

MATHS NOTES

Chap 01 : Quadratic Functions

$$f(x) = ax^2 + bx + c$$

→ +ve U / -ve A
follow

→ completed square form:

$$(a \pm \frac{b}{2})^2 - (\frac{b}{2})^2 + c$$

$$= (x-h)^2 + k \rightarrow (h, k) \text{ turning pt}$$

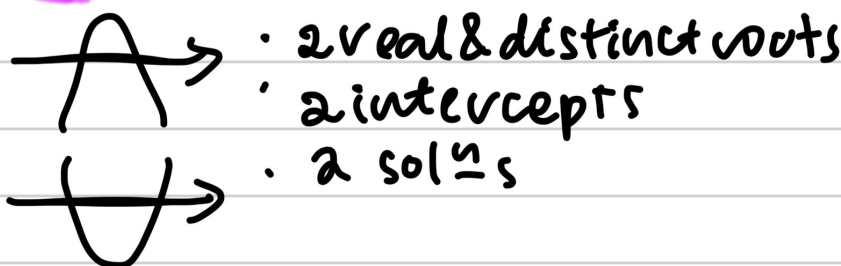
→ factorised form:

$$a(x-p)(x-q) \rightarrow x \text{ intercepts}$$

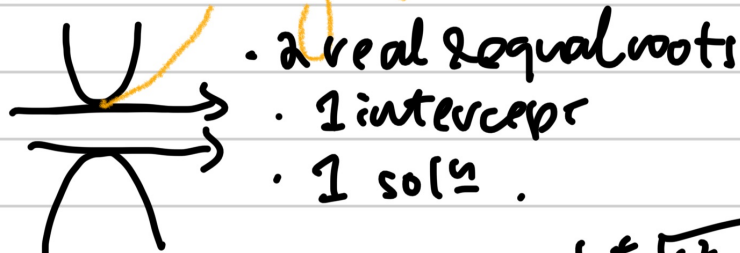
$$\text{discriminant: } b^2 - 4ac$$

→ discriminant

$$\textcircled{1} D > 0$$



$$\textcircled{2} D = 0 \text{ tangent}$$



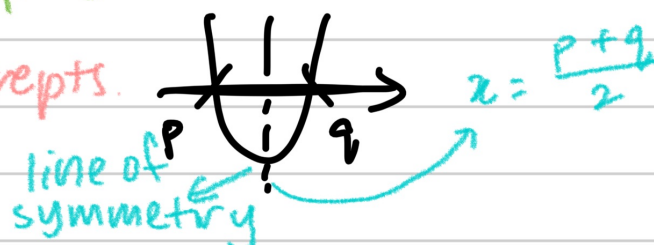
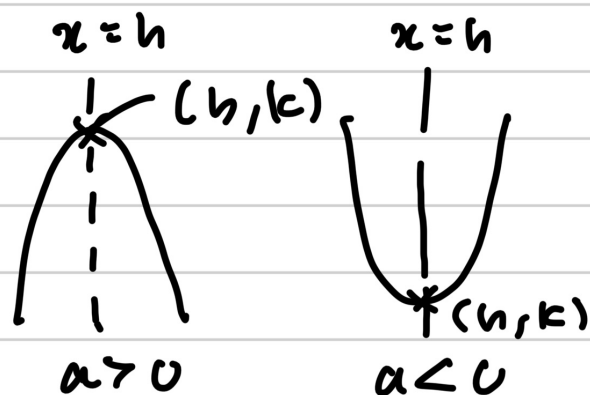
$$\text{quad formula } \star : \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

When U
 $D < 0, a > 0 \rightarrow$

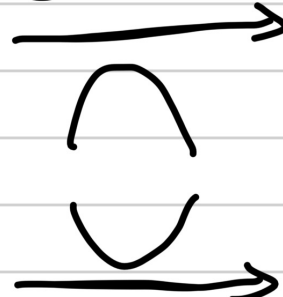
$$y = ax^2 + bx + c, y > 0$$

$$D < 0, a < 0 \rightarrow$$

$$y = ax^2 + bx + c, y < 0$$



$$\textcircled{3} D < 0$$



EX: Show $-x^2 - 4x - 10$ is -ve for all real values of x (e.g. $f(x) < 0$)

$$\begin{aligned} f(x) &= -x^2 - 4x - 10 \\ &= -(x^2 + 4x) - 10 \\ &= -[(x+2)^2 - 4] - 10 \\ &= -(x+2)^2 + 4 - 10 \\ &= -(x+2)^2 - 6 \end{aligned}$$

complete the sq

$$\begin{aligned} \text{since } (x+2)^2 &\geq 0 \\ -(x+2)^2 &\leq 0 \\ -(x+2)^2 - 6 &\leq -6 \\ f(x) &\leq -6 \end{aligned}$$

since $f(x) \leq -6$, then $-x^2 - 4x - 10$ is -ve for all real values of x (shown)

Chap 02 Eqs & Inequalities

l_2 l_1 Find ④: sub ① into ② } simultaneous eqns.
A → pt of intersection

Quad inequality

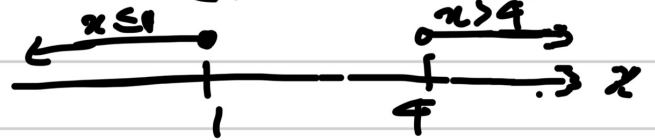
$$a > b \rightarrow a + c > b + c$$

$$a > b \text{ \& } c > 0, a \cdot c > b \cdot c$$

$$a > b \text{ \& } c < 0, a \cdot c < b \cdot c$$

Number line

$$\text{ex } x \leq 1 \text{ or } x > 4$$



Discriminant

> 0 2 real & distinct roots

$= 0$ 2 real & equal roots

> 0 2 real roots

< 0 no real roots

2 intersection pts

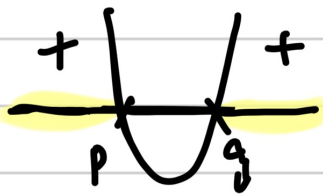
1 intersection pt

1/2 int.

0 int.

①

$$ax^2 + bx + c > 0$$



$$a > 0 \quad x < p \text{ or } x > q$$

②

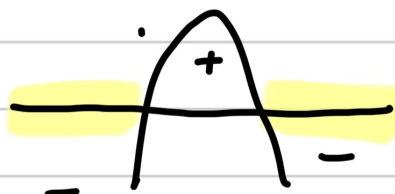
$$ax^2 + bx + c < 0$$



$$a > 0 \quad p < x < q$$



$$a < 0 \quad p < x < q$$



$$a < 0 \quad x < p \text{ or } x > q$$

Chap 03 Surds

$$\sqrt{8} = \sqrt{2 \times 2 \times 2} = \sqrt{2^2 \times 2} = 2\sqrt{2}$$

ladder method

start from lowest prime no.

2	48
2	24
2	12
2	6
3	3
	1

$$\sqrt{48} = \sqrt{2^2 \times 2^2 \times 3}$$

$$= 4\sqrt{3}$$

addition & subtraction

- $3\sqrt{2} + 4\sqrt{2} = 7\sqrt{2}$
- $6\sqrt{3} - 2\sqrt{3} = 4\sqrt{3}$
- $2\sqrt{5} + 4\sqrt{5} - 3\sqrt{5} = 3\sqrt{5}$
- $\sim 2\sqrt{18} + 4\sqrt{8} - 3\sqrt{2}$
 $= 6\sqrt{2} + 8\sqrt{2} - 3\sqrt{2}$
 $= 11\sqrt{2}$
- $\sim 5\sqrt{2} + 2\sqrt{12}$
 $= 5\sqrt{2} + 4\sqrt{3}$

same surds

division

- $\frac{\sqrt{5}}{\sqrt{5}} = 1$
- $\frac{4(3+\sqrt{7})}{(3+\sqrt{7})} = 4$
- $\frac{\sqrt{12}}{\sqrt{3}} = \sqrt{\frac{12}{3}} = 2$
- $\frac{\sqrt{12}}{\sqrt{3}} = \frac{2\sqrt{3}}{\sqrt{3}} = 2$

* Rationalise denominator

$$-\frac{3}{4+\sqrt{5}} \times \frac{4-\sqrt{5}}{4-\sqrt{5}} = \frac{12-3\sqrt{5}}{(4)^2-(\sqrt{5})^2}$$

use a^2-b^2 formula

$$= \frac{12-3\sqrt{5}}{11}$$

* simplify ans

$$\frac{6-2\sqrt{5}}{4} = \frac{3-\sqrt{5}}{2}$$

- $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$
- $\sqrt{a-b} \neq \sqrt{a} - \sqrt{b}$
- $a\sqrt{b} + c \neq a+c\sqrt{b}$
- $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$
- $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$
- $\sqrt{a} \times \sqrt{a} = a$

multiplication

- $\sqrt{10} \times \sqrt{2} = \sqrt{10 \times 2} = \sqrt{20} = 2\sqrt{5}$
- $\sqrt{2}(5-3\sqrt{2}) = 5\sqrt{2} - 3(2)$
 $= 5\sqrt{2} - 6$
 $= -6 + 5\sqrt{2}$
 $a + b\sqrt{c}$ (rational + surd)

- $(5-\sqrt{3})(7+2\sqrt{3})$
 $= 5(7) + 10\sqrt{3} - 7\sqrt{3} - 6$
 $= 29 + 3\sqrt{3}$
- $(4+\sqrt{3})(4-\sqrt{3}) = (4)^2 - (\sqrt{3})^2$
 $= 16 - 3$
 $= 13$

$a^2 - b^2$

a) $(2-\sqrt{3})^2 = (2)^2 - 2(2)(\sqrt{3}) + (\sqrt{3})^2$
 $= 7 - 4\sqrt{3}$

b) $\pm \sqrt{28-16\sqrt{3}} = \pm \sqrt{4(7-4\sqrt{3})}$
 $= \pm 2\sqrt{7-4\sqrt{3}}$
 $= \pm 2(2-\sqrt{3})$

solutions: $2(2-\sqrt{3})$ or $-2(2-\sqrt{3})$
 $= 4-2\sqrt{3}$ or $-4+2\sqrt{3}$

Solving w/surds

① $\sqrt{6-x} = 3$ } sq both sides
 $6-x=9$
 $-x=3$
 $x=-3$

When $x = -3$, LHS = $\sqrt{6-x}$
 $= \sqrt{6-(-3)} = \sqrt{9} = 3$
 $\therefore x = -3$ check if satisfies RHS

* Note:

$\sqrt{3-4x} + 2x = 0$
 $\sqrt{3-4x} = -2x$
 MUST square over then square!

Chapter 4 Polynomials

- Polynomials: sum/diff of multiple terms

ex: $2x^2 + 5x + 3$, $3x - 1$

coeff. $a x^n \rightarrow$ non-ve integer

- Degree of polynomial (highest power)

$4x^3 - 5x^2 + 2 \rightarrow$ degree = 3

- identity: eqs that are true for all values of x

$x^2 + 2x = x(x+2) \checkmark$

$x^2 + 2x = 4x + 3$ $\times$ { only true for some values of x }

when there are unknowns:

M1: sub values of x into identity

M2: equate coeff. of corresponding terms

example:

$x^3 - 2x^2 + 5 = a(x-1)^2 + b(x-1) + c$

M1: when $x=1$, $(1) - 2(1)^2 + 5 = c$

eliminate a & b $c = 4$

when $x=0$, $(0)^3 - 2(0)^2 + 5 = a(0)(-1)^2 + b(-1) + \text{sub } c$
 $5 = -b + 4$ $\textcircled{4}$

eliminate a $5 = -b + 4$
 $b = -1$

when $x=2$, $(2)^3 - 2(2)^2 + 5 = a(2)(1)^2 + (-1)(1) + 4$

choose any value (usually -1, 0, 1, 2) $5 = 2a + 3$
 $a = 1$

$\therefore a = 1, b = -1, c = 4$

M2: By comparing coeff. of x^3 : $a = 1$

By comparing coeff. of x : $0 = a + b$
Sometimes this method will need simul. eqs $0 = 1 + b$
 $b = -1$

By comparing constant: $5 = b(-1) + c$

$5 = 1 + c$

$c = 4$

$\therefore a = 1, b = -1, c = 4$

long div

5

Quotient
 $Q(x)$

example

$$2x^2 - x$$

Add if needed
for clarity

$$4x^3 - 5x + 5$$

$$\begin{array}{r}
 2x+1 \overline{) 4x^3 + 0x^2 - 5x + 5} \\
 \underline{-(4x^3 + 2x^2)} \\
 -2x^2 - 5x \\
 \underline{-(2x^2 - x)} \\
 -4x + 5 \\
 \underline{-(-4x - 2)} \\
 7
 \end{array}$$

$$= (2x+1)$$

$$(2x^2 - x - 2) + 7$$

$$\begin{array}{r}
 3 \overline{) 15} \\
 \underline{-15} \\
 0
 \end{array}$$

Divisor
 $D(x)$

Divident
 $P(x)$

Remainder $R(x)$

$$P(x) = Q(x) \cdot D(x) + R(x)$$

$$\deg P(x) = \deg D(x) + \deg Q(x)$$

$$\deg D(x) \geq \deg R(x)$$

$$\text{if } \deg D(x) = 2,$$

$\deg$ of $R(x)$ can be 1/0.

$$\text{ex: } 4x^3 - 6x^2 + 1 = (x-2) \cdot Q(x) + ax - 3$$

$$\deg = 3$$

$$\deg = 1$$

$$\deg = 2 \quad (3-1)$$

$$(ax^2 + bx + c)$$

remainder theorem

$$\text{ex: let } f(x) = x^4 - 4x^3 + 3x^2 + x + 3$$

$$\text{divisor: } x+1$$

using remainder theorem

$$\text{let } x = -1$$

$$f(-1) = \dots$$

$$= 10$$

Remainder

factor theorem

$$\text{ex: when } x=2$$

$$f(2) = (2)^3 - 3(2) + 2 = 0$$

since rem = 0, by factor theorem,

$$x-2 \text{ is a factor of } f(x)$$

(OR)

since is a factor of $f(x)$,

by factor theorem, remainder = 0

$$\therefore f(c) = 0$$

cubic polynomials

deg's

$$\text{Recall: } a^2 - b^2 = (a-b)(a+b)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

same

oppo
alw
tve

SOAP: same, oppo,

alw tve

Factorising cubic polynomials

Factorise $2x^3 + 3x^2 - 11x - 6$ completely

Step 1: Find linear factor of $f(x)$ using calculator

let $x = \boxed{2}$ ← found from calculator

$$\text{let } f(x) = 2x^3 + 3x^2 - 11x - 6$$

$$f(2) = 2(2)^3 + 3(2)^2 - 11(2) - 6 \\ = 0$$

using factor theorem, $x - 2$ is a factor of $f(x)$

Step 2: Find $ax^2 + bx + c$

m1: long div ...

m2: compare w/ $f(x)$
 $f(x) = (x - 2)(ax^2 + bx + c)$
deg = 3
deg = 2
deg = 1

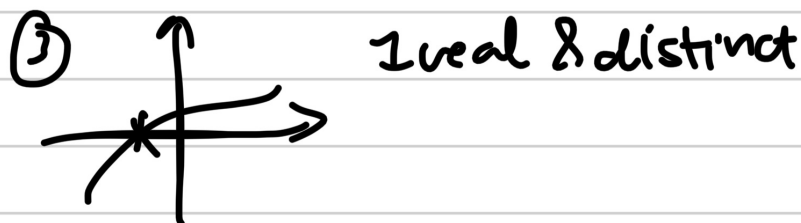
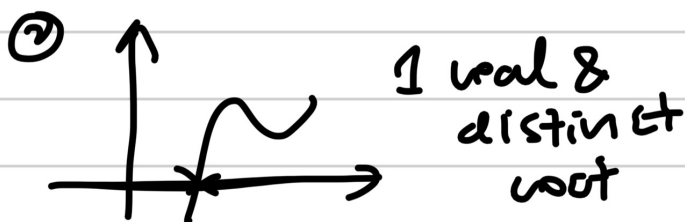
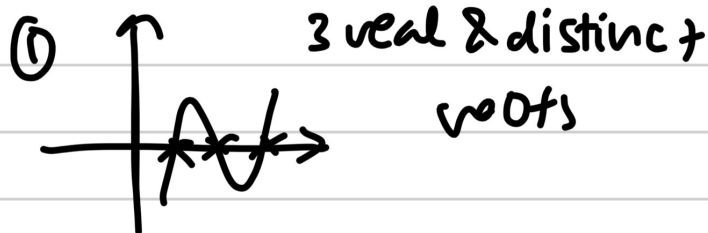
* If there are repeated ans, write out.

* If they ask 'leave in surds form',

use formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

recap



↳ Always has at least 1 solⁿ.

Example

$$x(x^2 + 6) = 3x^2 + 8$$

$$x^3 - 3x^2 + 6x - 8 = 0$$

$$\text{let } f(x) = x^3 - 3x^2 + 6x - 8$$

$$\text{let } x = 2$$

$$f(2) = \dots = 0$$

using factor theorem, $x-2$ is a factor of $f(x)$

$$x-2 \overline{) \dots}$$

$$f(x) = (x-2)(x^2 - x + 4)$$

$$\text{when } f(x) = 0$$

$$(x-2)(x^2 - x + 4) = 0 \quad \rightarrow$$

$$x-2=0 \text{ or } x^2 - x + 4 = 0$$

$$x=2$$

$$D = b^2 - 4ac$$

$$= (-1)^2 - 4(1)(4)$$

$$= -15$$

since $D < 0$, $x^2 - x + 4$ has no real roots. $\therefore$ $x=2$ is the only solⁿ of $f(x)$. Thus $f(x)$ only has 1 real solⁿ (shown)

$$\star \text{ when } x = \frac{1}{2}$$

cannot be a fraction.

$$f\left(\frac{1}{2}\right) = 0$$

$$\hookrightarrow 2x-1 \text{ is a factor of } f(x) \quad \checkmark$$

$$\hookrightarrow x - \frac{1}{2} \text{ is a factor of } f(x) \quad \times$$

Partial Fractions

proper: deg of num < deg of denom

improper: deg of num $\geq$ deg of denom

proper frac

$$\textcircled{1} \frac{px+q}{(ax+b)(cx-d)} = \frac{A}{ax+b} + \frac{B}{cx-d}, \quad A \text{ \& B constants}$$

$$\textcircled{2} \frac{px+q}{(ax+b)^2} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2}, \quad A \text{ \& B constants}$$

$$\textcircled{3} \frac{px^2+qx+c}{(ax+b)(x^2+c^2)} = \frac{A}{ax+b} + \frac{Bx+C}{x^2+c^2}, \quad A, B \text{ \& C are constants}$$

non-reducible

improper fractions

↳ use long div

ex: $\frac{2x^3 - x^2 - 8x + 16}{(x-2)(x+1)}$ in partial frac

$$(x-2)(x+1) = x^2 - x - 2$$

$$\begin{array}{r} \overline{2x + 1} \\ x^2 - x - 2 \overline{) 2x^3 - x^2 - 8x + 16} \\ \underline{-(2x^3 - 2x^2 - 4x)} \\ x^2 - 4x + 16 \\ \underline{-(x^2 - x - 2)} \\ -3x + 18 \end{array}$$

$$\therefore \frac{2x^3 - x^2 - 8x + 16}{(x-2)(x+1)} = 2x + 1 + \frac{-3x + 18}{(x-2)(x+1)}$$

use proper frac steps

* If A-C is a fraction, write like this

ex: $A = \frac{1}{2}$

$$\frac{(\frac{1}{2})}{x-2} \quad \times \quad \frac{1}{2(x-2)} \quad \checkmark$$

* Always check if proper / improper

* check if reducible or not

Chapter 05 Exponential & Log Func

$$a^x = y \Leftrightarrow x = \log_a y$$

$$\log_{10} x = \lg x$$

$$\log_e x = \ln x$$

log func

$$1) \log_a a = 1$$

$$2) \log_a 1 = 0$$

$$3) \log_a xy = \log_a x + \log_a y$$

$$4) \log_a \frac{x}{y} = \log_a x - \log_a y$$

$$5) \log_a x^y = y \log_a x$$

$$6) \log_a x = \frac{\log_b x}{\log_b a}$$

$$\log_a x \rightarrow x > 0$$

$$\log_{10} x = -2 \neq \log_{10} -2$$

laws of indices

$$1) a^0 = 1$$

$$2) a^{-x} = \frac{1}{a^x}$$

$$3) a^{\frac{x}{y}} = y\sqrt[y]{a^x}$$

$$4) a^x \times a^y = a^{x+y}$$

$$5) \frac{a^x}{a^y} = a^{x-y}$$

$$6) (a^x)^y = a^{xy}$$

$$7) a^x \times b^x = (ab)^x$$

$$8) \frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x$$

methods

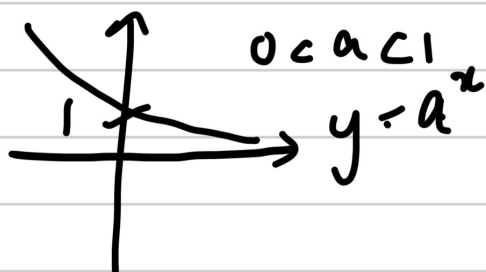
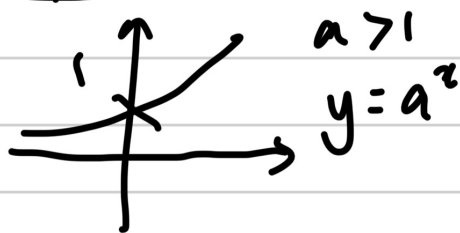
① Substitution

② Solving

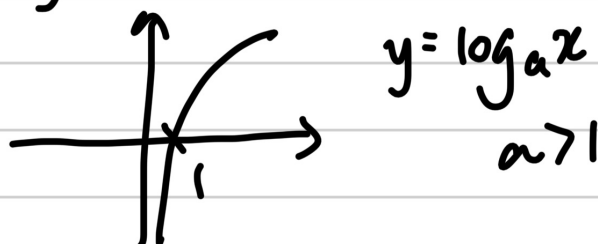
③ Graph Drawing

graphs

exponential



log



Chap 06 Binomial Theorem

! = factorial $0! = 1$

$$\binom{a}{1} = a$$

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$\binom{a}{0} = 1$$

$$\text{ex: } \binom{n}{3} = \frac{n(n-1)(n-2)\cancel{(n-3)!}}{3!\cancel{(n-3)!}}$$

$$= \frac{n(n-1)(n-2)}{3 \times 2 \times 1}$$

binomial formula

↳ used for : expand until _____ term

$$(a+b)^n = \binom{n}{0}(a)^n(b)^0 + \binom{n}{1}(a)^{n-1}(b)^1 + \dots + \binom{n}{n}(a)^0(b)^n$$

$$(1+b)^n = \binom{n}{0}(b)^0 + \binom{n}{1}(b)^1 + \dots + \binom{n}{n}(b)^n$$

general term formula

↳ used to find specific terms

$$T_{r+1} = \binom{n}{r}(a)^{n-r}(b)^r$$

*note!

Term indep of x
= constant

* Always simplify general term

$$T_{r+1} = \binom{n}{r}(a)^{n-r}(b)^r$$

$$= \binom{14}{r}(a)^{14-r}(x^2)^r$$

$$= \binom{14}{r}(a)^{14-r}(x^{2r})$$

① Simplify

② Group all x term together

③ Group constants

Q: why is _____ not a term in eqⁿ? power of the term
↳ find r & equate to _____

Since $r = \square$ is not a whole no., there is no term _____

$$\text{example: } T_{r+1} = \binom{10}{r}(1)^{10-r}\left(-\frac{x^2}{4}\right)^r$$

$$= \underbrace{\binom{10}{r}}_{\text{coeff}} \left(-\frac{1}{4}\right)^r (x^{2r})$$

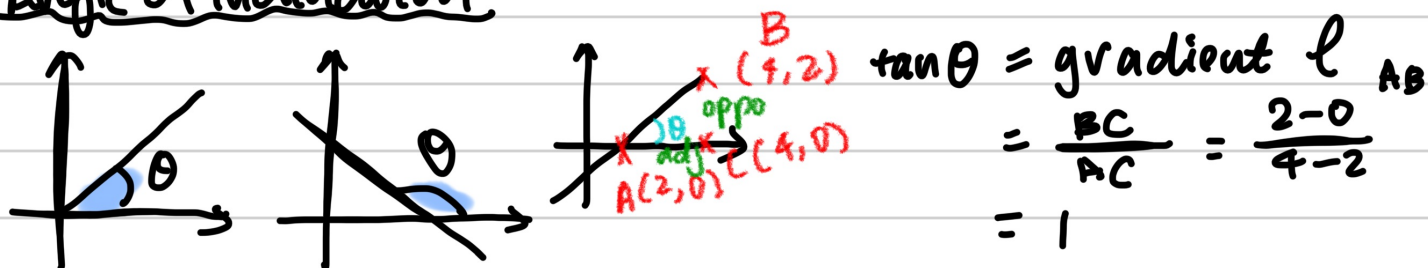
$2r = 3$ Since $r = 1\frac{1}{2}$, is not a whole no. there is no term in x^3 .
 $r = \frac{3}{2}$

Chap 07 coord Geometry

Tools

- ① Parallel lines $\rightarrow m_1 = m_2$
- ② Perpendicular Lines $(m_1 \times m_2 = -1 / m_1 = -\frac{1}{m_2})$
- ③ Gradient $\rightarrow$ eqⁿ of line
- ④ Gradient $(m = \tan \theta)$
- ⑤ coords of pt $\rightarrow$ 2 intersecting lines
- ⑥ eqⁿ : $y - y_1 = m(x - x_1)$
- ⑦ Shoelace formula (Area) $\odot$
- ⑧ Mid pt formula $(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$
- ⑨ Distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- ⑩ Gradient = $\frac{y_2 - y_1}{x_2 - x_1}$

Angle of Inclination



Collinear

ex: prove that $P(2, k+2)$, $Q(-2, k+2)$ & $R(3, k+3)$ are collinear

① gradient of $PQ = \frac{k+2 - (k+2)}{-2 - 2}$ gradient of $QR = \frac{k+2 - (k+3)}{3+2}$
#1 $\square$ & $\square$ hv same gradient $= \frac{-4}{-4} = 1$ $= \frac{k+2 - (k+3)}{3+2} = 1$

#2 indicate common pt.

② Since grad of $PQ =$ gradient QR & Q is the common pt.
 $\therefore$ Pts PQR are collinear

Midpt of line
 midpt, $M = \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2} \right)$

or let coords of A be (a, b)

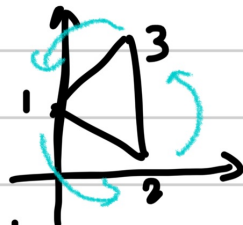
$\rightarrow [3, 5] = \left(\frac{6+a}{2}, \frac{7+b}{2} \right)$ midpt given and word given

By comparing, $3 = \frac{6+a}{2}$ & $5 = \frac{7+b}{2}$

Area of Δ & Quads

- anti clockwise $\curvearrowright$

- $\frac{1}{2} | x_1 \dots x_n x_1 |$ go back to orig pt.



ex: Area $\Delta ABC = \frac{1}{2} | -2 \times 7 \times 5 - 2 |$

$= \frac{1}{2} [(-12 + 84) - (21 + 30 - 24)] = \frac{1}{2} (87 - 27)$

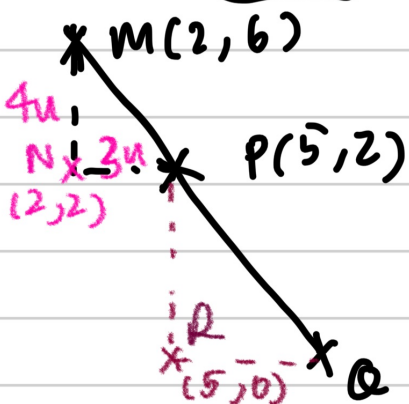
$= 30 \text{ units}^2$ Remember units!

Perpendicular bisectors

l is the $\perp$ bisector of AB as $AM = MB$
 & l is $\perp$ to AB
 $\hookrightarrow$ midpt formula
 $\hookrightarrow l \perp l_{AB}$ to find gradient.

} where to use??

Similar Δ s commonly used *



Δ similar Δ since ΔMNP & ΔPRQ

$\Downarrow$
 Ratio of $\frac{MN}{PR} = \frac{MP}{PQ} = \frac{6-2}{2-0} = 2$
 corr. sides

Ratio of $MP: PQ$ is $2:1$

chap 08 circles

standard form: $(x-a)^2 + (y-b)^2 = r^2$

general form: $x^2 + y^2 + 2gx + 2fy + c = 0$

↳ centre $(a, b) / (-g, -f)$

↳ radius = $\sqrt{g^2 + f^2 - c}$

① complete the sq

$$\left(x - \frac{a}{2}\right)^2 - \left(\frac{a}{2}\right)^2 + \left(y + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + \square = 0$$

$$\textcircled{2} \begin{array}{ll} 2g = \square & 2f = \square \\ g = \square & f = \square \end{array}$$

Always

-ve

* common qns

1) Reflected circles

a) centre

b) Radius

c) Eqⁿ

} find ...

2) Lie inside/on/outside

a) Find dist b/w pt & centre

b) compare w/ radius

— larger → outside

— equal → lie on

— smaller → inside

} 3 steps.

3) Coordinate

↳ intersecting (simultaneous)

Chap 09 Linear Law

objective : make eqs into linear eqs tip 2

Function

horizontal axis

vert. axis

gradient

vert. int

$$y = ax^n + b$$

$$x^n$$

$$a$$

$$b$$

$$y = a + \frac{b}{x^n}$$

$$\frac{1}{x^n}$$

$$b$$

$$a$$

$$\frac{1}{y} = ax^n + b$$

$$x^n$$

$$\frac{1}{y}$$

$$a$$

$$b$$

$$y = a + b\sqrt{x}$$

$$\sqrt{x}$$

$$b$$

$$a$$

$$y = ax + bx^2 \Rightarrow \frac{y}{x} = a + bx$$

$$\frac{y}{x}$$

$$b$$

$$a$$

$$y = ax + bx^2 \Rightarrow \frac{y}{x^2} = \frac{a}{x} + b$$

$$\frac{y}{x^2}$$

$$a$$

$$b$$

$$y = ab^x \Rightarrow \lg y = \lg a + x \lg b$$

$$\lg y$$

$$\lg b$$

$$\lg a$$

$$y = ab^x \Rightarrow \lg y = \lg a + b \lg x$$

$$\lg y$$

$$b$$

$$\lg a$$

ex: $y\sqrt{x} = -2x^{\frac{5}{2}} + \frac{1}{\sqrt{x}}$

$$xy = -2x^{\frac{5}{2}} + 1$$

$$y = xy, X = x^{\frac{5}{2}}$$

or

$$x - 4y = 3xy$$

$$\frac{x}{y} - 4 = 3x$$

$$\frac{y\sqrt{x}}{\sqrt{x}} = \frac{-2x^{\frac{5}{2}}}{\sqrt{x}} + \frac{(\frac{1}{\sqrt{x}})}{\sqrt{x}}$$

$$y = -2x^{\frac{3}{2}} + \frac{1}{x}$$

$$\text{grad} = -2$$

$$\text{vert. int.} = 1$$

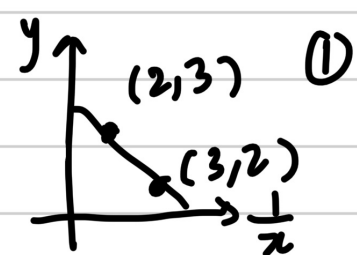
$$\frac{x}{y} = 3x + 4$$

$$w/w$$

$$y$$

$$\text{grad } X$$

$$Y \text{ int.}$$



$$y = mx + c$$

$$m = \frac{2-3}{3-2} = -1$$

$$y = -x + c$$

$$\text{when } x = 2, y = 3$$

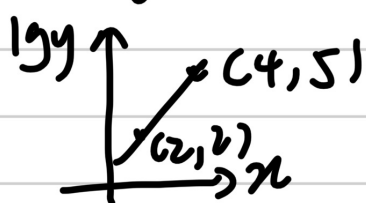
$$3 = -2 + c$$

$$c = 5$$

$$y = -x + 5$$

$$y = -\left(\frac{1}{x}\right) + 5$$

② Find A & B if str. line graph rep
 $y = Ab^x$



$$\text{let } y = \lg y \text{ let } x = x$$

$$m = \frac{5-2}{4-2} = \frac{3}{2}$$

$$y = \frac{3}{2}x + c$$

$$\text{when } x = 2, y = 2$$

$$2 = \left(\frac{3}{2}\right)(2) + c$$

$$c = -1$$

$$y = \frac{3}{2}x - 1$$

$$\lg y = \frac{3}{2}x - 1$$

$$\lg y = \lg(Ab^x)$$

$$= \lg A + x \lg b$$

$$\lg A = -1$$

$$A = 10^{-1}$$

$$= \frac{1}{10} //$$

$$\lg b = \frac{3}{2}$$

$$b = 10^{\frac{3}{2}} //$$

★ Always draw your graph with not manipulated variables (ex $\frac{1}{x}$, $\lg y$)

★ Your coords of (3, 2) / (2, 3) are always your 'y' & 'x' ($\lg y$ & $\lg x$)

★ Find the easiest way to manipulate eqns.

I Trigonometry

II Further Trigo

Recaps:

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

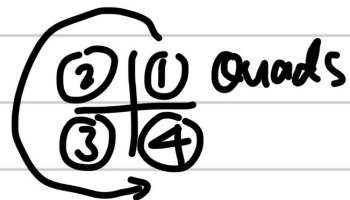
$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

TOA
CAH
SOH

sin +ve	All +ve
tan +ve	cos +ve

$$\frac{s}{T} \frac{A}{C}$$



2nd Quad

$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x}$$



only sin
+ve

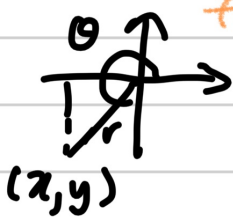
only
tan
+ve

3rd Quad

$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x}$$

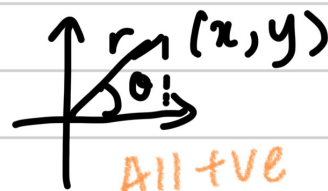


1st Quad

$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x}$$



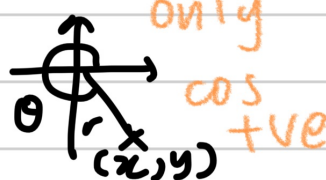
All +ve

4th Quad

$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x}$$



only
cos
+ve

$$\textcircled{*} \sin A = \sin(180^\circ - A)$$

$$\textcircled{*} \cos A = -\cos(180^\circ - A)$$

ex: $\sin x = \dots$

$\cos x = -\dots$

since $\sin x$ is +ve, x lies in quad $\dots$

since $\cos x$ -ve, x lies in quad $\dots$

ref $\angle$, $\alpha = \sin^{-1} \dots$

$x = \dots$

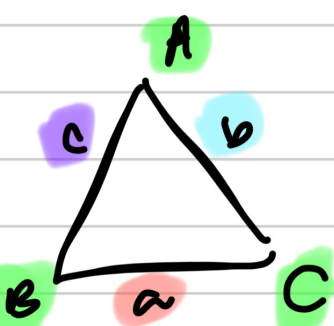
$x = \dots, \dots$

Area of Δ

$$\text{Area of } \Delta = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2} bc \sin A$$

$$= \frac{1}{2} ac \sin B$$



included $\angle$

Area of Δ

$$= \frac{1}{2} (\text{product of 2 sides}) \sin (\text{included } \angle)$$

$\rightarrow$ Always round 1 dp (final ans)

$\rightarrow$ Intermediate: 3-5 dp

$\rightarrow \sin \theta > 1$ } no soln.
 $\cos \theta > 1$

Sine Rule

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

ex



$$\frac{x}{\sin 55^\circ} = \frac{3}{\sin 30^\circ}$$

$$x = \frac{3}{\sin 30^\circ} \times \sin 55^\circ$$

Remember to use intermediate ans!!

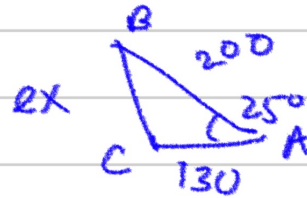
Cosine Rule

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$



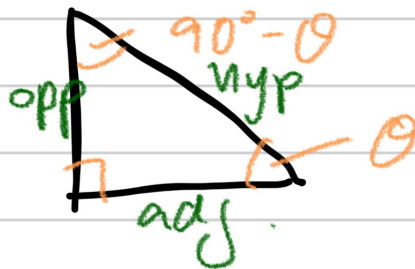
$$BC^2 = 200^2 + 130^2 - 2(200)(130)\cos 25^\circ$$

$$BC = 98.9 \text{ km (3sf)}$$

$$\sin(90^\circ - \theta) = \cos \theta$$

$$\cos(90^\circ - \theta) = \sin \theta$$

$$\tan(90^\circ - \theta) = \frac{1}{\tan \theta}$$



$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

Remember to always convert when necessary

Deg → Radian

$$\pi \text{ rad} = 180^\circ$$

$$\frac{\pi}{2} \text{ rad} = 90^\circ$$

$$\frac{\pi}{3} \text{ rad} = 60^\circ$$

$$\frac{\pi}{4} \text{ rad} = 45^\circ$$

$$\frac{\pi}{6} \text{ rad} = 30^\circ$$

	0°	30°	45°	60°	90°
sin	0	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2} = 1$
cos	$\frac{\sqrt{4}}{2} = 1$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	0
tan	$\frac{0}{\sqrt{4}} = 0$	$\frac{\sqrt{1}}{\sqrt{3}} = \frac{1}{3}$	$\frac{\sqrt{2}}{\sqrt{2}} = 1$	$\frac{\sqrt{3}}{\sqrt{1}} = \sqrt{3}$	$\frac{\sqrt{4}}{0}$



sin → below → √ fingers below

cos → above → √ fingers above

tan → flip your hand

√ fingers above ②

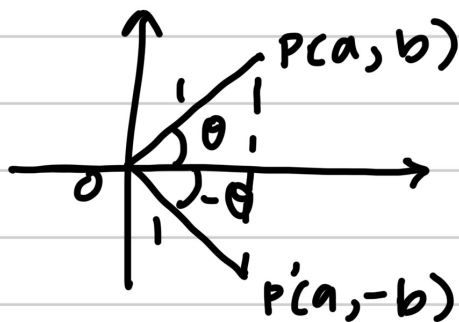
√ fingers below

Negative Angles

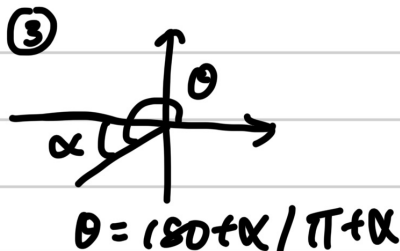
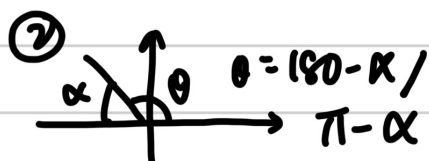
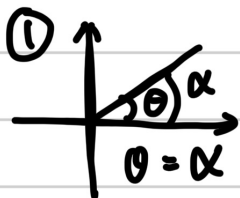
$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

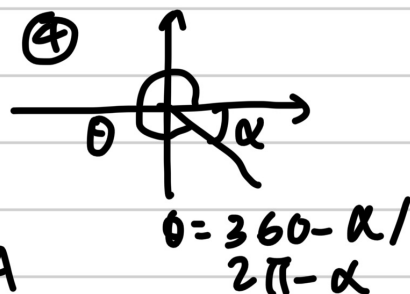
$$\tan(-\theta) = -\tan \theta$$



BS b/w $x + \theta$

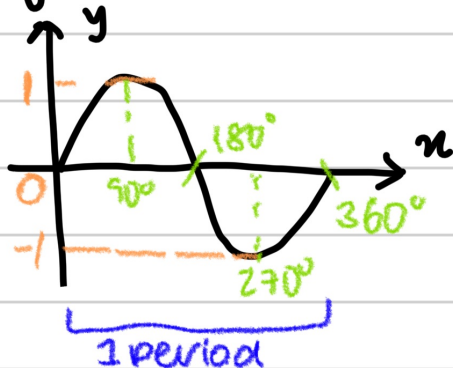


$$\frac{S}{T} \mid \frac{A}{C}$$

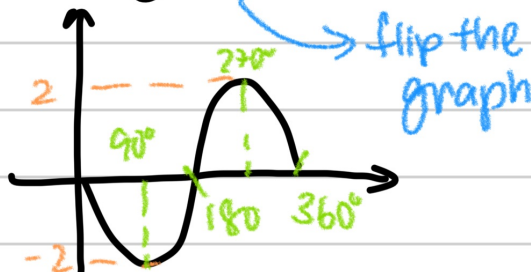


Graphs

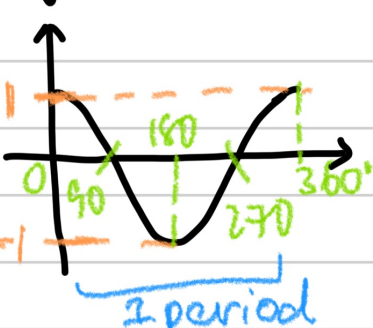
$$y = \sin x$$



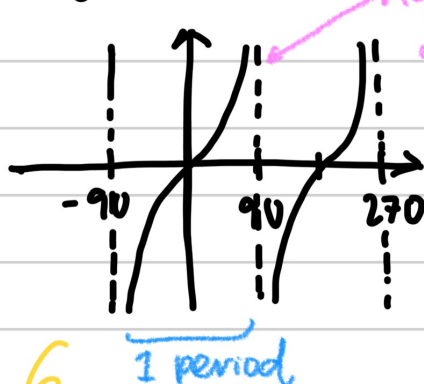
$$y = -2 \sin x$$



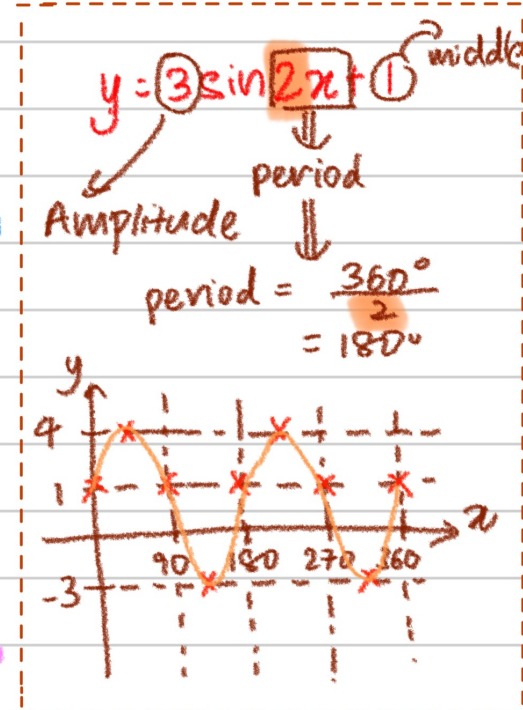
$$y = \cos x$$



$$y = \tan x$$



Amplitude undefined.



Principle values

$$-90^\circ \leq \sin^{-1} x \leq 90^\circ$$

$$0^\circ \leq \cos^{-1} x \leq 180^\circ$$

$$-90^\circ < \tan^{-1} x < 90^\circ$$

$\leq$ wrong bc asymptote

$\text{cosec } \theta = \frac{1}{\sin \theta}, \sin \theta \neq 0$
 $\sec \theta = \frac{1}{\cos \theta}, \cos \theta \neq 0$
 $\cot \theta = \frac{1}{\tan \theta}, \tan \theta \neq 0$

SINE GRAPH

$y = \sin x$



Period
 $= 360^\circ (2\pi)$
 Amplitude = 1

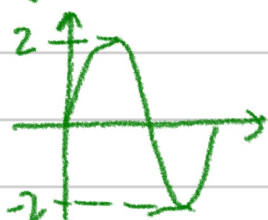
special values

$\sin x = 0$ at $x = 0^\circ, 180^\circ, 360^\circ$

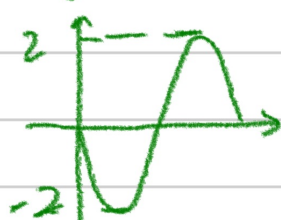
$\sin x = 1$ at $x = 90^\circ$

$\sin x = -1$ at $x = 270^\circ$

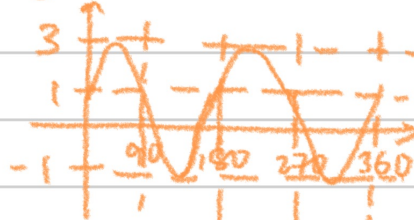
$y = 2 \sin x$



$y = -2 \sin x$



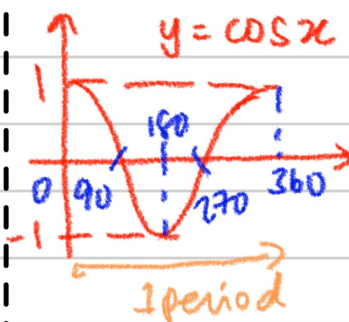
$y = 2 \sin 2x + 1$



repeats twice in 360°

COS GRAPH

$y = \cos x$



period =
 $2\pi = 360^\circ$
 Amplitude = 1

$-1 \leq \cos x \leq 1$

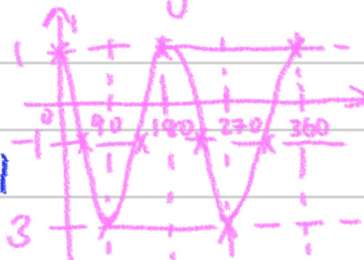
special values

$\cos x = 0$ at $x = 90^\circ, 270^\circ$

$\cos x = 1$ at $x = 0^\circ, 360^\circ$

$\cos x = -1$ at $x = 180^\circ$

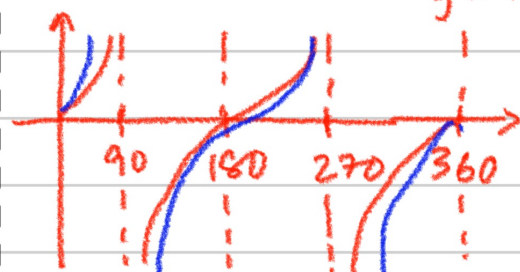
$y = 2 \cos 2x - 1$



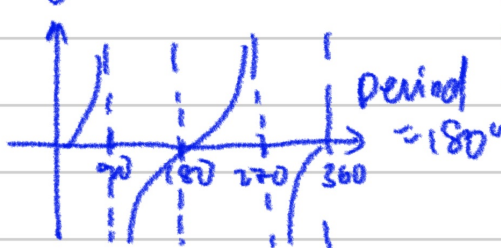
amplitude = 2
 period = $\frac{360}{2}$
 $= 180$
 mid pt = -1

TAN GRAPH

$y = \tan x$



$y = \tan 2x$



special values

$\tan x = 0$ at $x = 0^\circ, 180^\circ, 360^\circ$

$\tan x = 1$ at $x = 45^\circ, 225^\circ$

$\tan x = -1$ at $x = 135^\circ, 315^\circ$

Amplitude
 x defined

Formulas

$$\textcircled{*} \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Pythagorean

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$$

Addition

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

R formula

$$a \sin \theta \pm b \cos \theta = R \sin(\theta \pm \alpha)$$

$$a \cos \theta \pm b \sin \theta = R \cos(\theta \mp \alpha)$$

$$R = \sqrt{a^2 + b^2}$$

$$\tan \alpha = \frac{b}{a}, \alpha \text{ is acute}$$

Double

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Trig Expression

$$\sin^2 \theta = \sin \theta \cdot \sin \theta$$

$$4 \cos \theta - 3 \cos \theta + 2 = \cos \theta + 2$$

$$\sin^2 \theta - \cos^2 \theta = (\sin \theta + \cos \theta)(\sin \theta - \cos \theta)$$

$$2 \tan^2 \theta - 3 \tan \theta - 2 = (2 \tan \theta + 1)(\tan \theta - 2)$$

$$\frac{\cos^2 \theta - 1}{\cos^2 \theta - \cos \theta} = \frac{(\cos \theta + 1)(\cancel{\cos \theta - 1})}{\cos \theta (\cancel{\cos \theta - 1})}$$
$$= 1 + \frac{1}{\cos \theta}$$

commonly asked

$$\sin 2x = 2 \sin x \cos x$$

$$\sin^2\left(\frac{x}{2}\right) = 2 \sin \frac{x}{2} \cos \frac{x}{2}$$

$$\sin x = 2 \sin \frac{x}{2} \cos \frac{x}{2}$$

} half
x.

12: Differentiation

function

derivative

$$y = x^{-3}$$

$$\frac{dy}{dx} = -3x^{-4}$$

$$y = x^{-2}$$

$$\frac{dy}{dx} = -2x^{-3}$$

$$y = x^n$$

$$\frac{dy}{dx} = n x^{n-1}$$

op power -1
bring down

Remember that can separate fractions

$$\text{ex: } \frac{6x^2 - x^{\frac{1}{2}} + 2}{2x}$$

$$= \frac{6x^2}{2x} - \frac{x^{\frac{1}{2}}}{2x} + \frac{2}{2x}$$

$$= 3x - \frac{1}{2}x^{-\frac{1}{2}} + x^{-1}$$

$\frac{dy}{dx}$ = gradient

Chain rule

$$\frac{d}{dx}(u^n) = n u^{n-1} \left(\frac{du}{dx} \right)$$

$$\text{ex: } y = (x^4 + 3x^2)^{20}$$

$$\frac{dy}{dx} = 20(x^4 + 3x^2)^{19} (4x^3 + 6x)$$

$$= 40(2x^3 + 3x)(x^4 + 3x^2)^{19}$$

Differentiate inside the bracket.

Qn: $y = \frac{\pi\sqrt{x}}{2} + \frac{3}{\sqrt{x}}$ Find gradient of tangent to curve at pt where $x = -1$

$$y = \frac{\pi\sqrt{x}}{2} + \frac{3}{\sqrt{x}}$$

$$= \frac{\pi}{2}(x)^{\frac{1}{2}} + 3x^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{\pi}{4}x^{-\frac{1}{2}} - \frac{3}{2}x^{-\frac{3}{2}}$$

when $x = -1$

$$\frac{dy}{dx} = \frac{\pi}{4}(1)^{-\frac{1}{2}} - \frac{3}{2}(1)^{-\frac{3}{2}}$$

$$= \frac{\pi}{4} - \frac{3}{2}$$

grad of tangent line =

gradient of line at that P

tangent



$$y = mx + c$$

grad of curve at pt A $\left(\frac{dy}{dx} \right)$

Product Rule

$$\frac{d}{dx}(uv) = v \frac{du}{dx} + u \frac{dv}{dx}$$

x diff 2nd, diff 1st +

x diff 1st, diff 2nd.

$$\text{ex: } y = x^2(4x-1)^2$$

$$u = x^2$$

$$v = (4x-1)^2$$

$$\frac{dy}{dx} = [4x-1]^2(2x) + x^2[2(4x-1)(4)]$$

$$= (4x-1)^2(2x) + 8x^2(4x-1)$$

⋮

sim plify

Quotient Rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\left(\frac{du}{dx}\right) - u\left(\frac{dv}{dx}\right)}{v^2}$$

$$= \frac{x \text{ diff 2nd, diff 1st} - x \text{ diff 1st, diff 2nd}}{(2nd)^2}$$

ex: $y = \frac{4x+1}{2x^2-1}$ (1st) (2nd)

$$\frac{dy}{dx} = \frac{(2x^2-1)(4) - (4x+1)(4x)}{(2x^2-1)^2}$$

$$= \vdots$$

✳ Always factorise out smallest term if poss.

13: Tangents, Normals & Rate of Change

TANGENTS & NORMAL

eqⁿ of str. line:

$$y - y_1 = m(x - x_1)$$

$\uparrow$ grad = $\frac{dy}{dx}$

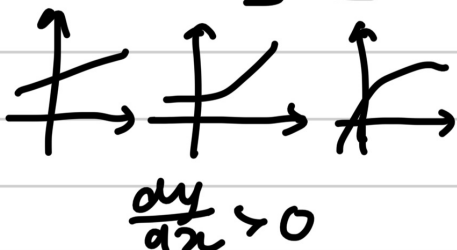
perpendicular lines:

$$m_1 \times m_2 = -1$$
$$m_2 = \frac{-1}{m_1}$$

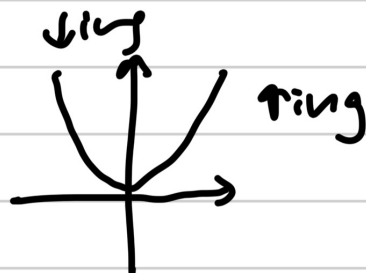
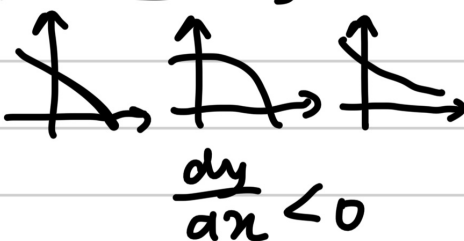
parallel lines: $m_1 = m_2$

INCREASING & DECREASING FUNC

increasing



decreasing



Qn Types

① Find if Ting / Ding

ex: $y = 2x^2, x > 0$ $4x > 0$

$$\frac{dy}{dx} = 4x$$

$$\frac{dy}{dx} > 0$$

when $x > 0$, $\therefore$ Ting func

② Find Range of values

$$y = 2x^3 + 3x^2 - 12x + 7$$

$$\frac{dy}{dx} = 6x^2 + 6x - 12$$

For y to be ting, $\frac{dy}{dx} > 0$
factorise $\frac{dy}{dx}$:

③ Proving if True / Lie

$$y = (x-1)^{-1}$$

$$\frac{dy}{dx} = \frac{-1}{(x-1)^2}$$

i) show $\frac{dy}{dx} < 0$ when $x > 1$

$$x > 1$$

$$x-1 > 0$$

$$(x-1)^2 > 0$$

$$\frac{1}{(x-1)^2} > 0$$

$$-\frac{1}{(x-1)^2} < 0$$

$$\therefore \frac{dy}{dx} < 0, \text{ (shown)}$$

ii) for $x < 1$

$$x-1 < 0$$

$$(x-1)^2 < 0$$

$$\frac{1}{(x-1)^2} < 0$$

$$-\frac{1}{(x-1)^2} < 0$$

$$\therefore \frac{dy}{dx} < 0, \text{ (shown)}$$

RATE OF CHANGE

given: — $\left(\frac{\omega}{dt}\right)$

find: —

* Rate of change +ve/-ve

* Remember write units

* Review all formulas

(vol/SA / similar Δ s etc.)

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

* Always look at

find: —

units to work
towards.

14: Maxima / Minima Nature of Stationary pts

min / max / pt of inflexion

1st Derivative test

ex: $y = x^2 - 4x + 3$

$$\frac{dy}{dx} = 2x - 4$$

At stationary pt, $\frac{dy}{dx} = 0$

use values like 1.9 / 1.8
 $2x - 4 = 0$
 $x = 2$

use values like 2.2 / 2.3

x	2^-	2	2^+
$\frac{dy}{dx}$	-ve	0	ve
slope	$\searrow$	$-$	$\nearrow$

sub x into $\frac{dy}{dx}$ to find

$\therefore$ Curve is at min pt at $x = 2$

2nd Derivative Test

$$\frac{d^2y}{dx^2} / f''(x)$$

ex:

$$\frac{d^2y}{dx^2} = 6x - 24$$

when $x = 6$

$$\begin{aligned}\frac{d^2y}{dx^2} &= 6(6) - 24 \\ &= 12 \\ &> 0\end{aligned}$$

Since $\frac{d^2y}{dx^2} > 0$ the stationary pt is a min pt at $x = 6$

min: $\frac{d^2y}{dx^2} > 0$

max: $\frac{d^2y}{dx^2} < 0$

unclear: $\frac{d^2y}{dx^2} = 0$

use 1st derivative test

① Formulate eqn

② Find Stationary pt

③ Determine / Show stationary pt is maxima / minima

15: Diffⁿ of Trigu, Exponential & log

Trigo:

$$\frac{d}{dx} \sin x = \cos x$$

$$\frac{d}{dx} \cos x = -\sin x$$

$$\frac{d}{dx} \tan x = \sec^2 x$$

need
to
know

$$\frac{d}{dx} \sec x = \sec x \tan x$$

$$\frac{d}{dx} \cot x = -\csc^2 x$$

good to
know

$$\text{ex: } \frac{d}{dx} \sin 3x = \cos 3x \cdot 3 \\ = 3 \cos 3x$$

① Diff 'sin'

② Diff '3x'

$$\text{ex: } \frac{d}{dx} \tan^2 2x = 2 \tan x + \tan^2 2x$$

$$= 2 \sec^2 x + 2 \tan 2x (\sec^2 x) (2)$$

$$= 2 \sec^2 x + 4 \tan 2x \sec^2 x$$

use
chain rule

Logarithmic Func:

$$\frac{d}{dx} \ln x = \frac{1}{x}$$

$$\frac{d}{dx} \ln u = \frac{1}{u} \cdot \frac{du}{dx}$$

$$\text{ex: } y = \ln(2x+4)$$

$$\frac{dy}{dx} = \frac{1}{2x+4} \cdot 2$$

put values over 1

diffⁿ
in
bracket

Product/quotient
rule still applies

Know how to solve
trig problems,
often come as
a pair!

16: Integration (Indefinite)

integration $y = 3x$ $\frac{dy}{dx} = 3$ diff =

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \text{ } C \text{ is a constant}$$

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{(n+1)a} + C$$

$$\int kf(x) dx = k \int f(x) dx$$

$$\times \int f(x)g(x) dx \neq \int f(x) dx \cdot \int g(x) dx$$

$$\int \frac{f(x)}{g(x)} dx \neq \frac{\int f(x) dx}{\int g(x) dx}$$

$$\int [f(x) + g(x)] dx = \int f(x) dx + \int g(x) dx$$

basics

ex: Show $\frac{d}{dx} \left(\frac{x}{2x+1} \right) = \frac{1}{(2x+1)^2}$, hence find $\int \frac{2}{(2x+1)^2} dx$

$$\textcircled{1} \frac{d}{dx} \left(\frac{x}{2x+1} \right) = \frac{(2x+1)(1) - (x)(2)}{(2x+1)^2} = \frac{1}{(2x+1)^2} \text{, (shown)}$$

$$\textcircled{2} \int \frac{1}{(2x+1)^2} dx = \frac{x}{2x+1} + C$$

$$2 \int \frac{1}{(2x+1)^2} dx = 2 \left(\frac{x}{2x+1} + C \right)$$

use ans in part (i) & convert to qns given eqn.

$$\int \frac{2}{(2x+1)^2} dx = \frac{2x}{2x+1} + C, \text{ where } C' = 2C \text{ \& is a constant.}$$

TRIG

$$\int \cos x dx = \sin x + C$$

$$\int \sec^2(ax+b) dx$$

$$= \frac{\tan(ax+b)}{a} + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \cos(ax+b) dx = \frac{\sin(ax+b)}{a} + C$$

$$\int \sin(ax+b) dx = -\frac{\cos(ax+b)}{a} + C$$

$$\int e^x dx = e^x + C$$

$$\int e^{ax+b} dx = \frac{e^{ax+b}}{a} + C$$

$$\int \frac{1}{x} dx = \ln x + C$$

$$\int \frac{1}{ax+b} dx = \frac{\ln(ax+b)}{a} + C$$

Remember
constant
always!

E
x
p
o
n
e
n
t
i
a
l

work
backwards
to get
ans

Qns to take note!

$$\text{ex: } \int 2 \tan\left(\frac{x}{2}\right) dx = 2 \int \tan\left(\frac{x}{2}\right) dx$$

$$= 2 \int \frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)} dx$$

$$= 2 \int \sin\left(\frac{x}{2}\right) \cdot \frac{1}{\cos\left(\frac{x}{2}\right)} dx$$

$$= -4 \int \frac{-\frac{1}{2} \sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)} dx$$

$$\begin{aligned} \frac{d}{dx} \left\{ \ln \left[\cos\left(\frac{x}{2}\right) \right] \right\} &= \frac{1}{\cos\left(\frac{x}{2}\right)} \cdot -\sin\left(\frac{x}{2}\right) \cdot \frac{1}{2} \\ &= -\frac{\frac{1}{2} \sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)} \end{aligned}$$

$$\therefore \int \frac{-\frac{1}{2} \sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)} = \ln \left[\cos\left(\frac{x}{2}\right) \right] + C$$

$$\text{ex: } \int \frac{2}{\cot 2x} dx = \int \frac{2}{\left(\frac{\cos 2x}{\sin 2x}\right)} dx$$

$$= \int \frac{2 \sin 2x}{\cos 2x} dx$$

$$= - \int \frac{-2 \sin 2x}{\cos 2x} = -\ln(\cos 2x) + C$$

$$\begin{aligned} \frac{d}{dx} \ln[\cos(2x)] &= \frac{1}{\cos 2x} \cdot -\sin 2x \cdot 2 \\ &= -\frac{2 \sin 2x}{\cos 2x} \end{aligned}$$

$$\therefore \int \frac{-2 \sin 2x}{\cos 2x} = \ln[\cos(2x)] + C$$

17: Definite Integral

$$\int_a^b f(x) dx = [F(x)]_a^b \\ = [F(b)] - [F(a)]$$

$\int_0^{\frac{\pi}{2}}$ ← radian mode

$$\Rightarrow e^{\ln 3} = 3$$

↑ quick trick!

ex: $\int_1^4 (\sqrt{x} - \frac{x}{2}) dx = \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{2} \left(\frac{x^2}{2} \right) \right]_1^4$

$$= \left[\frac{2}{3}(4)^{\frac{3}{2}} - \frac{(4)^2}{4} \right] - \left[\frac{2}{3}(1)^{\frac{3}{2}} - \frac{1^2}{4} \right]$$
$$= \frac{1}{3} - \frac{5}{12} \quad \text{MUST show evaluation}$$
$$= \frac{11}{12} \quad \text{can round / simplify if needed}$$

Take note!

$$\int_a^a f(x) dx = [F(x)]_a^a \\ = F(a) - F(a) \\ = 0$$

$$\textcircled{1} \int_1^5 f(x) dx = \int_1^{2.5} f(x) dx + \int_{2.5}^5 f(x) dx$$

Top must be larger

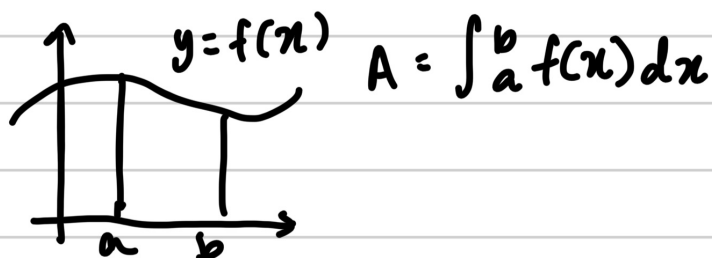
$$\int_a^b f(x) dx + \int_b^a f(x) dx = [F(x)]_a^b + [F(x)]_b^a \\ = 0$$

$$\int_b^a f(x) dx = - \int_a^b f(x) dx$$

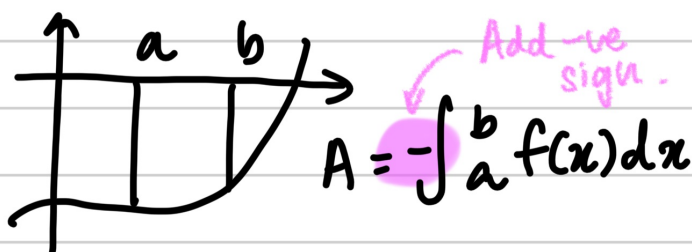
$$\textcircled{2} \int_a^b f(x) dx + \int_b^a f(x) dx = \int_a^a f(x) dx = 0$$

$$\int_2^1 f(x) dx = - \int_1^2 f(x) dx$$

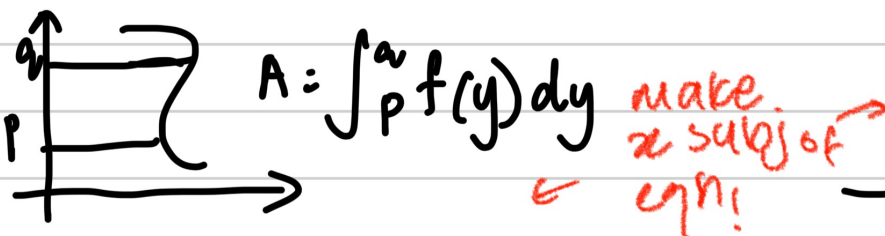
Area of region above x axis



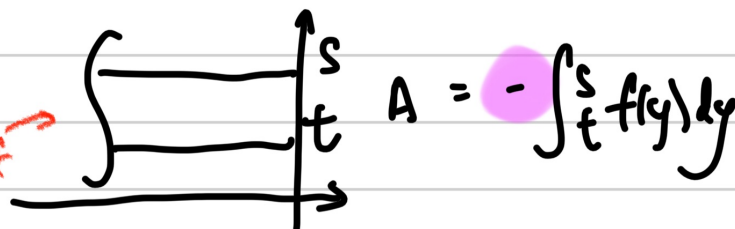
Area of region below x axis



Area of region right of y axis



Area of region left of y axis



18: Kinematics

① speed = $\frac{\Delta \text{dist}}{\Delta \text{time}}$ (scalar)

② velocity = $\frac{\Delta \text{disp}}{\Delta \text{time}}$ (vector)

③ Displacement (vector)

④ Distance (scalar)

s : displacement

$s > 0$: along the s axis

$s = 0$: at ref. pt.

$s < 0$: along -ve s axis

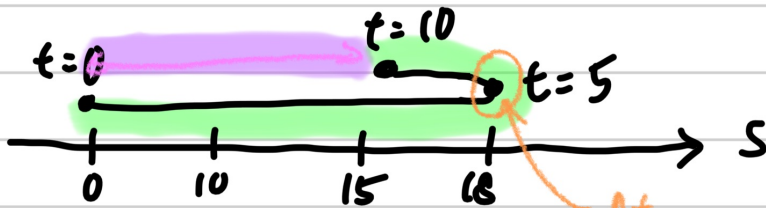
$v > 0$: direc of +ve disp

$v = 0$: instantaneous rest

$v < 0$: direc of -ve disp

purple : tot. displacement

green : tot. distance

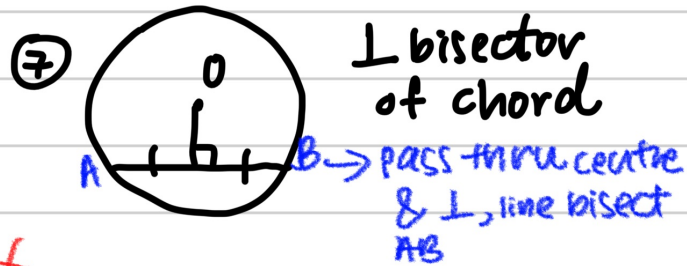
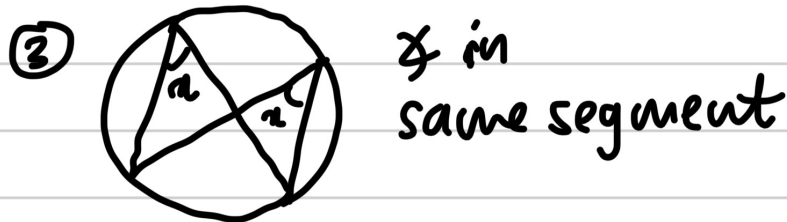
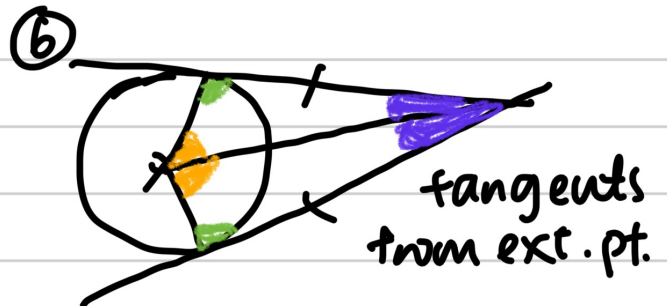
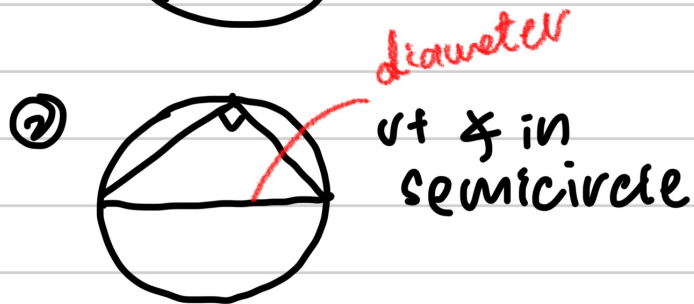
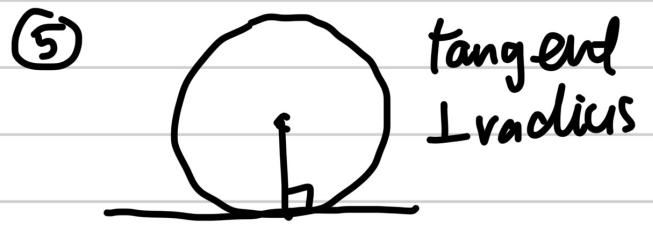
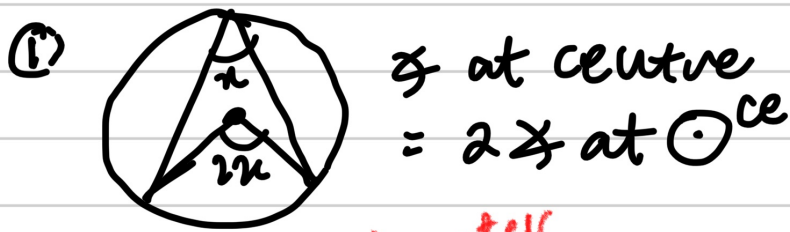


At
instantaneous rest
(changes direction)

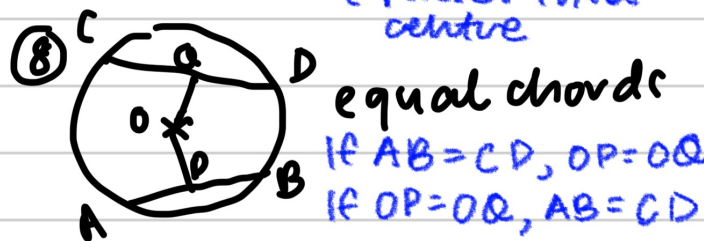
displacement (s) $\xrightarrow{v = \frac{ds}{dt}}$ velocity (v) $\xrightarrow{a = \frac{dv}{dt}}$ Acceleration (a)

$s = \int v dt$ $v = \int a dt$

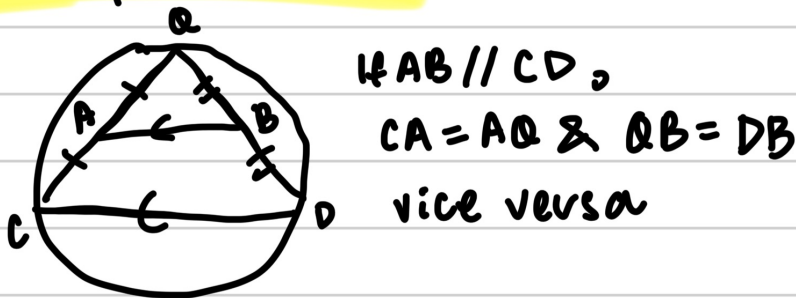
19: Plane Geometry



$\rightarrow$ pass thru centre
& bisect, $\ell \perp AB$
 $\rightarrow$ bisect AB & $\perp$,
 ℓ passes thru
centre



⑨ Midpt Theorem



⑩ Alternate Segment Theorem



ℓ tangent to circle

new
to Amath!

$\star$ Need to know
how to proof.

$\star$ Use similar
 $\triangle$ s $\rightarrow$ often
tested.