BEATTY SECONDARY SCHOOL
END-OF-YEAR EXAMINATION 2025
SECONDARY THREE EXPRESS / G3

CANDIDATE NAME			
CLASS		REGISTER NUMBER	

ADDITIONAL MATHEMATICS

4049

7 October 2025

Setter:

2 hours 15 minutes

Candidates answer on the Question Paper
Additional Materials: Nil

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Given non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is 90.

For Examiner's Use

This document consists of **19** printed pages and **1** blank pages.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 **(a)** Express $1-6x-2x^2$ in the form of $p(x+q)^2+r$ where p, q and r are constants. [2]

(b) Find the range of values of the constant k for which the equation $1-6x-2x^2=kx+3$ does not have two distinct roots. [3]

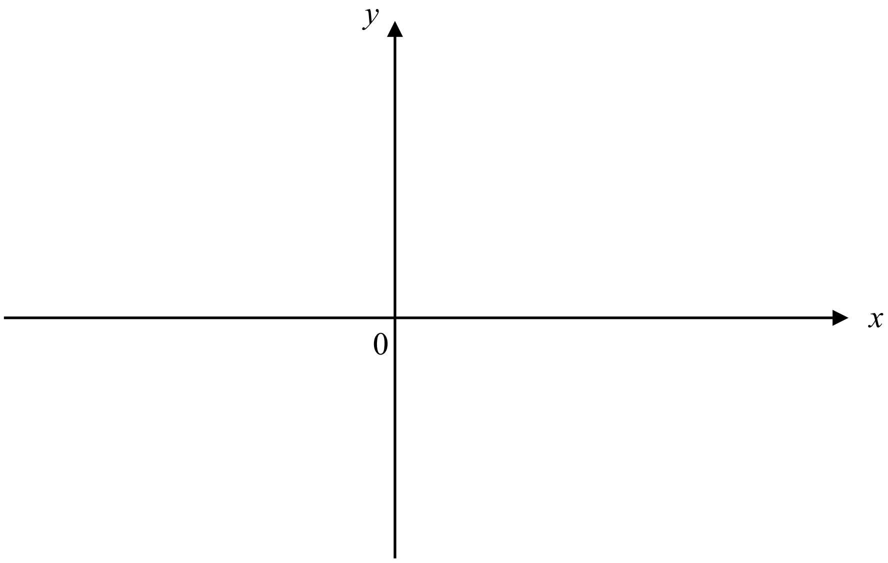
2 **(a)** Given $\log_m a = x$ and $\log_m b = y$, express $\log_{ab} m$ in terms of x and y. [3]

(b) Without using a calculator, evaluate $\log_2 3 - \log_4 36 + \log_{\frac{1}{2}} 8$. [4]

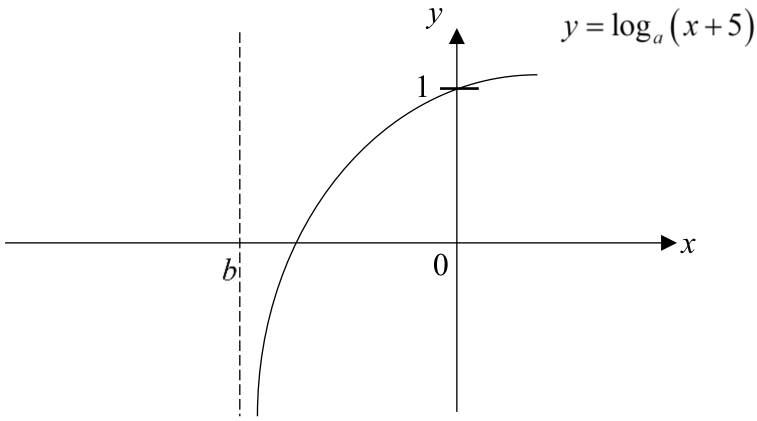
3 **(a)** The equation of a curve is $y = a(x+b)^2 + c$ where a, b and c are constants.

The solutions of $a(x+b)^2 + c = 0$ on the graph is -2 and 8, and the maximum value of y is 3. Find the values of a, b and c. [3]

(b) Using the values of a, b and c found in **(a)**, sketch the graph of $y = a(x+b)^2 + c$. [2]

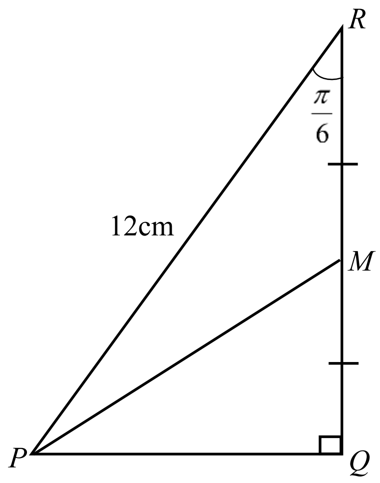


4 **(a)** The diagram shows part of the graph of $y = \log_a(x+5)$. The vertical asymptote is b and the y-intercept is 1, where a and b are constants. Find the values of a and b. [2]



(b) Solve $2^{\frac{x}{2}+3} + 21 = 5(2^x)$. [4]

5



The diagram shows a triangle PQR in which $PR = 12$ cm, M is the mid-point of QR, angle $PRQ = \frac{\pi}{6}$ radians and angle PQR is a right angle.

Without using a calculator, find the value of p such that angle $RPM = \sin^{-1}\left(\frac{\sqrt{p}}{14}\right)$. [5]

6 **(a)** The lengths of the diagonals *PR* and *QS* of the rhombus *PQRS* are $\left(4+2\sqrt{3}\right)$ cm and $\left(\dfrac{6}{2+\sqrt{3}}\right)$ cm respectively.

Find the value of PQ^2, leaving your answer in the form $a+b\sqrt{c}$. [4]

(b) Determine if the curve $y = px^2 + 8x + p - 6$ lies completely above or below the x-axis for $p > 8$. [3]

7 Express $\dfrac{11x^2+9x+4}{(x+3)(2x^2+1)}$ in partial fractions. [5]

8 It is given that $\mathrm{f}(x) = x^3 + ax^2 + bx + 28$, where a and b are constants, has a factor of $x+4$ and leaves a remainder of 30 when divided by $x-1$.

(a) Find the values of a and b. [4]

(b) Using the values of a and b found in **(a)**, show that the equation $\mathrm{f}(x) = 0$ has only one real root. [3]

9 **(a)** It is given that $y = a\cos 3x + b$, where $a > 0$, has a minimum value of -3 and a maximum value of 5.

(i) Without solving for b, explain why $a = 4$. [1]

(ii) Hence, or otherwise, find the value of b. [1]

(iii) State the period of y in radians. [1]

9 **(b)** **(i)** Sketch the graph of $y = 3\sin\left(\frac{x}{2}\right) - 1$, for $0 \le x \le 3\pi$. [3]

(ii) Sam claimed that the equation $6\pi \sin\left(\frac{x}{2}\right) - 3x = 0$ has only one solution for $0 \le x \le 3\pi$.

Do you agree with him? Draw a suitable line on the same axes in part **(b)(i)** and justify your answer. [3]

10 (a)(i) Write down and simplify the first four terms, in descending powers of x, in the expansion of $\left(x-\frac{2}{x}\right)^{10}$. [2]

(ii) Hence, find the coefficient of x^{10} in the expansion of $(2+3x)^2\left(x-\frac{2}{x}\right)^{10}$. [3]

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(b) Find the value of n given that the coefficient of x and x^2 in the expansion of $(2+3x)^n$ are in the ratio 4 : 57. [5]

11 A particle is heated until it reaches a temperature of C degrees Celsius. It is then left to cool. Its temperature, T degrees Celsius, when it has been cooling for time t minutes, is given by the equation $T = 22 + 48e^{-\frac{t}{5}}$.

(a) Find

(i) the value of C, [1]

(ii) the value of T after 16 minutes. [1]

(b) The particle must be heated again if it's temperature drops to 25°C.

Calculate the time taken for this to first happen. [2]

(c) Sketch the graph of $T = 22 + 48e^{-\frac{t}{5}}$ for $t \geq 0$. [2]

[Turn over

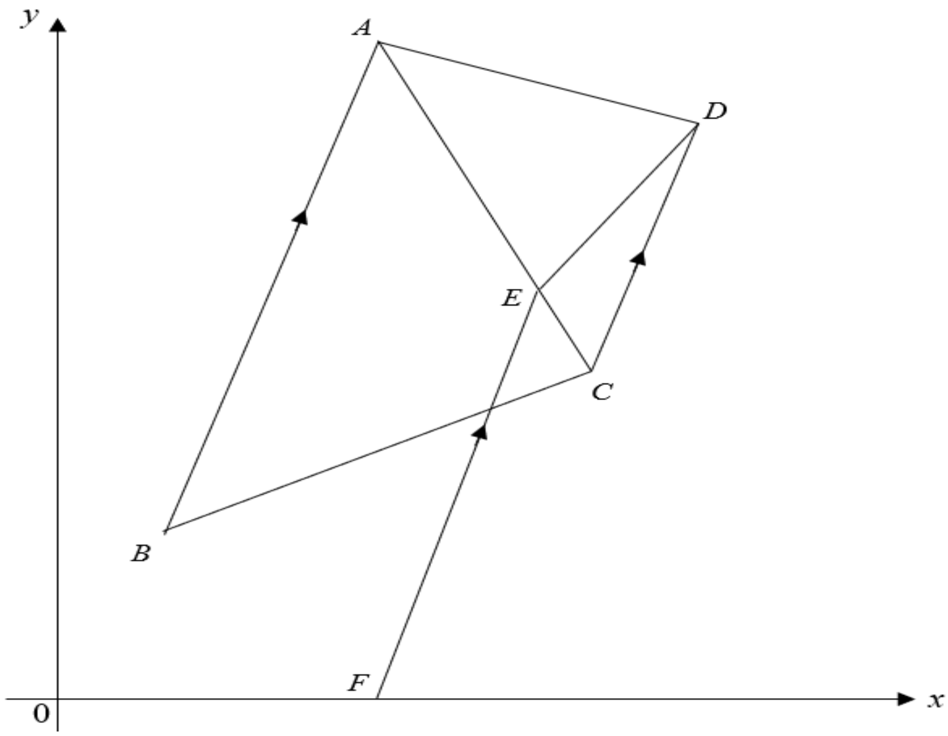
12 The equation of a circle C_1 is $(x+1)(4x+3)=(3-y)(4y+1)$.

(a) Express the equation of circle C_1 in its standard form $(x-a)^2+(y-b)^2=r^2$. [2]

(b) Another circle C_2 is formed when C_1 is reflected about the x-axis.
Find the equation of the circle C_2, leaving your answer in its general form
$x^2+y^2+2gx+2fy+c=0$. [1]

(c) Hence, find the length of the chord formed when the line $y = 2x$ intersects the circle C_1. [5]

13



The diagram shows a trapezium $ABCD$ in which $A(3,\ 8)$, $B(1,\ 2)$ and $C(5,\ 4)$, angle $BAD = 90°$ and BA is parallel to CD.

(a) Show that the coordinates of D is $(6,\ 7)$. [5]

(b) The point E lies on AC such that $AE = 3EC$.
Given $E(p, 5)$, show that $p = 4.5$. [1]

(c) The point F lies on the x-axis such that FE is parallel to CD.
Find the coordinates of F. [2]

(d) Hence or otherwise, find the area of $CDEF$. [2]

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Answer (BTY G3 AM 2025)

1a) $-2(x+1.5)^2+5.5$

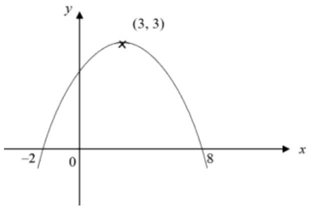
1b) $-10 \le k \le -2$

2a) $\dfrac{1}{x+y}$

2b) -4

3a) $a=-0.12$, $b=-3$, $c=3$

3b)



4a) $a=5$, $b=-5$

4b) 3.17

5 21

6a) $70-32\sqrt{3}$

6b) For $p>8$, $p-8>0$, $p+2>0$ and

$-4(p-8)(p+2)<0$.

Since discriminant < 0, the curve

$y=px^2+8x+6-p$ lies completely above

the x-axis for $p>8$.

7 $\dfrac{11x^2+9x+4}{(x+3)(2x^2+1)}=\dfrac{4}{x+3}+\dfrac{3x}{2x^2+1}$

8a) $a=2$, $b=-1$

8b)

$x=-4$ Discriminant $=(-2)^2-4(1)(7)$

$=-24<0$

$\therefore$ no real root

$\therefore$ there is only one real root for solution for

$f(x)=0$.

9ai) Since minimum value and maximum values

of y are -3 and 5 respectively,

$$\text{amplitude}=\frac{5-(-3)}{2}$$

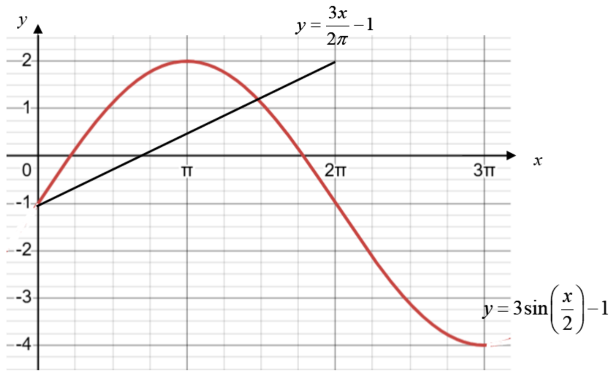
$$=4$$

$$\therefore a=4$$

9aii) 1

9aiii) $\dfrac{2\pi}{3}$

9bi)



9bii) I disagree with him.

When $y=\dfrac{3x}{2\pi}-1$ and $y=3\sin\left(\dfrac{x}{2}\right)-1$

are drawn on the same axes, there are 2 points of intersection, hence 2 solutions for $0 \le x \le 3\pi$.

10ai) $x^{10}-20x^8+180x^6-960x^4+......$

10aii) 1540

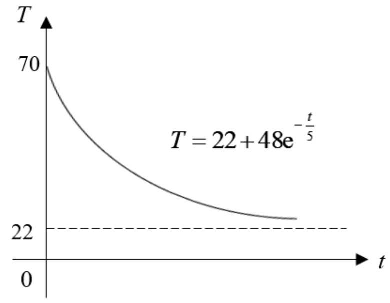
10b) 20

11ai) 70

11aii) 24.0

11b) 13.9

11c)



12a) $\left(x+\dfrac{7}{8}\right)^2+\left(y-\dfrac{11}{8}\right)^2=\dfrac{85}{32}$

12b) $x^2+y^2+\dfrac{7}{4}x+\dfrac{11}{4}y=0$

12c) 1.68 units

13c) $F\left(2\dfrac{5}{6},0\right)$

13d) $3\dfrac{1}{3}$ units^2