x is determined by the position of the last digit of its actual error Δx.

Eg: If g has been calculated to be 9.80645 m s^{-2} & Δg has been calculated to be 0.04848 m s^{-2},
then, Δg should be *recorded* as 0.05 m s^{-2} {1 sf } & g must then be recorded as (9.81 $\pm$ 0.05) m s^{-2}.
If Δg = 1.23 m s^{-2}, then g should be recorded as (10 $\pm$ 1) m s^{-2}.
If the actual uncertainty is recorded (to 1sf) to its *tenth or hundredth place*, the number of sf for its calculated quantity should follow to its tenth or hundredth place respectively.

Eg: If speed of sound, v has been calculated to be 330.80645 m s^{-1} &
Δv has been calculated to be 11.12 m s^{-1}, Δv must be recorded as 10 m s^{-1} {1 sf };
v must then be recorded as (330 $\pm$ 10) m s^{-1}.
If Δv = 102 m s^{-1}, then v should be recorded as (300 $\pm$ 100) m s^{-1}.

14. A **scalar** quantity is a quantity which has only magnitude but no direction.
A **vector** quantity has both magnitude and direction and thus is only completely described if both its magnitude & direction are known.

15. Vector Addition



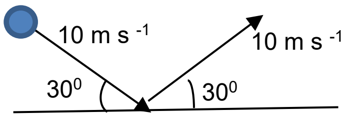
Vector addition obeys:

- commutative law, i.e. $A + B = B + A$ (i.e. order of addition is not important)
- associative law, i.e. $(A + B) + C = A + (B + C)$

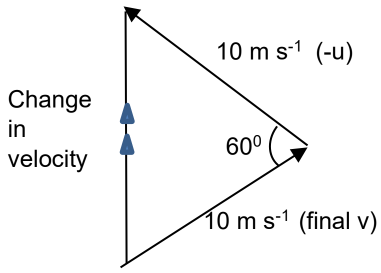
Note: in equations used for problem-solving, such as kinematics, forces equations, take care to substitute negative signs for vectors that are pointing in opposite directions to the sign convention taken for the positive direction!

16. Vector Subtraction: Final Vector + (<u>negative</u> of Initial Vector)

Eg: A ball hits the surface of the road with an initial speed of 10 m s^{-1}, 30^0 to the surface and rebounds with a speed of 10 m s^{-1}, 30^0 to the surface. Determine the magnitude of the change in velocity.



Soln: **Change in velocity** = final velocity – intial velocity {by definition, not "initial – final" }
= **final velocity + (negative of initial velocity)**



{The **sequence of drawing the 3 vectors** is important, using triangle law method}
By cosine rule,
change in velocity = $(10^2 + 10^2 - 2(10)(10) \cos 60^0)^{1/2}$
= 10 m s^{-1}

17. Resultant vector calculation via resolution & addition of component vectors (Components Method):

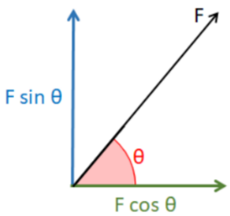
Step 1: Resolve vectors into their 2 perpendicular components (need not always be in x- and y- directions, esp for objects on a *slope. When there is acceleration, it is in dir of acceleration and perpendicular to it*).

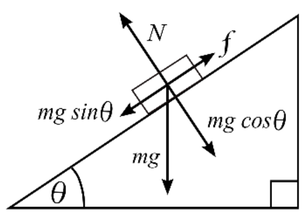
Step 2: Sum all the x-components (Ax & Bx) to obtain vector Rx ; and sum all the y-components (Ay & By) to obtain vector Ry.

Step 3: Rx and Ry are then added "vectorially" using Pythagoras theorem to obtain the magnitude of resultant R.

Step 4: The direction, angle $\propto$ of R (with respect to the horizontal) is obtained using the formula,

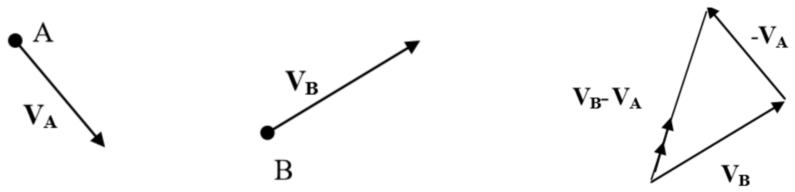
$\tan \propto = \dfrac{|R_y|}{|R_x|}$





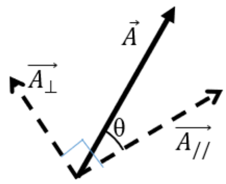
18. Relative velocity:

The apparent velocity of B when observed from A is called the "**velocity of B relative to A**", i.e. $\boldsymbol{V}_{BA} = \boldsymbol{V}_B - \boldsymbol{V}_A$

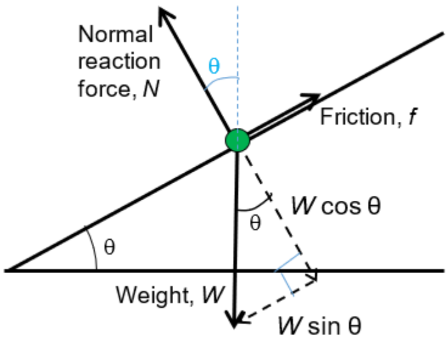


19. Resolution of vectors into other directions (eg. Slopes)

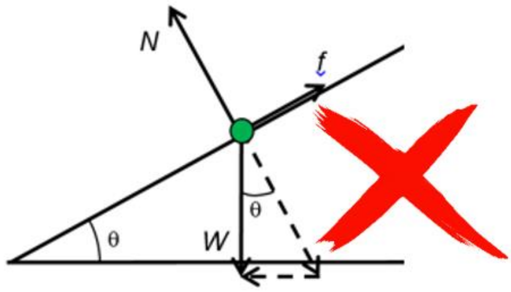
Note that a vector may be resolved into any 2 perpendicular directions that may not include the horizontal and vertical directions (see right diagram).



Thus for objects on a (rough) slope:



Common mistake!!!



Topic 2: Forces and Moments

1. **Field of force:** a region of space within which a force is experienced.
   - **Gravitational field:** A region of space in which a *mass* experiences an attractive force due to the effect of another mass.
   - **Electric field:** A region of space where *an electric charge* experiences an attractive or repulsive force due to the effect of another charge.
   - **Magnetic field**: A region of space in which a *moving electric charge* or a current-carrying conductor experiences a force (that is perpendicular to the magnetic field).

2. Egs of Types of Forces:
   - Normal reaction: always perpendicular to the surface.
   - **Contact**/Reaction force: vector sum of normal reaction and friction.
   - **Viscous force** (drag): present when an object moves through a fluid. E.g. air resistance. Magnitude of viscous force always increases with the speed of the object (BUT it is is NOT necessarily $\propto v$ unless question states so)
   - Friction: always opposes relative motion; dissipative in nature.
   - Tension and compression: act along the string, away from for tension/ towards for compression the object.

3. **Weight:** force experienced by a mass in a gravitational field. $W = mg$
   Mass: a measure of the amount of matter in a body; hence**,** a measure of the **inertia** the body
   Inertia is the property of a body which **resists change in its motion** (2022 P3Q1(a))

   Apparent weight refers to the Normal reaction N that the surface/ weighing scale exerts on a body.

An astronaut in a spacecraft orbiting the Earth may be described as weightless. Explain why this is so. [2]
• Astronaut and spacecraft are both accelerating at the same value towards the Earth. • No (normal) reaction force experienced by the astronaut.

4. Hooke's law:
   If the limit of proportionality {Examiner may use "elastic limit"} is not exceeded, the extension (produced in a material) is directly proportional to the force/ load applied.

   i.e. $\boxed{F = k\,x}$ {k: force constant; spring constant, for a spring. x can also be the compression}

5. Pressure **difference** betw 2 pts within a liquid & separated by a vert dist h $\boxed{p = \rho\, g\, h}$

6. **Upthrust**: **Origin**: an upward force exerted by a fluid on a body in a fluid due to the **difference in pressure** between the upper and lower surfaces of the object.

Explain why the liquid exerts an upthrust on the sphere.
• There is upward pressure on the bottom of the sphere and downward pressure on the top. • Since (hydrostatic) pressure increases with depth, the pressure at the bottom is greater than the pressure at the top. • Due to the pressure difference between the bottom of the sphere and the top of the sphere, there is a net upward force exerted by the liquid known as upthrust.

7. When an object floats, the upthrust acting on it must be equal in magnitude and opposite in direction to the weight of the object since it is **in vertical equilibrium**.
   i.e. Upthrust = weight of object (assuming no other forces present)

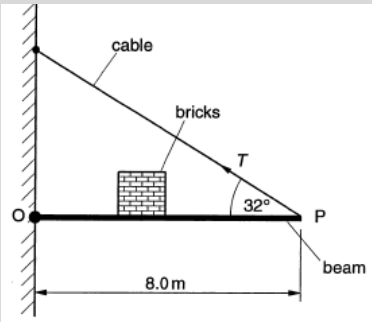
8. **Moment of a force:** is defined as the product of the force and the perpendicular distance of its **line of action from the pivot/ axis of rotation.**

A couple is a pair of equal and opposite forces, whose lines of action do not coincide; hence they are parallel forces. (Hence it tends to produce rotation only.)
Torque of a couple = product of **one of the forces** of the couple and the perpendicular distance between the lines of action of the forces.

9. **2 Conditions for Equilibrium** of an "extended" object {i.e. not a point mass} (N06P2Q6a):
   (1) The resultant force is zero.
   (2) The resultant moment **about any point** equals zero.

Principle of Moments:
For a body to be in rotational equilibrium, the sum of all the anticlockwise moments *about any point* must be equal to the sum of all the clockwise moments about that same point. {Must explicitly state the point used for pivot in problem solving!!}

a. **Exam Tip**: If a force, of either **unknown magnitude or direction**, acts on a body, it is often necessary/convenient to take moments about **a certain pivot through which this unknown force acts** if it is not the unknown to be determined first (so that its moment would not need to be considered).}
b. To find the unknown force later in the question, the next step is usually to apply translational equilibrium (i.e. $F_{net} = 0$ in 2 perpendicular directions).



The figure below shows a uniform metal beam pivoted at one end and held in equilibrium in a horizontal plane by a cable attached to its other end.

The beam has mass M and length 8.0 m. The cable makes an angle of 32° to the beam and has a tension T. A pile of bricks of total mass m is placed on the beam.

Describe and explain what would happen to the tension T in the cable as the pile of bricks is slowly moved towards the end P of the beam.

- By principle of moments, taking pivot at O, as bricks moves towards the end *P*, clockwise moments increases.
- Hence the anticlockwise moments by *T* have to increase.
- So *T* will increase since the perpendicular distance from *T* to O is fixed.

10. Centre of gravity of an object is defined as that single point through which the entire weight of the object may be considered to act.

11. If a mass is acted upon by 3 forces *only* and is in equilibrium, then
   - the **lines of action** of the **3 forces** must pass through a **common point** (i.e. the 3 forces are concurrent).
   - When a vector diagram of the three forces is drawn, the forces will form a closed triangle (vector triangle), with the vectors pointing in the same orientation around the triangle.

Topic 3: Motion and Forces

1. **Displacement** is defined as the distance moved in a specific direction.
   Distance travelled is the total length covered irrespective of the direction of motion.

2. Speed is defined as the distance travelled per unit time OR the rate of change of distance travelled
   {N04P3Q1: Explain why it is technically **incorrect to define speed as distance *per second***. 1 m}

   Velocity is defined as the displacement per unit time OR the rate of change of displacement
   {**NOT**: **displacement "over time", nor displacement "per second", nor "rate" of change of displacement "per unit time", nor speed in a particular direction, nor rate of change of "distance"**.}

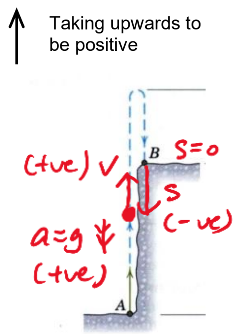
   The gradient of a displacement-time graph is the instantaneous velocity.
   The area under a velocity-time graph is the change in displacement {$s \neq vt$ unless v = constant}.

3. Acceleration is defined as the rate of change of velocity.

   The gradient (tangent for curve) of a velocity-time graph is the instantaneous acceleration.
   The area under an acceleration-time graph is the change in velocity.

4. Equations of motion for straight line motion at **constant** acceleration:

   (1) $\boxed{v = u + a\,t}$ derived from definition of acceleration: $a = (v - u) / t$
   (2) $\boxed{s = \frac{1}{2}\,(u + v)\,t}$ derived from the area under the *v-t* graph
   (3) $\boxed{v^2 = u^2 + 2\,a\,s}$ derived from equations (1) and (2)
   (4) $\boxed{s = u\,t + \frac{1}{2}\,a\,t^2}$ derived from equations (1) and (2)



   Two conditions to apply these equations: motion in a straight line and magnitude of the acceleration is constant. {Eg, air resistance must be negligible}

   Problem solving:
   - Set a sign convention (the direction where vectors are taken to be positive). If the vectors (velocity, acceleration, displacement) are opposite to the direction taken as positive, add a negative sign when substituting the values into the equation. Note that displacement vector points from the position which is defined as zero to the position of the object.

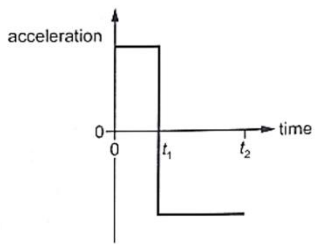
   - Consider *s, u, v, a, t* and note the values given in the question and what you want to find and what you do not have and is not finding. Then select the right equation to use eg $v = u + at$ has no *s.*

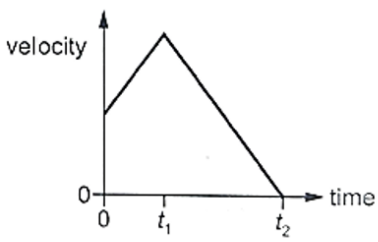
   - If object A is moving with object B and released, the initial velocity of object A is the velocity of object B. eg a ball thrown from a hot air balloon moving upwards with velocity v will have an initial velocity v upwards too.

5. Graphs

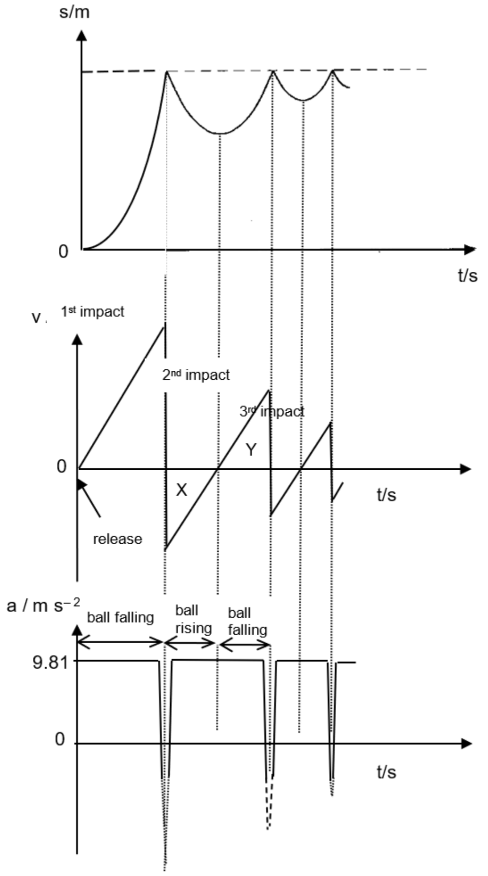
   Questions on *x-t* , *v-t* and *a-t* graphs are important

   Problem solving: it is important to be proficient in drawing one graph given another using gradient (it is harder to use area under the graph as it does not tell you the shape of the graph BUT area is sometimes calculated to get the exact values for precision of the drawing). If you are given a-t graph, deduce from the graph, the sign of a and how it is changing and that is the sign of the gradient of v (as a = dv/dt) and how the gradient is changing. You can then draw the shape of the v-t graph. The change in v on the v-axis is given by the area under the a-t graph.





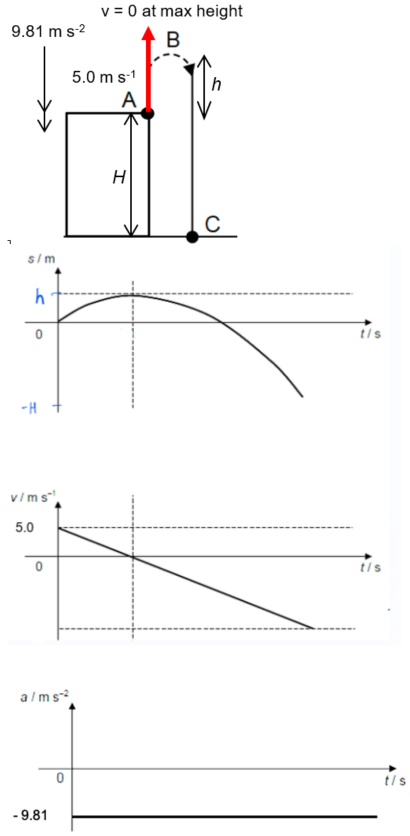
5. Graphs representing motion of a bouncing ball



{the velocity of the ball just before it hits the ground is NOT zero}

This is often described as "ball hitting the ground" but means "just before hitting the ground" when the only force acting on it is still its weight (thus acceleration still g = constant and can use the kinematic equations as you consider free all) without the large upwards normal contact force when it hits the ground.

6. Graphs for an object thrown vertically upwards from a height above the ground



7. **Linear momentum p** of a body is defined as the product of its mass and velocity. i.e. $p = m\,v$
Units: kg m s^{-1}

8. **Newton's First Law:**
A body at rest will stay at rest, and a body in motion will continue to move at constant velocity, unless acted on by a resultant external force

9. **Newton's Second Law**:
The rate of change of momentum of a body is (directly) proportional to the resultant force acting on the body and is in the same direction as the resultant force

10. **Force** on a body is defined as the rate of change of momentum of the body {i.e. $F=\frac{d(mv)}{dt}$ } and it acts in the direction of the change in momentum**.** {N2010 P3 Q6a, 2 m}

System	Equation	Example
Constant mass with varying velocity	Net force, $F = \frac{m(v-u)}{t} = ma$ acting over a finite time interval t	Forces acting on a point mass Forces acting on connected bodies
Constant velocity with varying mass	Thrust, $F = v\frac{dm}{dt}$ with mass varying at dm/dt and moving at velocity v	Expelled fuel from a rocket providing a forward thrust (in opp dir to the dir of expelled fuel) on the rocket due to Newton's 3rd law as the mass of the rocket decreases as dm/dt when the fuel is expelled at velocity v

Problem solving:

- Draw FBD (where the object with mass m is isolated and ALL forces acting on it are identified) and apply Fnet = ma. Remember the forces involved are usually weight W, tension T, normal contact force N, friction f, hinge force F (can be called contact force), viscous force (air resistance usually), upthrust U.

- For connected bodies, the acceleration has to be the same for both bodies. FBD can be drawn for the system or for each of the mass involved.

11. **Newton's Third Law:**
The force exerted by one body on a second body is equal in magnitude and opposite in direction to the same type of force simultaneously exerted by the second body on the first. {Ref: Examiner's report for N01P3Q1}

    - These two forces formed an action-reaction pair, always act on **different bodies**; hence, they cannot cancel each other out.
    - They are of the **same type** of force {so if the action force is a gravitational force, the reaction must also be gravitational in nature}
    {Must know how to identify forces which form an action-reaction pair. Note: ***weight and the normal reaction are not an action-reaction pair as they are different type of force!*** }
    - If body A acts on body B, the F-t graph of body A and body B are reflection of each other in the time-axis as $F_{AB} = F_{BA}$ to reflect Newton's 3rd law.

Topic 4: Projectile Motion

1. Projectile Motion

At any instant: **instantaneous velocity, *v* =** $\sqrt{v_x^2 + v_y^2}$

tan θ = $\dfrac{v_y}{v_x}$ (θ: direction of tangential velocity wrt horizontal) {NOT: tan θ = $\dfrac{s_y}{s_x}$ }

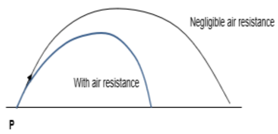
Problem solving:

- Consider vertical and horizontal direction separately. Set sign convention.
- $KE = ½ mv^2$ where v = $\sqrt{v_x^2 + v_y^2}$
- *KE* at max height is NOT zero as there is still the horizontal component of the velocity

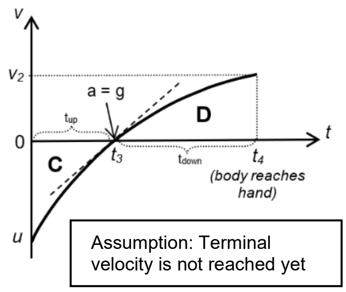
2. Effect of Air resistance

- Projectile motion path with air resistance: Max height & Range reduced. Path is asymmetrical.

- Explanation for the difference in paths:
  Air resistance opposes the motion of the body and acts against both the horizontal and vertical motions. Hence the body's maximum height is reduced and the maximum range is also reduced.
  As the body's horizontal velocity keep decreasing, the horizontal displacement per unit time keeps decreasing. The path is thus asymmetric.



- Suggest a reason, how time taken for object to reach maximum height t_{up} < time taken for object to return to ground from maximum height t_{down}:
  Upward journey, net (decelerating) force = Weight + Air resistance > Weight, while on downward journey, net (accelerating) force = Weight – Air resistance < Weight since air resistance acts opposite to direction of motion.
  Since upward displacement (Area C) = downward displacement (Area D), the time for upward motion < time for downward motion



3. Terminal Velocity (v_t) of object falling in air:

At terminal velocity, Weight = Upthrust + $k\, v_t^n$ { $k\, v_t^n$: *viscous force*, *n*: unknown constant of the medium it is travelling in}

In the case of object falling through air where upthrust is insignificant compared to the viscous force, usually v_t is reached when Weight = Air resistance where air resistance is the viscous force (equation for air resistance will be given if calculation is needed. Generally, air resistance increases with speed)

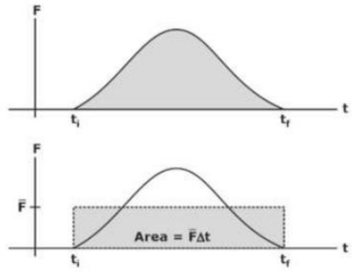
Topic 5: Collisions

1. Impulse of a force ***I*** is defined as the product of the force and the time Δt during which it acts.

Impulse is a vector.

Case 1: For a constant net force over the duration Δt -Impulse, $\boxed{I = F \times \Delta t}$

Case 2: For a variable force – Impulse, I = $\boxed{\text{area under the } F\text{-}t \text{ graph} = \int F\, dt}$



$\overline{F}$ = average of the varying force

- It is <u>equal in magnitude</u> to the <u>change</u> in momentum of the body. <u>{Incorrect to define impulse as change in momentum}</u>
- Hence the change in momentum of the body is equal in magnitude to the area under the (net) force-time graph. {For *F-t* graph of irregular shape, use "<u>count-the-squares</u>" method to estimate the area}
- An average force $\overline{F}$ can be found (Δp = area under *F-t* graph = $\overline{F}t$ where t is duration of impulsive force)

Discuss how seat belts and air bags in a car ensure greater safety. [2]
• Seat belts and air bags <u>increase</u> the time for which the <u>same</u> change in momentum of the person occurs, thus reducing the decelerating force acting on him. • Air bags also reduce the pressure on the driver as a result of its larger area over which the force acts.

2. Principle of <u>C</u>onservation of <u>L</u>inear <u>M</u>omentum (PCM):

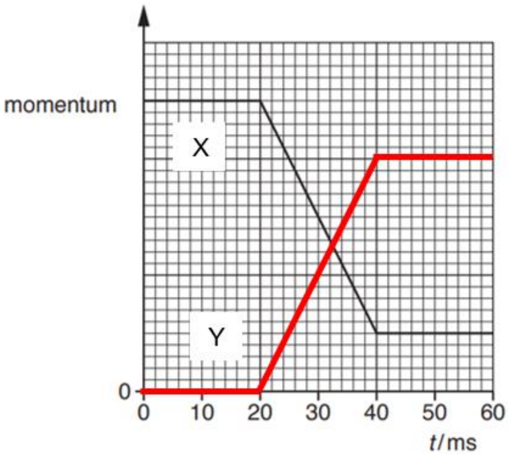
For a <u>system</u> of <u>interacting bodies</u>, the <u>total momentum</u> of the bodies (i.e. momentum of the system) remains constant, provided <u>no net force</u> acts on the system. {incorrect to say "no external force", incorrect to say "momentum is conserved" which is a circular definition repeating the stem of the question 2023 P3 Q9}

or, the total momentum of a closed system is constant,

i.e. $\boxed{m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2}$, if net $F = 0$ {applicable for all interactions}

Note: Total momentum <u>*during*</u> the interaction is also conserved. {See N08P1Q6}

A stone is dropped from a point a few metres above the Earth's surface. Considering the system of the stone and the Earth, discuss briefly how the principle of conservation of momentum applies before the impact of the stone with the Earth.
• Initial total momentum of the system of stone and Earth is zero. • As the stone falls and accelerates towards the Earth, its downward momentum increases. • Since there is no net external force acting on the system of stone and Earth, by the principle of conservation of momentum, the total momentum of the stone-Earth system is unchanged and thus zero. • Thus the Earth must accelerate upwards towards the falling stone, with the magnitude of upward momentum of Earth equals downward momentum of stone at all times. {Due to the large mass of the Earth, its velocity is negligible and hence, its movement is not noticeable.}
When a man falls from a height and undergoes free fall, his momentum increases. Explain if principle of conservation of momentum is violated.
• If the system consists of only the man, the gravitational force on the man by the Earth will be an external force. Since there is a net external force acting on the system, momentum is not conserved. • If the system consists of the man and the Earth, gravitational force on the man by the Earth will be an internal force. Since there is no net external net force, momentum is conserved.
Does the principle of conservation of momentum apply in cases where two colliding bodies lose kinetic energy as a result of sticking to one another at the point of collision? Explain your answer with reference to Newton's third law. [2]
• Yes. According to Newton's Third Law, the two bodies exert equal and opposite forces on each other. • Hence the impulse (or change in momentum) of body A will be equal but opposite to that of body B. • Taken as a whole system, the total momentum of the colliding bodies remains unchanged, and the principle applies.



A block X slides in a straight line along a horizontal frictionless surface collides with a stationary block Y. Both blocks have the same mass. Block X makes contact with block Y at time t = 20 ms. Duration of collision is from 20 to 40 ms.

No net force (frictionless surface) – velocity and thus momentum (just) before and (just) after collision is constant. Pf = Pi = 25 units (Conservation of momentum)

Note: Block Y may not have zero velocity initially. Block Y can have negative velocity initially if block X and Y are moving in opposite direction before collision. The total momentum is still conserved (taking into account the direction as momentum is a vector)

3. Types of collisions:

(Perfectly) elastic collision	Both momentum and (total) kinetic energy of the system before and after collision are conserved. **During** collision, KE is converted into other forms of energy (e.g. deformation of rubber balls during collision) but is recovered (balls regaining their shapes) such that KE **before and after** collision are the same.
Inelastic collision	Only momentum is conserved before and after collision, total kinetic energy is not conserved.
Perfectly inelastic collision {no need to memorise as question will tell you bodies moved together as one after collision}	Only momentum is conserved before and after collision, and The particles stick together {move with the same velocity} after collision.

For all **elastic collisions**,

relative speed of approach = relative speed of separation**:** $\boxed{u_1 - u_2 = v_2 - v_1}$

{this is a vector equation where u_1, u_2, v_1 and v_2 are all vectors and the signs for the four vectors must be substituted correctly}

conservation of kinetic energy: $\boxed{\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2}$ more tedious, not recommended

{this is a scalar quadratic equation and there will be two soln. The correct soln must be chosen based on context given}

4. Problem Solving:

For inelastic collision: Use PCM
For elastic collision: Use PCM and the “relative speed equation”.

Note:

- PCM applies to all interacting systems not subjected to a net force including explosion e.g. two magnet with north pole facing repelling each other, charges of same sign repelling, not just collision.
- Momentum and velocity are vectors thus direction must be taken into account in applying PCM and relative speed equations
- Always draw diagram of before and after collision/explosion to help keep track of the sign convention.

Topic 6: Energy and Fields

1. **Energy** is the ability to do work. It exists in many possible forms: Mechanical (i.e. kinetic, potential energies), chemical and thermal. It is a scalar quantity. S.I. unit of energy: joule (J).

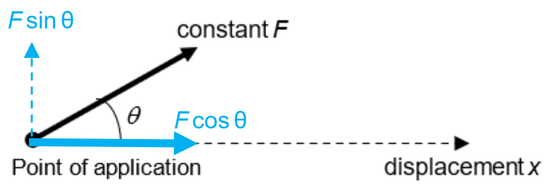
2. **Principle of conservation of energy** (PCE) : The total energy of an isolated system is constant. Energy can neither be created nor destroyed but can transferred from one form /store to another.

3. **Work done** by a force is defined as the product of the force and the displacement in the direction of the force. It is a scalar quantity.

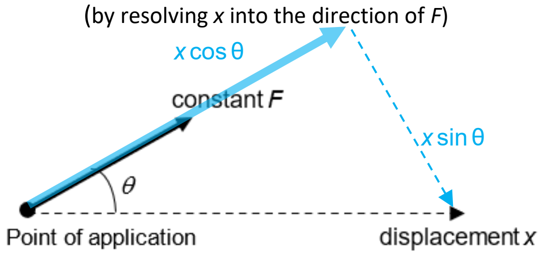
4. Work done by a constant force F:

<table>
<tr>
<td>Product of displacement and component of force ($F\cos\theta$) along the same direction as displacement (draw in below diagram):

$W = (F\cos\theta)(x)$ ....(6.2a)
(by resolving F into the direction of x)


</td>
<td>Product of force and component of displacement ($x\cos\theta$) along the same direction as force (draw in below diagram):

$W = (F)(x\cos\theta)$........(6.2b)
(by resolving x into the direction of F)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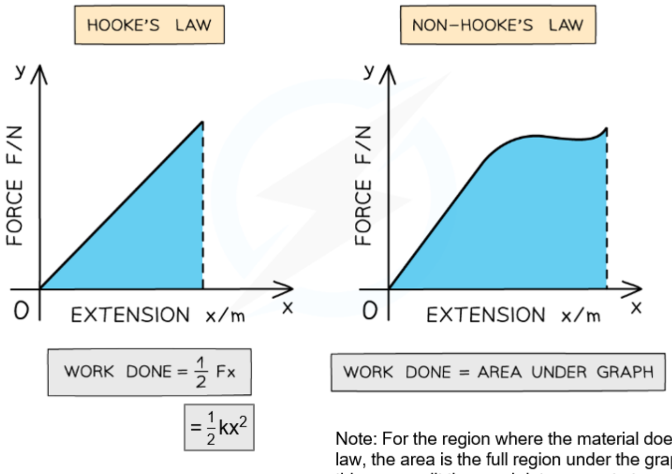
5. Work done by a variable force F parallel to x:
   $W = \int_{x_i}^{x_f} F\,dx$ = Area under F –x graph
   Area under the F – x graph can be determined (approximately) by method of "counting the squares".

6. Work done by a variable (elastic) force – **Elastic Potential Energy (EPE) / Strain energy**



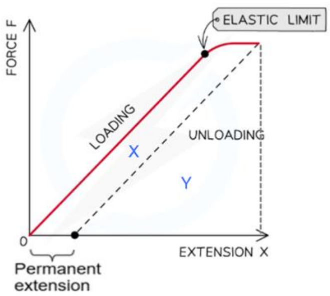
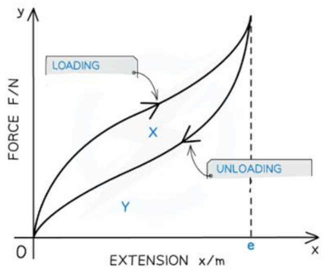
Note: For the region where the material doesn't obey Hooke's law, the area is the full region under the graph. To calculate this area, split the graph into separate trapeziums and add up the individual areas of each, OR use method of "counting the squares".

Note: F-x axes may be swapped in the A-level exam. (See Tut Q11) and x-axis may be the total length instead of extension .

Problem-solving:
Many students made the mistake of calculating change in EPE as $\frac{1}{2}k(\Delta x)^2$

Change in EPE should be = ½ kx_f^2 – ½ kx_i^2
Mathematically, $\frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2 = \frac{1}{2}k(x_1^2 - x_2^2) \neq \frac{1}{2}k(x_1 - x_2)^2$

Areas under F-x graphs (with loading & unloading curves):

Materials that can suffer permanent deformation (e.g. Metal)	Elastic materials (e.g. Rubber)
	

Area	What it represents
X	Work done to cause permanent deformation in the spring; and this energy is not recoverable
Y	Elastic potential energy stored in material & recoverable when force applied is removed
X + Y	Work done by the force to stretch the material to extension e

7. **Kinetic energy**, $\boxed{KE = ½\, m\, v^2}$

- <u>need to derive</u> using: definition of work by net force & work by net force converted to KE **only .** This requires the force to be a net force in the horizontal direction so the mass does not experience a change in GPE, & $v^2 = u^2 + 2\,a\,s$

8. To distinguish **Gravitational PE** & **Elastic PE** {Potential Energy (PE) is defined as the stored energy available to do work}
   - GPE is the potential energy possessed by a mass due to its <u>position</u> {or height or distance} *in the gravitational field of another mass*
   - EPE of a system is due to its <u>deformation</u> {or stretching or compression}.

9. **GPE** $\boxed{GPE = mgh}$ where *g* remains constant over distance *h {near Earth's surface}* and h is measured with reference to a position defined as h = 0, GPE = 0.
   - <u>need to derive</u>, using: definition of work & work converted to GPE only
     This requires the force exerted by the external agent to be = *mg* in magnitude but in opposite direction to weight, so that the mass does not experience a change in KE.

10. **PE, *U***, of a body in a force field {whether gravitational or electric field} is related to the force *F* it experiences by:
    $\boxed{F = -\frac{dU}{dx}}$ {see also Topic 8}

11. Relationship between Work Done, Change in Energy:

Work done by an applied (external) force, WD, on a system/object is related to the energy changes by:

WD = ΔGPE + ΔKE + ΔEPE + Thermal Energy

where:

- WD = Work done by the applied (external) force F on the system
- ΔKE = Change in kinetic energy (½ mv2 - ½ mu2)
- ΔGPE = Change in gravitational potential energy (mgΔh)
- ΔEPE = Change in elastic potential energy (½ kxf 2 - ½ kxi 2 for a spring)
- Thermal Energy = Energy lost as heat due to friction, air resistance, or internal energy dissipation

12. Problem Solving involving Conservation of Energy

Two approaches are:

1. Loss in GPE/KE/EPE = Gain in KE/GPE/EPE, if no dissipative forces (e.g. friction) are involved.
2. Total initial energy = Total final energy

13. Power is defined as the work done per unit time {Not "work done per *second*"}, or the rate at which energy is transferred, i.e. $\boxed{P = \frac{W}{t} = \frac{E}{t}}$

Average Power (over a given finite time interval) = $\langle P \rangle = \frac{W}{t} = \frac{E}{t}$

Instantaneous Power, $P = \frac{dW}{dt} = \frac{dE}{dt}$

If a constant force F is applied and does work over a displacement s in time t, then the power applied

$\boxed{P = F v}$

14. Efficiency is defined as the ratio of (useful) output energy of a machine to the input energy.

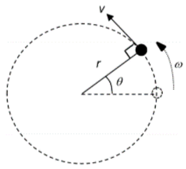
$$\boxed{\text{Efficiency} = \frac{\text{(Useful) Output Energy}}{\text{Input Energy}} \times 100\,\% = \frac{\text{(Useful) Output Power}}{\text{Input Power}} \times 100\,\%}$$

15. Energy Consumption in **kWh**

Energy consumed over a time t is $E = P\,t$

Energy in units of kilowatt-hours (kWh) is the product of power in kilowatts and time in hours.

Topic 7: Circular Motion



1. Angular displacement θ is defined by $\boxed{\theta = \frac{s}{r}}$

Unit: radian (rad). Vector quantity.

- One (the) radian is defined as the angle subtended at the centre of a circle by an arc equal to the radius of the circle.
- For one complete revolution, an object rotates through 360°, or 2π rad.

2. Angular velocity ω is defined as the rate of change of angular displacement (about the centre of the circle)

i.e. $\boxed{\omega = \frac{\theta}{t} = \frac{2\pi}{T}}$ (for a complete revolution)

3. Linear [or tangential] velocity, $\boxed{v = r\,\omega}$
is the instantaneous velocity at any point in its circular path.

The velocity of a body moving in a circle at a <u>constant speed</u> changes since its <u>direction changes</u>. It thus experiences an acceleration**,** a net force and a change in momentum.

4. *Explain why an object moving with constant speed in a circle is said to be accelerating.*
Ans: The <u>velocity</u> of the object <u>changes</u> since its <u>direction changes</u>. It thus experiences an acceleration since by definition, acceleration is the rate of change of velocity.

- Candidates are expected to explain qualitatively why the acceleration is <u>radially towards the centre</u> of the circle {hence, called the **centripetal acceleration**}

- Having an acceleration must mean there's a <u>net force</u> (which must similarly be directed towards the centre of the circle by Newton's 2nd law).

5. Centripetal acceleration, $\boxed{a_c = r\,\omega^2 = \frac{v^2}{r}}$ (in magnitude)

6. Centripetal force:

$\boxed{F_c = m\,r\,\omega^2 = m\frac{v^2}{r}}$ gives the **magnitude** of centripetal force

refers to the <u>**resultant** of all the forces that act on a system</u> in circular motion.
{"centripetal" simply means "centre-seeking"}

- Hence, when asked to draw a diagram showing all the forces that act on a system in circular motion, it is <u>wrong to include a force that is labelled as "centripetal force"</u> as that amounts to 'double-counting'.
- In presenting a solution involving centripetal force, it is essential to begin with:
  "The centripetal force is <u>provided by….</u> {resultant of some named forces, e.g., tension, lateral friction, normal reaction, gravitational force, depending on situation}.

7. Candidates are expected to solve problems involving **horizontal** circular motion, e.g., on a <u>banked road (friction), aeroplane turning (lift), conical pendulum (tension)</u> by <u>forming **2 equations**,</u>

e.g., for conical pendulum:

- **Horizontally**, there's centripetal acc; hence, $T_x = m\frac{v^2}{r}$ ---------- eqn (1)
- **Vertically**, there is <u>equilibrium</u> of forces; hence, $T_y = mg$ ---------- eqn (2)

8. Candidates are expected to solve problems involving **vertical** circular motion, e.g.
- "**looping the loop**" in a **roller coaster ride** {Example 13, p 16};
- car moving over a hump such that it **does not lose contact** with road {Example 11, p 14}.

Topic 8: Gravitational Fields

1. **Newton's law of gravitation**:
The gravitational force of attraction F between two **point** masses M and m separated by a distance r is given by

$$\boxed{F = \frac{GMm}{r^2}}$$

where G is the Universal gravitational constant (given in data list), 6.67 x 10^{-11} N m^2 kg^{-2}

or, the gravitational force of attraction between two point masses is proportional to the product of their masses & inversely proportional to the square of their separation.

- Must include "point masses" {N09P3Q5aii}
- Note that r refers to the distance between the *centres* of the masses, not the *radius* of masses.

2. **Gravitational field**
is a region of space in which a mass experiences an attractive force due to the presence of another mass.

Features of a gravitational field: arrows used to rep *directions* of field (i.e. towards mass creating the field), while density of field lines rep *magnitude* of the field strength.

3. **Gravitational field strength, *g***
at a point is defined as the gravitational force per unit mass at that point. {NOT: "force *on* unit mass"}

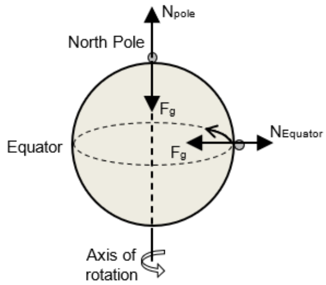
$$\boxed{g = \frac{F}{m}}$$ Unit: N kg^{-1}. Vector quantity.

Field Strength due to a point mass *M*, $\boxed{g = \frac{GM}{r^2}}$

where M = Mass of object "creating" the field. (Notice that m does not appear in this formula)
r = distance measured from centre of M to that point.

- Need to derive using Newton's law of gravitation & definition of g (N09P3Q5a);
- Near the Earth's surface {up to say, about 1000 m from the Earth's surface}, g can be taken to be constant (= 9.81 N kg^{-1}).

4. Explain the **effect** of Earth's **rotation** on the value of **acceleration of free fall** {N07/P3/Q5c}



- Concept: At **equator**, object is both falling freely AND moving in a circle, so it experiences both acceleration of free fall AND centripetal acceleration,

i.e., $F_{net} = ma_{net}$ where $a_{net} = a_{free\ fall} + a_{centripetal}$
$F_g = m(a_{free\ fall} + a_{centripetal})$

ie $F_g/m = a_{free\ fall} + a_{centripetal}$

$\Rightarrow$ g-field strength = acceleration of free fall + centripetal acceleration
$\Rightarrow$ acceleration of free fall = g-field strength – centripetal acceleration
$\Rightarrow$ acceleration of free fall < 9.81 m s^{-1} (gravitational field strength)
i.e. Part of the gravitational field strength is needed to provide for the centripetal acceleration.

- Thus, at the **poles**, where centripetal acceleration = 0, the acceleration of free fall is larger than the acceleration of free fall at the equator.

Explain why **apparent weight** of a mass m at the surface at equator < at poles:

- Concept: At equator, resultant of the gravitational force and the normal reaction N provides the centripetal force to keep the body in circular motion. i.e. $\frac{GMm}{r^2} - N_{equator} = mr\omega^2$

  $\Rightarrow N_{equator} = \frac{GMm}{r^2} - mr\omega^2$

  where $N_{equator}$ is considered the "apparent weight"

  At the poles: $\omega = 0$

  $\Rightarrow N_{poles} = \frac{GMm}{R^2}$

  Thus apparent weight at equator < apparent weight at the poles.

Explain why the **acceleration of free fall near the Earth's surface** is constant: {2015 P3Q6(a)(ii) [3]}

- Use $g = \frac{GM}{r^2}$ where r = radius of Earth (R_E) + height above surface (h)
- Near the Earth's surface, ***h* << *R***
- Thus $g = \frac{GM}{R_E^2}$ = a constant , independent of h

By reference to the pattern of the lines of gravitational force near to the surface of the Earth, explain why the **acceleration of free fall** near to the Earth's surface is approximately constant (CIE M18P42, 3 m)

Any 3 from:

- The lines are radial
- Near the surface the lines are (approximately) **parallel**
- Parallel lines means field strength (g) is constant
- Constant field strength, hence constant acceleration of free fall

Explain why, near the surface of the Earth, the **gravitational field strength is equal to the acceleration of free fall**. Assume effects of Earth's rotation is negligible. {2021 P3 Q4c [3]}

- The acceleration of free fall is the acceleration, *a,* of a mass *m* near the Earth's surface where the net force is provided by the gravitational force of the Earth. (A) [1]
- The gravitational field strength at a point, *g*, near the Earth's surface is defined as the gravitational force per unit mass at that point. [1]
- Since the net force in (A) is provided by the gravitational force exerted by Earth, g = a [1]

5. Satellites in Circular Orbits (round a mass *M*, e.g. the 9 or so planets round the Sun)

$$\boxed{\frac{GMm}{r^2} = mr\omega^2}$$

$$r^3 = \frac{GM}{4\pi^2} T^2$$

$\boldsymbol{T^2 \propto r^3}$ (since $\frac{GM}{4\pi^2}$ is a constant) (Kepler's 3rd law)

- Not allowed to recall & use this eqn
- The r in GMm/r^2 represents **centre-to-centre separation**. The r in $mr\omega^2$ represents **radius of the circular motion**. They are two different things, although they may share the same values. This distinction is exceptionally crucial when we look at **binary star problems**.

State and explain whether the rock will fall to the planet's surface or go into orbit around the planet or travel off into space. [N19 P3 Q2]

- For an object in orbit at height h around the planet of radius R, since $F_G = F_c$, it can be derived that the linear velocity is $v = \sqrt{\frac{GM}{R+h}}$
- For a rock moving past the planet at height h, starting from rest at infinity, its speed can be derived {from PCOE} as $V = \sqrt{\frac{2GM}{R+h}}$
- Since V is **not equal** to v, the rock will not go into circular orbit.

6. Geostationary Satellite

is one which is always above a certain point on the Earth as the Earth rotates about its axis.

For a geostationary **orbit**, satellite must satisfy 3 conditions:

1. have a period = period of Earth's rotation {i.e. 24 hours}
2. rotate from west to east
3. lie on the equatorial plane of Earth

Because the period is 24 h, the satellite will

- be at a certain/particular height from the Earth's surface
- have a certain/particular orbital speed ($v = \sqrt{\frac{GM}{r}}$ which can be derived from $F_G = F_C$)

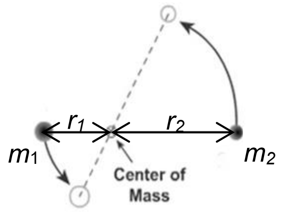
Note: the mass of the satellite does not have to be any particular value & hence its KE, GPE, & the centripetal force are also not particular values (since their values depend on the mass of the geostationary satellite.)

Explain why a geostationary orbit must lie on the **equatorial plane** of the earth:

- The force of attraction to the Earth is to its centre so the circular orbit must be centred at the Earth's centre.
- The plane of the orbit must lie on the equatorial plane of the Earth; otherwise it would have a varying latitude (and thus not be geostationary). {N2003 P3Q2 c}

7. Binary Star system

$$\boxed{F = \frac{G m_1 m_2}{R^2} = \frac{G m_1 m_2}{(r_1 + r_2)^2} = m_1 r_1 \omega^2 = m_2 r_2 \omega^2}$$



- angular velocity ω is the same for both stars
- orbital radius for each star is r_1 and r_2 respectively
- separation between both stars, $R = (r_1 + r_2)$ for the gravitational force equation

8. Gravitational potential, ϕ

at a point is defined as the work done per unit mass (by an external agent) in bringing a **small** test mass from infinity to the point (without a change in its kinetic energy).

$$\boxed{\phi = \frac{U}{m}}$$

Explain why gravitational potential values are *always negative*? {N2011P3Q6a} [2]

- Potential at infinity is defined to be zero.
- As the gravitational force is attractive, the force exerted by the external agent is opposite in direction to the displacement of the test mass, hence work done is negative.
- Thus the potential, (which is the work done per unit mass), always has a negative value.

Gravitational potential due to a point mass *M*, $\boxed{\phi = -\frac{GM}{r}}$ {negative sign must not be omitted

Relation between g and ϕ: $\boxed{g = -\frac{d\phi}{dr} = \text{- gradient of } \phi\text{-}r \text{ graph}}$

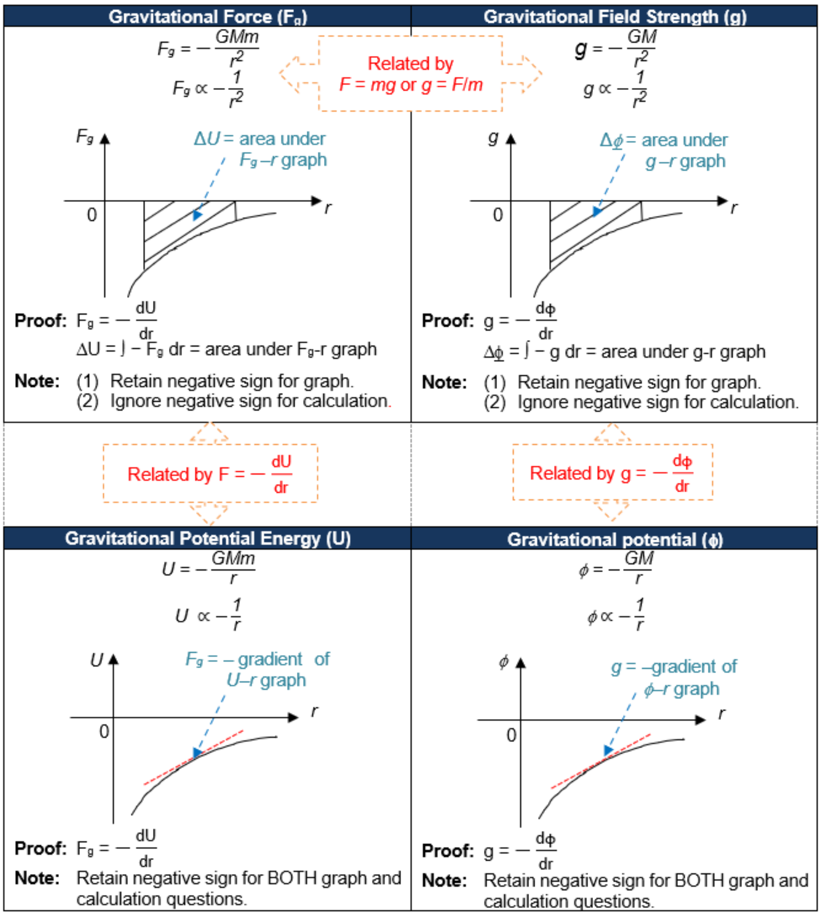
9. Gravitational potential energy *U* of any mass *m* at a point (in the gravitational field of another mass) *M*, is the work done (by an external agent) in bringing *that* mass *m* {NOT: "*unit mass*", or "*a mass*"} from infinity to that point (without a change in KE).

$$\boxed{U = m\,\phi = -\frac{GMm}{r}}$$

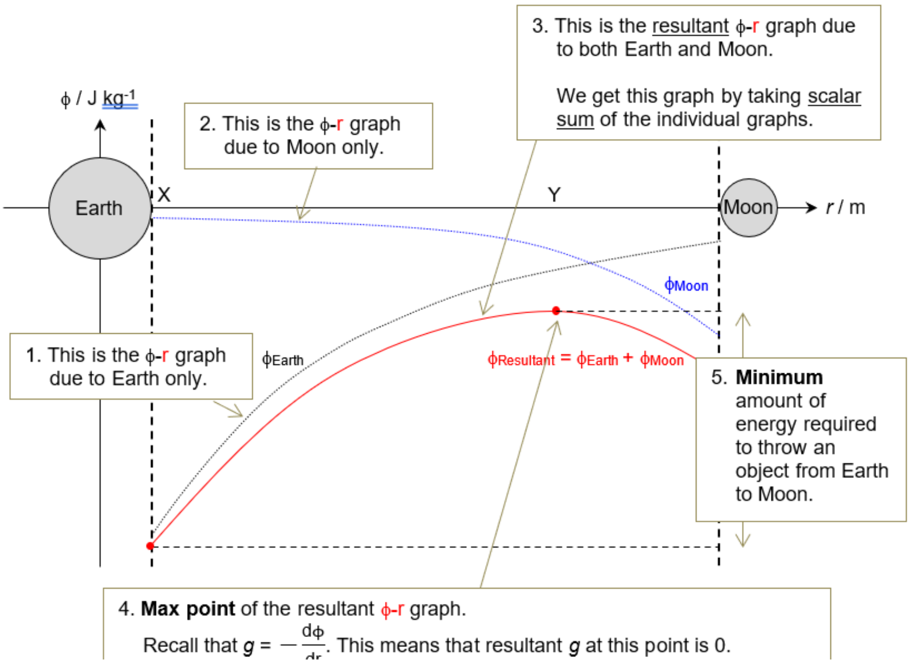
Note: negative sign is part of the magnitude, must be kept for ALL types of questions (both calculation and graphical). At infinity, GPE = 0 { as potential ϕ at infinity is defined to be zero}

Relation between F_g and *U*: $\boxed{F_g = -\frac{dU}{dr} = \text{gradient of } U\text{-}r \text{ graph}}$

10. Graphical Representations of *F, g, U,* ϕ



Potential-distance graphs in region betw 2 planets:



11. Change in GPE, $\boxed{\Delta U = m\,g\,\Delta h}$

{only where *g is (approx.)constant* over the distance Δh; hence applicable only for Δh << Earth's radius; otherwise, must use: $\Delta U = m\phi_f - m\phi_i$ }

Work done (by external agent) on a body of mass *m* to move it from a point to another point,
$\boxed{\Delta U = m\,\Delta\phi}$

12. Escape speed $\boxed{v_E = \sqrt{2gR} = \sqrt{\frac{2GM}{R}}}$, where *R* = radius of planet with mass *M*

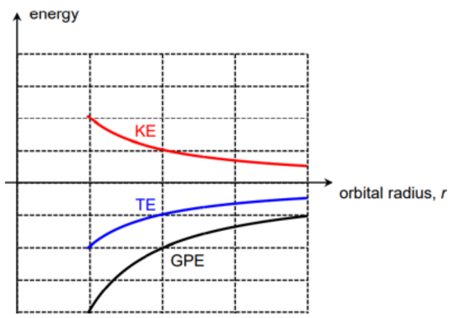
{Need to derive using PCE & definition of escape speed; recall & use of eqn is not allowed}

By Principle of Conservation of Energy,
Initial KE + Initial GPE = Final KE + Final GPE

i.e. $\frac{1}{2} m v_E^2 + (-\frac{GMm}{R}) = 0 + 0$

$v_E = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$ (since $g = \frac{GM}{R^2}$)

13. Energies of an orbiting satellite:



$\boxed{\textbf{KE} = \frac{GMm}{2r}}$ {should be derived from $F_G = F_c$}

$\boxed{\textbf{GPE} = -\frac{GMm}{r}}$

Total Energy $= \text{GPE} + \text{KE} = \frac{GMm}{2r} + (-\frac{GMm}{r}) = \boxed{-\frac{GMm}{2r}}$

Note: Total Energy, TE of a satellite is always negative.

14. Explain why a satellite does not move in the direction of the gravitational force:
i.e. why does it stay in its circular orbit, and not fall towards the Earth?

- gravitational force exerted by Earth on satellite is JUST sufficient to cause the centripetal acceleration
- Misconception: to think that motion (i.e. velocity) must always be in the direction of the net force. Newton's 2nd law only requires the (rate of) change in momentum, & hence the CHANGE in velocity, to be in the direction of the net force. {The direction of the vector, *change in velocity*, is not the same as that of the *initial velocity*, in general.}
  Illustration: direction of motion in a parabolic motion is not in the direction of the net force

Why satellites, as they gradually lose energy due to small resistive forces, may eventually 'burn up' in the Earth's atmosphere? [4m] {2011P3Q6d – an important question}

What happens to GPE, KE and TE of satellites, as they gradually lose energy due to atmospheric friction?

- Since there is work done against friction in the Earth's atmosphere, **total** energy of satellite ($-\frac{GMm}{2r}$) decreases (becomes more negative) and the radius r of orbit decreases.
- Since r decreases, kinetic energy ($\frac{GMm}{2r}$) increases and the satellite speeds up.
- Since r decreases, GPE ($-\frac{GMm}{r}$) deceases (becomes more negative).

15. Weightlessness:
Explain why a person in a satellite orbiting the Earth experiences "weightlessness" although the gravitational field strength {*g*} at that height is not zero.

- Since the person and the satellite would both have the same acceleration, the normal reaction on the person is zero.
- {Elaboration: the sensation of weight is due to the normal reaction exerted on the object. When the person & the floor of the satellite have the same acceleration, the contact force between them is zero, hence the normal reaction is zero. This is the state of "weightlessness".}

Topic 9: Oscillations

1. **Displacement** x, is the distance of the oscillating mass from its equilibrium position in a particular direction
Amplitude x_o, is the maximum displacement from the equilibrium position. (a scalar)
Period T, is the time taken for one complete oscillation.

Frequency f, is the number of oscillations per unit time. Hence $\boxed{f = \frac{1}{T}}$ [Hz or s^{-1}]

Angular frequency ω is defined by $\omega = 2\pi f$ where f is the number of oscillations per unit time.

Thus $\boxed{\omega = 2\pi f}$

2. Phase is an angle in radians which gives a measure of the fraction of a cycle that has been completed by an oscillating particle or by a wave. {One cycle corresponds to 2π rad.}

Phase difference/angle $\boxed{\phi = \frac{x}{\lambda} \times 2\pi = \frac{t}{T} \times 2\pi}$

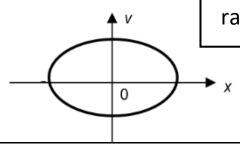
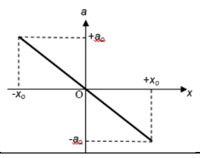
{x = separation between 2 wave particles, measured ***along the direction of wave motion***;
t = time difference between two waves or two particles in a wave}

3. **Simple harmonic motion** is an oscillatory motion in which the acceleration [or restoring force] is

- always proportional to, and
- opposite in direction to the displacement **from a certain fixed point / equilibrium position**

Hence. $\boxed{a = -\omega^2 x}$ (defining equation of S.H.M)

- The **negative sign** indicates that the 2 vectors, ***x* and *a*,** are always in **opposite directions**.

"Time Equations"	"Displacement Equations"
$x = x_o \sin(\omega t)$ Case 1 $x = x_o \cos(\omega t)$) Case 2 {these are most common or other variation depending on the initial condition given in the question and sign convention}	
$v = \frac{dx}{dt}$ $= x_o\omega \cos(\omega t)$ {for Case 1} $= v_o \cos(\omega t)$ If ωt is in units of radian, your calculator MUST be in radian mode when calculating sin (ωt) or cos(ωt)	$v = \pm\omega\sqrt{x_o^2 - x^2}$ {in List of Formulae. Impt equation that is often forgotten!} ($v - x$ graph is an ellipse)  ω must be in units of rad s^{-1} for this equation.
$a = -\omega^2 x$ $= -x_o\omega^2 \sin(\omega t)$ {Case 1} $= a_o \sin(\omega t)$	$a = -\omega^2 x$ (defining equation of SHM) 
KE = $\frac{1}{2} mv^2$ = $\frac{1}{2} m [x_o\omega \cos(\omega t)]^2$ {Case 1}	KE = $\frac{1}{2} mv^2 = \frac{1}{2} m\omega^2 (x_o^2 - x^2)$ (KE – x graph is an inverted parabola)
PE = $\frac{1}{2} m\omega^2 x^2$ = $\frac{1}{2} m\omega^2 [x_o \sin(\omega t)]^2$ {Case 1}	PE = $\frac{1}{2} m\omega^2 x^2$ {PE – x graph is a parabola}

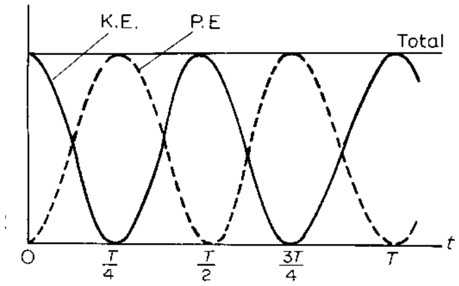
Problem solving: For "lose contact" question, this happens in vertical oscillations at the amplitude position coming down. Write down F_{net} = ma at that position and set N = 0 (lose contact).

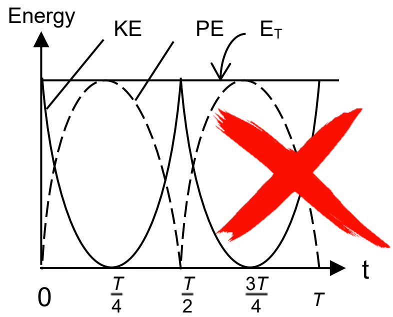
Max Velocity $\boxed{v_o = x_o\,\omega}$; **Max acceleration** $\boxed{a_o = x_o\,\omega^2}$

Note: Know $v = 0$ at amplitude and maximum at equilibrium, $a = 0$ at equilibrium and maximum at amplitude.

In general, **Total Energy**, E_{total} = KE + PE at any instant = max PE

Graphs of KE, PE and TE wrt time





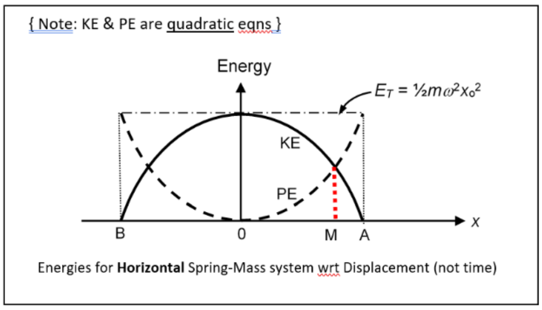
Suppose period of shm is T.
Then period of KE variation = ½ T

Common error: Students draw the "McDonald's" energy-time graph shown above where there are discontinuities.

- PE in SHM comprises "all" possible potential energies ie GPE & EPE. In solving problem involving PE, use the SHM formula for PE ($\frac{1}{2}\,m\omega^2x^2$), rather than finding the different types of potential energies separately (mgh , ½ kx^2) unless required by qn. This can often be complex.

- There are two cycles in the graphs of KE and PE in one period (T) of oscillation. The mistake is to often to take T/2 for the period of the oscillation when given a KE or PE graph.

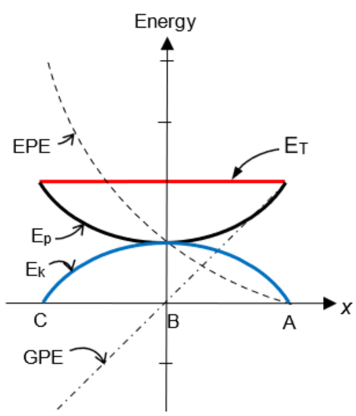
Graphs of KE, PE and TE wrt displacement



Energies for **Horizontal** Spring-Mass system wrt Displacement (not time)

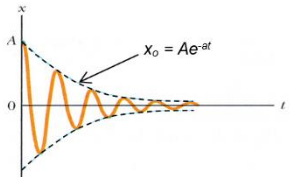
4. For **vertical spring-mass system**:

- extension e of the spring is **not** equal to the displacement x of the mass {In contrast, for a horizontal spring-mass system, x = e }
- EPE = ½ Fx = ½ kx^2 where x is the extension measured from the natural length without a load
- KE = ½ m v ^{2} = ½ m ω^2 (x$_o^2$ – x^2) where x is the displacement measured from the equilibrium position
- Find the extension from natural length at the equilibrium position first using F = kx where x is the extension
- Total energy of the spring-mass system is not equal to KE$_{max}$ although it is still equal to PE$_{max}$.
  E_T = E_K + (GPE + EPE).
- **If GPE is chosen to be zero at the equilibrium position** (ie B) and **EPE is zero at amplitude** (ie the mass max displacement upwards is the natural length position), the graph for the various energies will be as follows:
  (The graphs will be different if the zero GPE reference level is chosen differently and the position where EPE = 0 is different as stated in the question.)

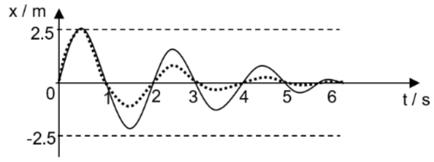


5. **Damping** refers to the loss of energy from an oscillating system to the environment, caused by dissipative forces, eg friction, viscous force.

Light Damping: The system oscillates about the equilibrium position with decreasing amplitude over a period of time.



- System still oscillates about equilibrium position
- Amplitude decreases exponentially over time (draw the dotted line to demonstrate you know it is an exponential decay.)
- Period will increase slightly (draw the period as the same as the undamped oscillation as this change is very slight)



Critical Damping: The system does not oscillate & damping is *just* adequate such that the system returns to its equilibrium position in the **shortest** possible time.

- Need to know the uses of it eg car suspension system, ammeter needle

Heavy Damping: The damping is so great that the displaced object does not oscillate but returns to its equilibrium position very very slowly.

{Need to illustrate these 3 degrees of damping with a displacement-time graph}

7. **Free Oscillation**: An oscillating system is said to be undergoing free oscillations if:

- its oscillations are not subjected to an external periodic driving force.
- Hence the system oscillates at its natural frequency.

Forced Oscillation: An oscillating system is said to undergo forced oscillations if:

- it is subjected to an external **periodic** driving force.
- As a result, the **frequency of the forced (or driven) oscillations will be at the frequency of the *driving force*** [called the driving frequency] i.e. no longer at its own natural frequency.

8. **Resonance**:

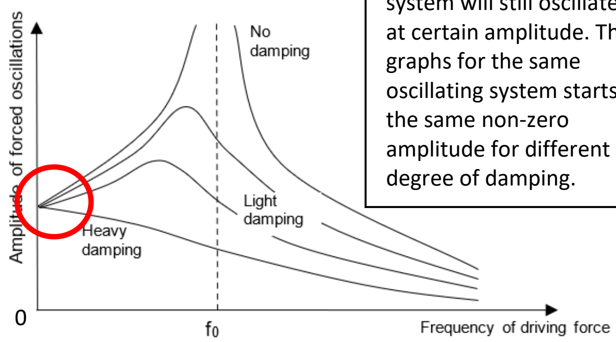
- a phenomenon whereby the amplitude of a system undergoing forced oscillations is at a maximum.
- It occurs when the frequency of the periodic driving force is equal to the natural frequency of the system.

Problem solving: Identify the system that is oscillating and identify the periodic driving force. Eg oscillating system - a car suspension system, periodic driving force – humps on road at regular interval, oscillating system – air column, perioding driving force – vibrating tuning fork

9. **Effects of damping** on the frequency response of a system undergoing forced oscillations:

When driving force is absent ie f = 0, the driven system will still oscillate at certain amplitude. The graphs for the same oscillating system starts at the same non-zero amplitude for different degree of damping.

1) resonant frequency decreases { causing the peak to shift slightly leftwards}
2) sharpness of resonance [resonant peak] decreases
3) amplitude of forced oscillations decreases



*A graph showing the amplitude of the forced oscillations at various driving frequencies f_d. The natural freq of the undamped system is f_o & the 4 curves are for the same oscillator at different **degrees of damping**. Magnitude of driving force (ie its amplitude) on the driven system is kept const.*

Topic 10: Wave Motion

1. Mechanical versus Electromagnetic wave

	Mechanical Waves	**Electromagnetic Waves**
Characteristic	require a material medium for their propagation	can propagate without a material medium
Examples	1) Water waves propagated through the oscillations of water molecules 2) Sound wave transported via the vibrations of air molecules	Radio waves, microwaves, infrared, visible light, ultraviolet, x-rays, and gamma-rays

2. Displacement: the distance of the oscillating mass from its equilibrium position in a particular direction.

3. Amplitude: The maximum displacement of an oscillating particle from its equilibrium position.

4. Period: Time taken for a particle to undergo one complete cycle of oscillation.

5. Frequency: Number of oscillations performed by a particle per unit time.

$$\boxed{f = \frac{1}{T}}$$

6. Wavelength λ: distance between 2 consecutive points on a wave which are in phase.

7. Wave speed of a wave, $\boxed{v = f\lambda}$ {derived from $v = \lambda / T$ and $f = 1 / T$}

{It refers to the speed of propagation of the energy {which is constant}, **NOT to the speed of *oscillation of a wave particle*** which is simple harmonic in nature, given by $v = \pm\omega\sqrt{x_o^2 - x^2}$ }

8. **Wavefront** is line or surface joining points which are at the same state of oscillation, i.e. in phase, e.g. the lines formed by crests of ripples on a pond

9. Phase is the angle which gives a measure of the fraction of a cycle that has been completed by an oscillating particle or by a wave. {One cycle corresponds to 2π rad.}

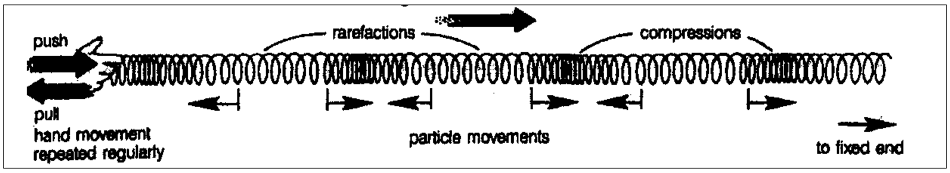
10. **Phase difference** (ϕ) is a measure of how much one wave is 'out of step' with another wave or how much one particle in a wave is out of step with another particle in the same wave. It is expressed in terms of angles from 0 to 2π radians.

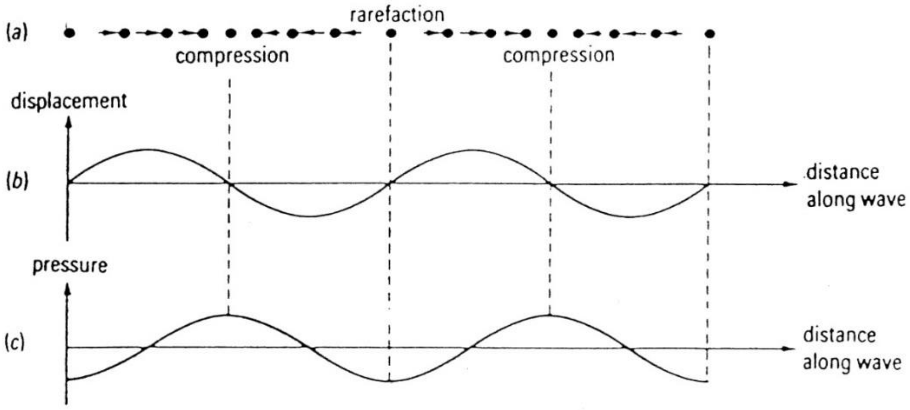
i.e. $\boxed{\phi = \frac{x}{\lambda} \times 2\pi}$ & $\boxed{\phi = \frac{t}{T} \times 2\pi}$

- x: separation betw the 2 particles measured **along the direction of wave travel**

11. **Transverse wave**: a wave in which the oscillations of the wave particles {**NOT: movement/motion**} are *perpendicular* to the direction of the **transfer of energy { NOT:** propagation of the wave }.

Longitudinal wave: a wave in which the oscillations of the wave particles are parallel to the direction of the transfer of energy.





12. The **displacement-time graph** shows how displacement of a <u>single particle</u> at a particular position (of the wave) varies with <u>time</u>. *("video recording" of the particle)*

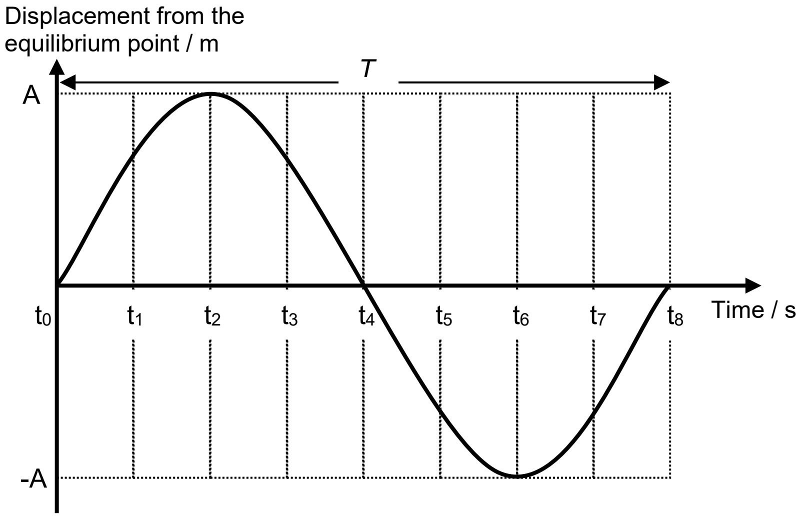


Fig.: Displacement-time graph

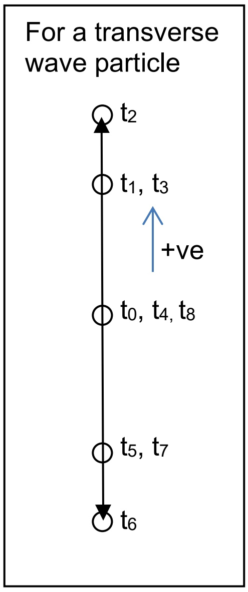


Fig.: Actual positions of particle at different times for a transverse wave.

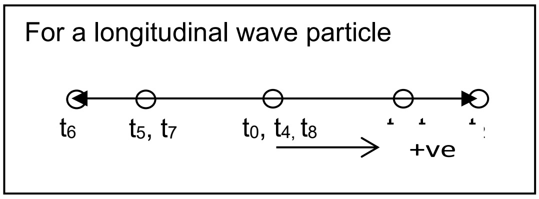


Fig.: Actual positions of particle at different times for a longitudinal wave.

13. The **displacement-distance graph** shows how the displacements of all the particles vary with the distance from the source at a particular instant in time. *("snapshot / photograph" in time)*

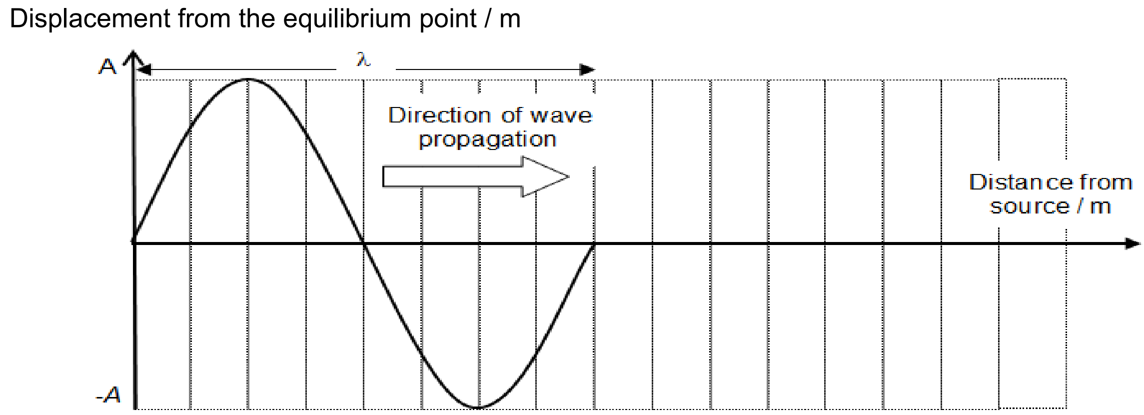


Fig. 8(a): Displacement-Distance graph

Note:
From the graph, the wavelength can be determined from the completion of one cycle of oscillation.

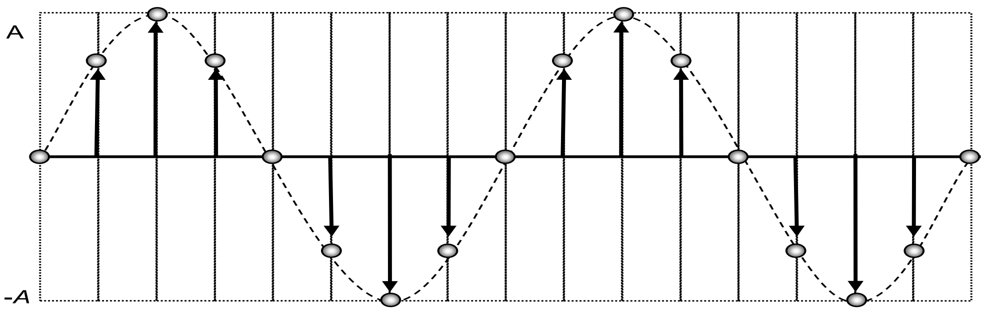


Fig: Actual positions of various particles at different locations along a transverse wave.

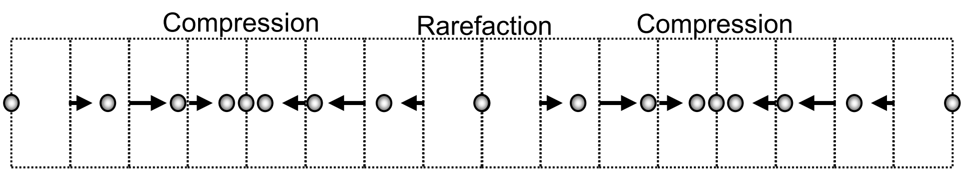


Fig: Actual positions of various particles at different locations along a longitudinal wave.

It is necessary to adopt some convention to present the direction of the vibration of the wave particle;

- For transverse wave, the positive axis usually refers to the upward displacement of a particle
- For longitudinal waves, the positive axis usually refers to the displacement of a particle to the right.

14. **Progressive wave**: is the movement of a disturbance from a source which transfers energy from the source to places around it by means of vibrations/oscillations. {Mark Scheme for N07P2Q3a, 2 m}

15. Intensity (of a wave at a location): defined as the rate of energy flow [i.e. power] per unit area **perpendicular** to the direction of wave propagation (at that location)

i.e. $\boxed{\text{Intensity} = \dfrac{\text{Energy}}{\text{Time} \times \text{Area}} = \dfrac{\text{Power}}{\text{Area}}}$ S.I. unit of intensity is W m^{-2}

- For **all** types of wave sources (with the same frequency), $\boxed{\text{Intensity} \propto (\text{Amplitude})^2}$ since Total Energy $\propto A^2$

- Only for a **point source** (which would emit spherical wavefronts),

$\boxed{\text{Intensity} = \dfrac{Power}{4\pi r^2}}$ where r = distance from the point source.

Area **perpendicular** to wave = surface area of a sphere = $4\pi r^2$

For constant power point source,
$I \propto \dfrac{1}{r^2}$
Since $I \propto A^2 \rightarrow A \propto \frac{1}{r}$

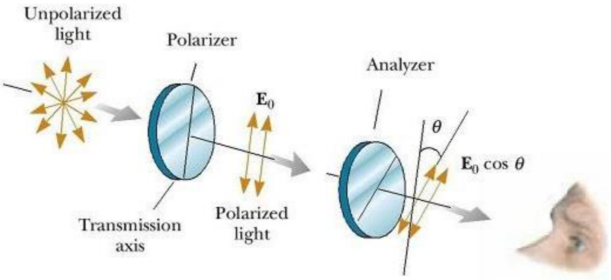
- $\boxed{\text{Intensity at the receiver} = \dfrac{\text{Power from the \textbf{source}}}{\text{Surface Area of the wavefront}} = \dfrac{\text{Power incident on the \textbf{receiver}}}{\text{Area of the \textbf{receiver}}}}$

16. State what is meant by ***plane polarisation*** of a wave. *(2019 P2 Q2, 1 m)*
- **Plane polarisation** is a process by which the oscillations/vibrations of the wave (particles) are confined to only one direction, in the plane normal to the direction of energy transfer.

17. *State why a sound wave cannot be polarised. (2019 P2Q2, 1 m)*
- For a sound wave (which is a longitudinal wave), its wave particles oscillate in a direction parallel to (ie NOT perpendicular to), the direction of energy transfer. Hence it's not possible to confine the oscillations to a plane which is normal to the direction of energy transfer.

18. Malus' Law $\boxed{I = I_o \cos^2 \theta}$

Intensity I of light transmitted by the analyzer is directly proportional to the square of the cosine of angle between the transmission axes of the analyzer and the polarizer.



2 polarising filters/sheets whose transmission axes make an angle of θ with each other

- Malus' Law is about the effect of a polaroid on *polarized* light, NOT on unpolarised light.
In set up above, the 1st polaroid allows only **½** the incident intensity to pass through it regardless of the direction of the polarizing axis, such that I_o= ½ of the intensity of the unpolarised light.

Of the polarised output from the 1st polaroid, the 2nd polaroid lets through a fraction ($\cos^2\theta$) . Hence the intensity transmitted by the 2nd polaroid I = $\boldsymbol{I_o \cos^2 \theta}$

Topic 11: Superposition

1. **Principle of Superposition**
   When two or more waves of the same type exist simultaneously at a point , the ***resultant displacement*** {NOT: amplitude} of the waves is equal to the *vector* **sum** of their (individual) displacements at that point.

2. **Coherence**
   Two waves are coherent if they have a constant phase difference (not *zero phase difference*) between them (with respect to time).
   - Common error: students wrongly think coherent waves must always be *in phase* with each other. The phase difference can be of any value, it is only required to be constant wrt time.

3. **Interference: NOT tested in FE**
   refers to the superposition of **coherent** waves which results in a change in the overall intensity.

   Constructive interference:
   This occurs when waves from two (or more) arrive at a point *in phase* (i.e. zero phase difference), producing a resultant wave with maximum amplitude (& hence, max intensity)

   Destructive interference:
   This occurs when waves from two (or more) coherent sources arrive at a point in *anti-phase* (i.e. phase difference of π radians), producing a resultant wave of minimum amplitude (and hence, minimum intensity.)

4. **Conditions for CI & DI: NOT tested in FE**
   (a) in terms of **Phase Difference** at the screen**:**
   (i) If ***constructive interference*** occurs at a pt P (on the screen),
   $\Rightarrow$ phase difference of the 2 waves at P = $(n)\,2\pi$ radians { 0, 2π, 4π, etc }

   (ii) If ***destructive interference*** occurs at P,
   $\Rightarrow$ phase difference of the 2 waves at P = $(n + ½)\,2\pi$ radians { ie phase diff = π, 3π, 5π etc }
   In both cases, n = 0, 1, 2, 3, ...

   (b) in terms of **Path Difference:** Here, **phase difference betw the 2 sources** must be known.
   (i) If ***constructive interference*** **occurs at P**,
   the path difference (for the two sources at S_1 and S_2) must be an integral multiple of the wavelength λ , ***if the two sources are in phase*****:**
   $\Rightarrow$ path difference $|S_1P - S_2P|$ = 0, 1 λ, 2 λ, 3 λ, ... = $\boldsymbol{n\lambda}$

   - *If the sources are in antiphase*:
     the condition changes to: path difference = $(n + ½)\,\lambda$ (odd multiples of half wavelengths) }

   (ii) If **destructive interference occurs at P,**
   the path difference will be an odd multiple of half wavelengths, if the **two sources are in phase:**
   $\Rightarrow$ path difference = 0.5 λ, 1.5 λ, 2.5 λ, 3 .5 λ, ...
   = $\boldsymbol{(n + 1/2)\,\lambda}$ or $(2n + 1)/2\,\lambda$

   - *If the sources are in antiphase*: path difference = $n\,\lambda$

5. **Conditions to produce a 'well-defined' interference pattern: NOT tested in FE**
   For an interference pattern to be clearly observable (well-defined), the waves must:
   (i) **meet / superpose**,
   (ii) be **coherent,** &
   (iii) have **equal or approximately equal amplitude**,

6. Young's Double Slit Interference – NOT in FE

Fringe separation, $\boxed{x = \frac{\lambda D}{a}}$

where x: distance between 2 successive *bright* fringes (or 2 *dark* fringes).
λ: Wavelength of light.
D: distance between the double slits and the screen.
a: distance between the 2 double slits.

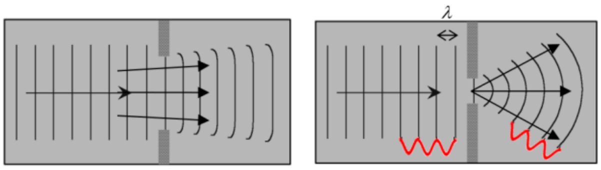
- applicable **only if a << D;** hence, generally applies only to double slit interference of light, i.e. NOT for microwaves, sound waves, water waves since condition of a << D is difficult to be met in lab. However, see 2009 P1Q24 on radiowaves.

7. NOT in FE
Resultant intensity due to interference of 2 waves with different intensities at
(a) a pt of constructive interference: see Worked Eg 11
(b) a pt of destructive interference see Worked Eg 12

Resultant intensity is NOT = sum (or difference) of the 2 individual intensities because that's not how waves interact when they superpose. Refer to Principle of Superposition.

8. Diffraction refers to the **spreading** {not: bending} of waves when they pass through a slit (opening/gap/aperture), or round an obstacle into the "shadow" region.

- For significant diffraction to occur, the size of the gap should be approximately equal to the wavelength of the wave.



- Note that the wavelength (spacing between the lines) stays the SAME before and after the gap.
- Diffraction of light provides evidence for the wave nature of light and all other electromagnetic radiation. Because the *wavelength* of *light* is very short (≃ 10^{-7} m), diffraction can only be observed through very narrow slits or small obstacles. Sound waves, however, are able to diffract around bigger objects because of their longer wavelength.

9. Single-Slit Diffraction – NOT in FE

Angular position of **First Minima**: $\boxed{\sin\theta_1 = \frac{\lambda}{b}}$ where b = slit width.

- when θ is small, then $\theta \approx \frac{\lambda}{b}$ {in radians}
- the central maximum is twice as wide ($\frac{2\lambda}{b}$ wide) as any of the other secondary maxima ($\frac{\lambda}{b}$ wide each).
- **Rayleigh criterion** states that 2 images are said to be ***just distinguishable*** {ie ***just* resolved**} if the central maximum of one image falls on the first minimum in the diffraction pattern of the other.
- **Angular position of the 1st minimum,** $\boxed{\theta_{min} \approx \frac{\lambda}{b}}$ when θ is small { which should be the case in the Exam}
- Thus **resolving power** of lens/aperture is high if θ_{min} is small. This can be achieved by using a large aperture diameter (width b) or a small wavelength λ.
  - If the ***actual* angle subtended by the two point sources at the slit**, $\boldsymbol{\theta_s = \theta_{min}}$, then the images are **just resolved** (by Rayleigh's criterion).
  - Hence, $\boldsymbol{\theta_s}$ **must be >** $\boldsymbol{\theta_{min}}$ if the images are to be (**obviously) resolved**.
  - Conversely, if $\boldsymbol{\theta_s < \theta_{min}}$, the images are **not resolved**.

- Typically θ_s is calculated using $\theta_s = \frac{s}{r}$
  where $r \approx$ distance of the two objects from aperture/lens,
  s (arc length) ≈ (linear) separation of objects (since θ_s is small)

10. Diffraction grating (Multiple Slit Diffraction)

maxima are located at angular positions defined by θ in : $\boxed{a \sin\theta = n\lambda}$

where: a: slit separation (**grating or line spacing**) is the distance between adjacent slits
θ: angle of diffraction /angular deviation /angle of deviation (measured from the normal)
n: order of diffraction (must be an integer or zero) , λ: wavelength

- The **maximum number of orders** can be found by substituting θ = 90° to 1st det the **highest order of diffraction**, $n_{highest}$
  where $n_{highest} = d / \lambda$ (rounded **down** to nearest integer)
  Hence **maximum number of orders/bright fringes** (displayed on screen) = $2n_{highest} + 1$
- When a *white* light passes through a diffraction grating:
  - for each order of diffraction, a longer wavelength {red} diffracts more than a shorter wavelength {violet} {as $\sin\theta \propto \lambda$ for a given n}.
  - the zeroth order (at centre of diffraction pattern) would remain white as all the colours/wavelengths in white light meet at this position.
  - Overlapping betw different orders of different wavelengths might also occur { eg if the 3rd order of violet is at a smaller angle than the 2nd order of red. }

11. Standing (stationary) wave

A stationary wave is formed when two progressive waves of the same frequency, amplitude and speed, travelling in opposite directions are superposed. {mnemonic: **FASO**}

{Assume "boundary conditions" are met, for eg, $n\frac{\lambda}{2}$ = distance between source & reflector, for standing waves in a stretched string}

Differences between standing waves & progressive waves:

	Standing Waves	Progressive Waves
Amplitude	Varies from maximum at the anti-nodes to zero at the nodes.	Same for all particles in the wave (assuming no energy is lost).
Phase [Phase angle]	Particles in the same loop/ between 2 adjacent nodes are in phase. Particles in adjacent loops are in anti-phase.	Particles which are "integer wavelengths apart" are in phase. All particles within one wavelength have different phases.
Wave Profile	The wave profile does not advance.	The wave profile advances.
Energy Propagation	No energy is transferred by the wave.	Energy is transferred in the direction of the wave propagation.

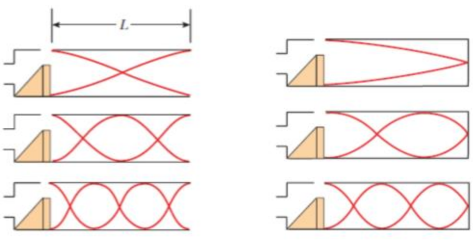
- **Node**: a region of destructive superposition where the waves always meet out of phase by π radians**.** Hence displacement here is permanently zero { zero only if the 2 amplitudes are equal; otherwise, it's described as a minimum}.

- **Antinode:** a region of constructive superposition where the waves always meet in phase**.** Hence a particle here vibrates with maximum *amplitude*.
{but it is NOT a point with a *permanent* large displacement! }

- **Distance between 2 successive nodes/antinodes = λ/2**

- **Boundary Conditions** for Standing Waves on Stretched Strings betw 2 fixed ends
  - at the 2 fixed ends: always a node

- **Boundary Conditions** for Standing Waves in Air Columns
  - at open end: displacement antinode (assuming no end corrections)
  - at closed end: displacement node



(a) Open at both ends (b) Closed at one end, open at the ot

- Need to know relation between λ, length of pipe L, & end correction in both open & closed pipes {N09P1Q22}

  Eg, for **fundamental** mode in closed pipe: $\frac{\lambda}{4}$ = L + c, if end correction c is not negligible.

- For standing sound waves: **Change in Pressure at Displacement Nodes & Displacement Antinodes** (over one period)

  Fig. 10.35 shows three snapshots of a longitudinal *standing* wave formed by the (left/ right) oscillations of air molecules (represented by vertical lines) at three instants of time. (The incident and reflected longitudinal waves are not included for clarity of diagram).

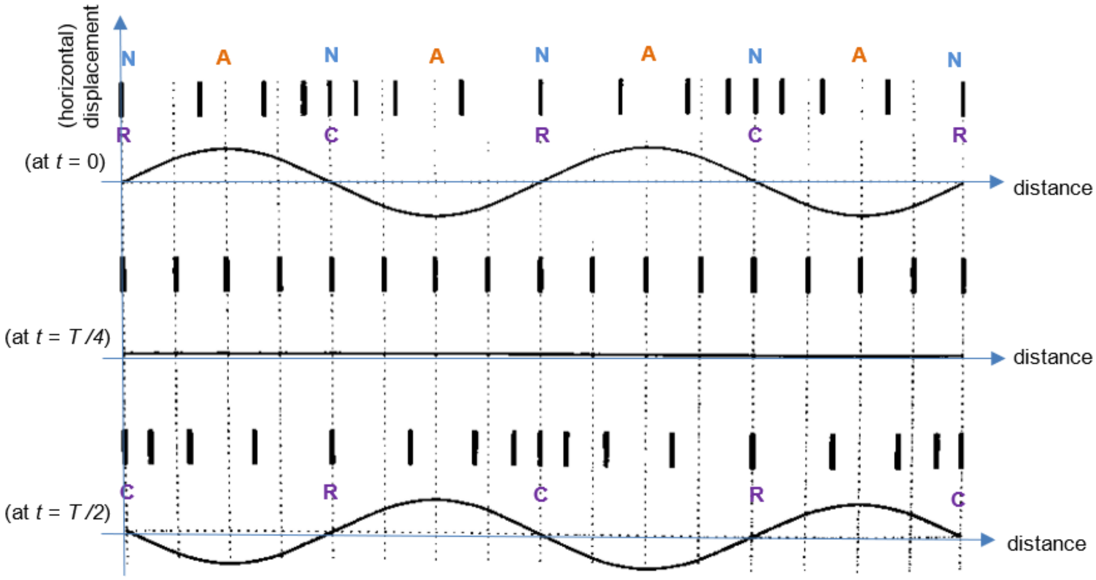


Fig. 10.35: Displacement-distance graphs at three-time instants and the corresponding positions of air molecules.

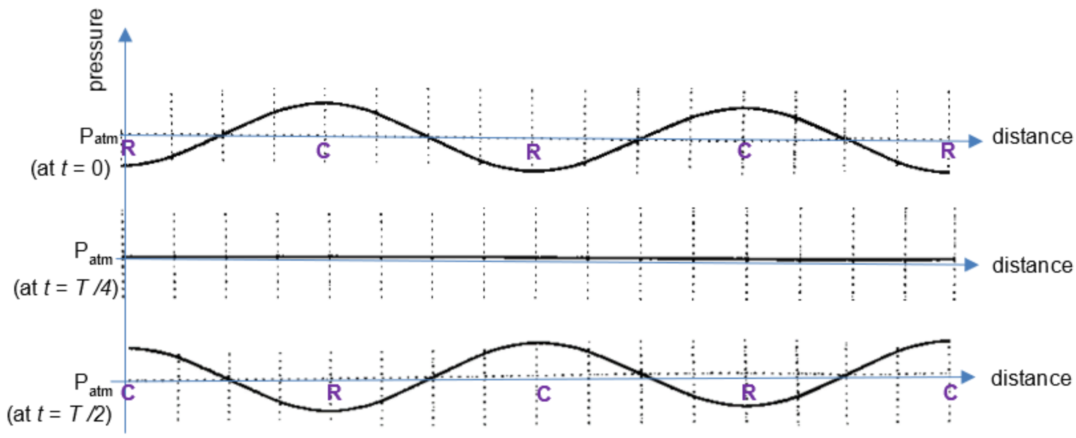


Fig. 10.36: Graphs of pressure against distance for stationary sound wave.

1) A pt of compression & a pt of rarefaction: both have zero displacement; must know how to deduce from a displacement-distance graph the <u>positions of the points of compressions & rarefactions.</u>

2) at <u>displacement</u> <u>nodes</u> : **maximum pressure change** occurs because <u>every displacement node changes from a point of compression to become a point of rarefaction half a period later</u> {N05P3Q3c; N08P1Q22 }

3) at <u>displacement</u> <u>antinodes</u>: there is **no change** in pressure.

- In describing the nodes (or antinodes) in an **<u>air column</u>**, candidates **<u>must</u> state explicitly whether they are referring to "*<u>displacement</u>* node" or "*<u>pressure</u>* node" .**

12. Intensity Graph for Single-slit Diffraction: NOT in FE

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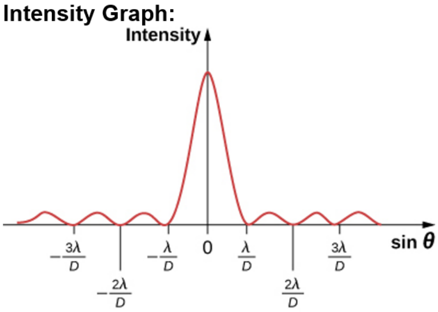
Single Slit

Formula: $\sin\theta_1 = \frac{\lambda}{b}$

Meaning of Symbols:
b = slit width
λ = wavelength
θ_1 = angular position for first minimum

Intensity Graph:



Note:
Central maximum is twice as wide as secondary maxima

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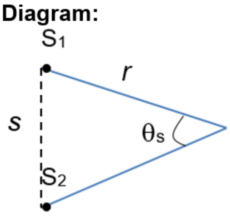
Rayleigh Criterion

Formula: $\theta_{min} \approx \frac{\lambda}{b}$, $\theta_s = \frac{s}{r}$,

Meaning of Symbols:
θ_s = angle subtended by 2 distant sources
θ_{min} = limiting angle of resolution
b = slit width
λ = wavelength
r = distance of the two objects from aperture,
s $\approx$ separation of objects

Diagram:



Note:
Resolved, $\theta_s > \theta_{min}$

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