RAFFLES INSTITUTION
2025 YEAR 5 PROMOTION EXAMINATION
Higher 2

MATHEMATICS 9758

3 hours

Additional Materials: Printed Answer Booklet
List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **8** printed pages.

1

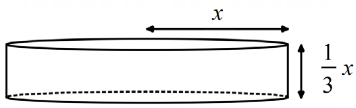


Fig. 1

A tablet is dissolving in water and is modelled as a cylinder, as shown in Fig. 1. At t seconds after being dropped into the water, the radius of the tablet is x mm, and its thickness is $\frac{1}{3}x$ mm. The circular cross-sectional area of the tablet is decreasing at a constant rate of 0.5 mm² per second.

When $x = 5$, find the exact value of

(a) $\frac{\mathrm{d}x}{\mathrm{d}t}$, [2]

(b) the rate of change of the volume of the tablet. [2]

2 **Do not use a calculator in answering this question.**

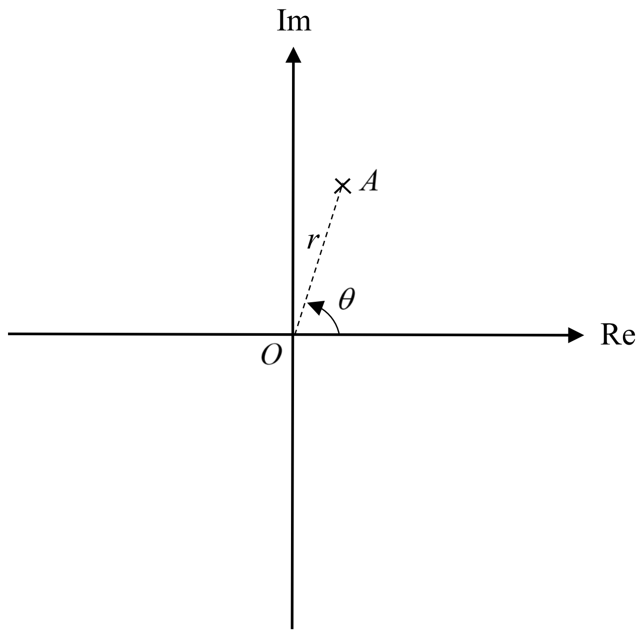
Showing your working, find the complex numbers z and w which satisfy the simultaneous equations

$$2\mathrm{i}z + w = 5 + 7\mathrm{i},$$
$$(4-\mathrm{i})w^* = z - 3\mathrm{i}.$$ [5]

3 The point A on the Argand diagram below represents the complex number w_1 with modulus r and argument θ. It is given that the complex numbers w_2 and w_3 satisfy the equations

$$w_2 = -w_1 \text{ and } w_3 = \mathrm{i}w_2 .$$

Let B, C and D represent the complex numbers w_2, w_3 and $w_2 + w_3$ respectively.

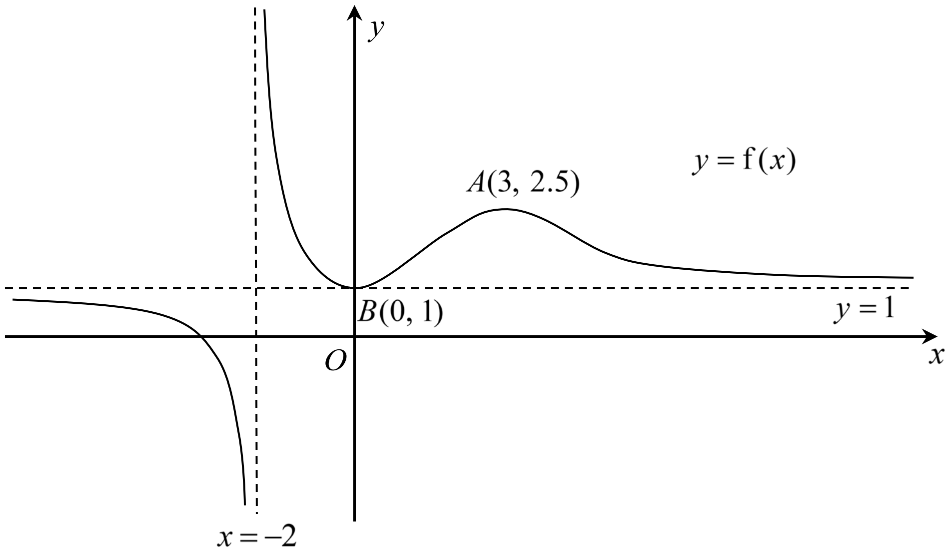


(a) On the copy of the Argand diagram in the Printed Answer Book, plot the points B, C and D, indicating clearly the modulus and argument of w_2 and w_3. [3]

(b) State, in radians, the angle BOC. [1]

(c) By considering the quadrilateral $OBDC$, find $|w_2 + w_3|$ in terms of r and $\arg(w_2 + w_3)$ in terms of θ. [2]

4 **(a)** The diagram shows the curve $y = \mathrm{f}(x)$, with turning points at $A(3,\ 2.5)$ and $B(0,\ 1)$. The lines $y = 1$ and $x = -2$ are asymptotes of the curve.



On separate diagrams, sketch the following graphs, clearly stating the equations of the asymptotes and the coordinates of the points corresponding to A and B where appropriate.

(i) $y = \mathrm{f}(x-2)+2$ [2]

(ii) $y = \mathrm{f}'(x)$ [3]

(b) The parametric equations of a curve are given by

$$x = t^2 - 2t, \qquad y = t^3 - 3t, \qquad \text{for } t > 0.$$

The curve is reflected in the y-axis. Find the exact coordinates of the point(s) where the reflected curve intersects the x-axis. [2]

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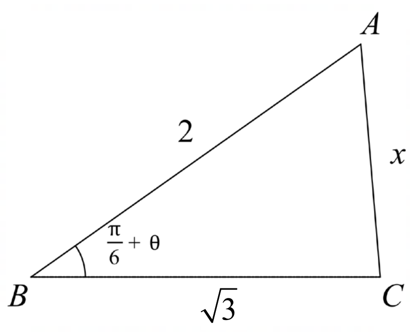

In the triangle ABC, $AB = 2$, $BC = \sqrt{3}$, $AC = x$ and angle $ABC = \dfrac{\pi}{6} + \theta$ radians (see diagram).

(a) Show that $x^2 = 7 - 6\cos\theta + 2\sqrt{3}\sin\theta$. [2]

(b) Show that $x\dfrac{\mathrm{d}^2x}{\mathrm{d}\theta^2} + \left(\dfrac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2 = 3\cos\theta - \sqrt{3}\sin\theta$. [2]

(c) Use the result from part **(b)** to find the Maclaurin expansion for x, up to and including the term in θ^3. Give the coefficients as exact values in their simplest form. [3]

(d) Using the result from part **(c)**, deduce the approximate value of x when angle ABC is 35°, giving your answer correct to 6 decimal places. [2]

6 **(a)** The non-zero vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are such that $3\mathbf{a}\times\mathbf{b} = 2\mathbf{b}\times\mathbf{c}$, where $3\mathbf{a} \neq -2\mathbf{c}$. Find a linear relationship between $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$. [3]

(b) The variable position vector $\mathbf{v}$ satisfies the equation

$$\mathbf{v}\times(-\mathbf{i}+2\mathbf{j}+\mathbf{k}) = a\mathbf{i}+\mathbf{j}+\mathbf{k},$$

where a is a non-zero constant.

(i) Show that $a = 3$. [1]

(ii) Hence, by finding the set of position vectors $\mathbf{v}$, describe geometrically the set of points represented by the position vectors $\mathbf{v}$. [3]

7 The curve C_1 has equation $y=\dfrac{ax^2+bx+3}{x+2}$, where a and b are constants. It is given that C_1 has an asymptote $y=x-3$.

(a) State the value of a and show that $b=-1$. [3]

(b) Using an **algebraic method**, find the set of values that y cannot take. [3]

The locus of a point is defined as the path traced out by that point as it moves.

(c) Let $P(x,y_1)$ be a point on the curve C_1 and $Q(x,y_2)$ be a point on the curve C_2 with equation $y=\dfrac{3x-x^2}{x+2}$. Find the cartesian equation of the locus of the midpoint of P and Q as x varies. [2]

8 The function g is defined by $\mathrm{g}:x\mapsto \ln\left(x-\sqrt{3}\right)$, for $x\in\mathbb{R}$, $x>\sqrt{3}$.

(a) Explain how you know g^{-1} exists. [2]

(b) Find $\mathrm{g}^{-1}(x)$ and write down its domain. [3]

(c) Solve $\mathrm{g}(x)=0$ exactly. [1]

(d) Hence, without using a calculator, solve the inequality $\dfrac{\mathrm{g}(x)}{(x-3)(x+1)}\geq 0$. [3]

9 **(a)** It is given that $\displaystyle\sum_{r=1}^{n}\frac{2}{r(r+1)(r+2)}=\frac{1}{2}-\frac{1}{(n+1)(n+2)}$.

(i) State the sum to infinity of the series. [1]

(ii) Find the sum of the first n terms of the series

$$\frac{2}{17\times18\times19}+\frac{2}{18\times19\times20}+\frac{2}{19\times20\times21}+\cdots.$$ [2]

(iii) Find $\displaystyle\sum_{r=2}^{n}\frac{1}{r\left(r^2-1\right)}$. [3]

(b) The sequence $u_1,u_2,u_3,\ldots$ is defined by $u_1=3$, $u_{n+1}=2-\dfrac{4}{u_n}$, $n\geq1$.

(i) Find the values of u_2, u_3, u_4 and u_{100}. [2]

(ii) Hence find $\displaystyle\sum_{r=1}^{3n-2}u_r$ in terms of n, simplifying your answer. [2]

10 With reference to the origin O, the point A has position vector $-\mathbf{i}-3\mathbf{j}+3\mathbf{k}$. The plane π and the line l have equations

$$\mathbf{r}=\begin{pmatrix}4\\1\\1\end{pmatrix}+\lambda\begin{pmatrix}-1\\3\\2\end{pmatrix}+\mu\begin{pmatrix}2\\3\\-4\end{pmatrix} \text{ and } \mathbf{r}=\begin{pmatrix}2\\1\\1\end{pmatrix}+\beta\begin{pmatrix}3\\t\\-2\end{pmatrix},$$

respectively, where λ, μ and β are parameters and t is a real value.

(a) It is given that l and π intersect at a point.

(i) Find the range of values of t. [2]

(ii) Find, in terms of t, the coordinates of the point of intersection, B, of l and π. [2]

(b) It is given that A lies on l.

(i) Show that $t=4$. [1]

(ii) Find the position vector of the foot of perpendicular, F, of A onto π. [3]

(iii) Find the cartesian equation of another line m through A, that lies on the plane containing A, B and F, and m makes the same angle with π as l makes with π. [4]

11 An arithmetic series has positive first term a and common difference d. The first, fifth and second terms of the arithmetic series are the first three consecutive terms of a geometric series respectively. It is also given that the terms of the arithmetic series are decreasing.

(a) Show that $d=-\frac{7}{16}a$. [3]

(b) Give a reason why the geometric series converges. [2]

It is given that the sum to infinity of the geometric series is $\frac{32}{7}$.

(c) Find the sum of the first twenty terms of the arithmetic series. [2]

(d) Given that the sum of all the terms after the nth term of the geometric series is greater than 2% of the sum of the first n terms of the geometric series, find the possible values of n. [4]

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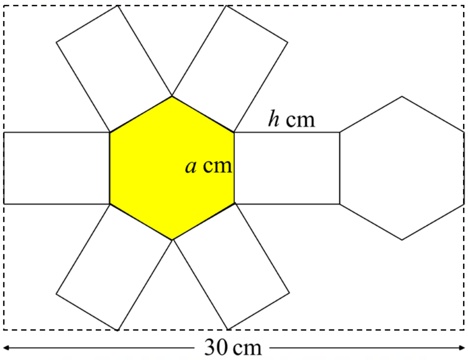

Fig. 1

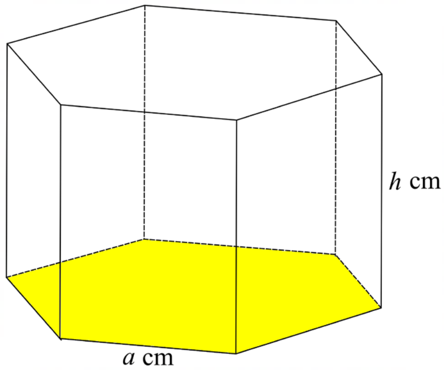

Fig. 2

Fig. 1 shows the net of a right regular hexagon-based prism cut from a rectangular cardboard of one side length 30 cm. The net consists of two regular hexagons of side length a cm and six rectangles, each with sides of length a cm and h cm. The net is folded to form a prism which has a hexagonal base of side length a cm and vertical height h cm, as shown in Fig. 2.

(a) Show that $h + a\sqrt{3} = 15$. [2]

(b) Show that the area of the hexagonal base is $\dfrac{3\sqrt{3}}{2}a^2$ cm^2. [1]

(c) Use differentiation to find the exact value of the maximum possible volume of the prism and prove that this value is a maximum. [6]

(d) Find the ratio of h to a that gives this maximum volume. [1]

(e) Sketch the graph showing the volume of the prism as the value of a varies. [2]