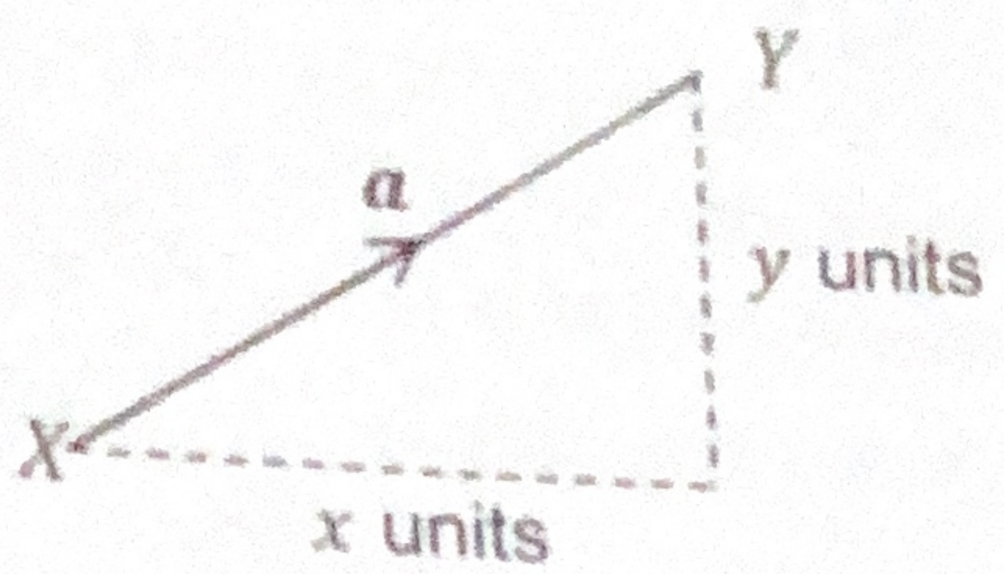
Summary (Recap)

1. **Definition of Vectors**: Quantity that has both **magnitude** and **direction**.

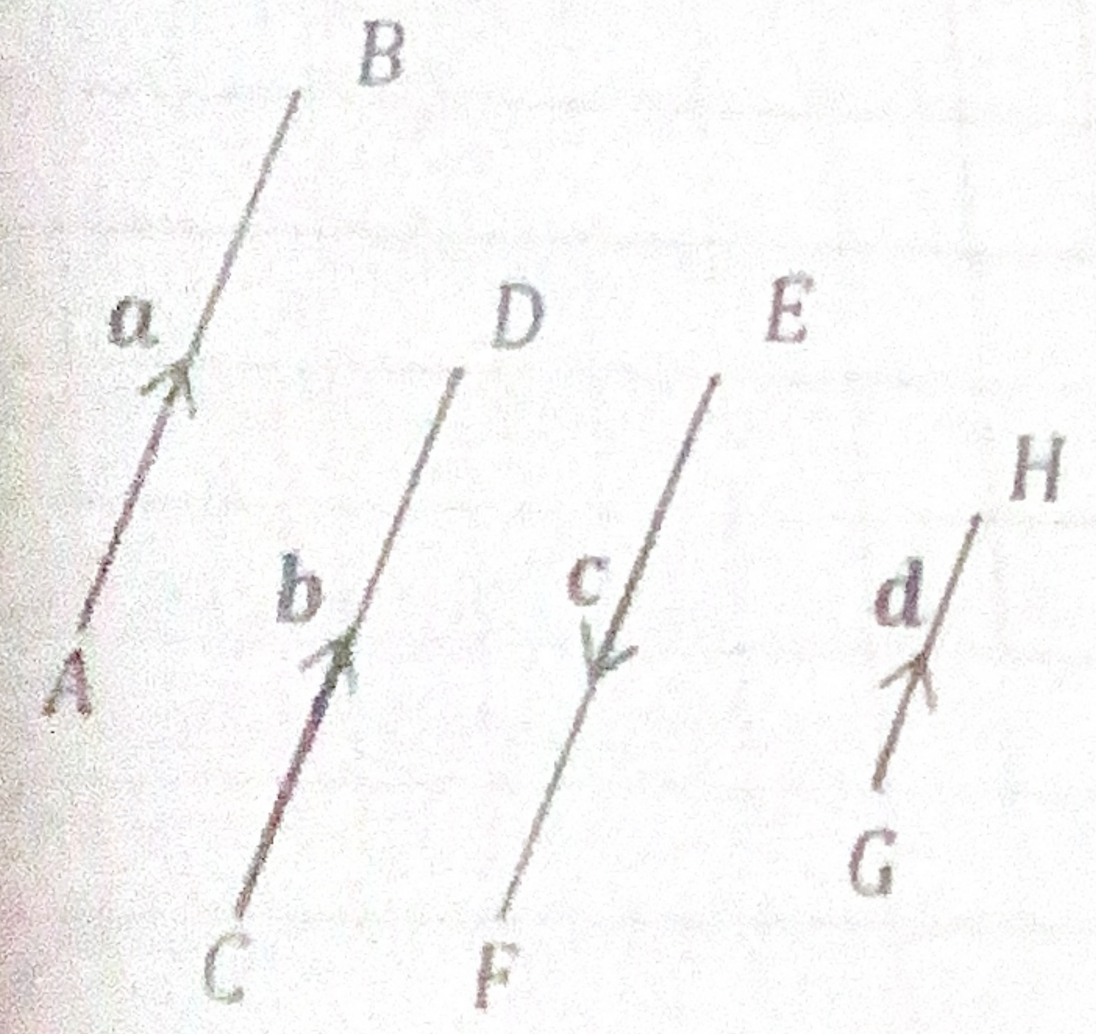
2. **Representation of Vectors**: It can be represented by a **directed line segment** where the **direction** of the line segment is that of the vector and the **length** of the line segment represents the **magnitude** of that vector.



- The vector above is denoted by $\overrightarrow{XY}$, $\boldsymbol{a}$ and $\begin{pmatrix} x \\ y \end{pmatrix}$. As we cannot bold the letter "$\boldsymbol{a}$" in writing, we use $\underset{\sim}{a}$.
- Point X is the **starting/initial** point and Point Y is the **ending/terminal** point.
- The magnitude of XY is denoted by $|\overrightarrow{XY}|$ or $|\boldsymbol{a}|$ or $\left|\begin{pmatrix} x \\ y \end{pmatrix}\right|$.
- $\begin{pmatrix} x \\ y \end{pmatrix}$ is a **column vector** where x represents the x units in x –direction and y represents the y units in y –direction. They are called the components of the vector $\begin{pmatrix} x \\ y \end{pmatrix}$ where x is the x –component and y is the y –components.
  - x –direction is the horizontal direction whereby the position moves to the right in positive units and left in negative units.
  - y –direction is the vertical direction whereby the position moves up in positive units and down in negative units.
- In general, the magnitude of a column vector $\boldsymbol{a} = \begin{pmatrix} x \\ y \end{pmatrix}$ is $|\boldsymbol{a}| = \sqrt{x^2 + y^2}$ by using Pythagoras' Theorem. Therefore,
  - Magnitude of a horizontal vector $\boldsymbol{b} = \begin{pmatrix} x \\ 0 \end{pmatrix}$ is $|\boldsymbol{b}| = x$
  - Magnitude of a vertical vector $\boldsymbol{c} = \begin{pmatrix} 0 \\ y \end{pmatrix}$ is $|\boldsymbol{c}| = y$

3. **Equal, Negative and Zero Vectors**
- **Equal vectors** are vectors with the **same magnitude** and direction.
  - For example, $\overrightarrow{AB} = \overrightarrow{CD}$ or $\boldsymbol{a} = \boldsymbol{b}$.
- **Negative vectors** are vectors that have **the same magnitude but in opposite directions**.
  - For example, $\overrightarrow{AB} = -\overrightarrow{EF}$ or $\boldsymbol{a} = -\boldsymbol{c}$

  Note: $|\overrightarrow{AB}| = |\overrightarrow{EF}|$

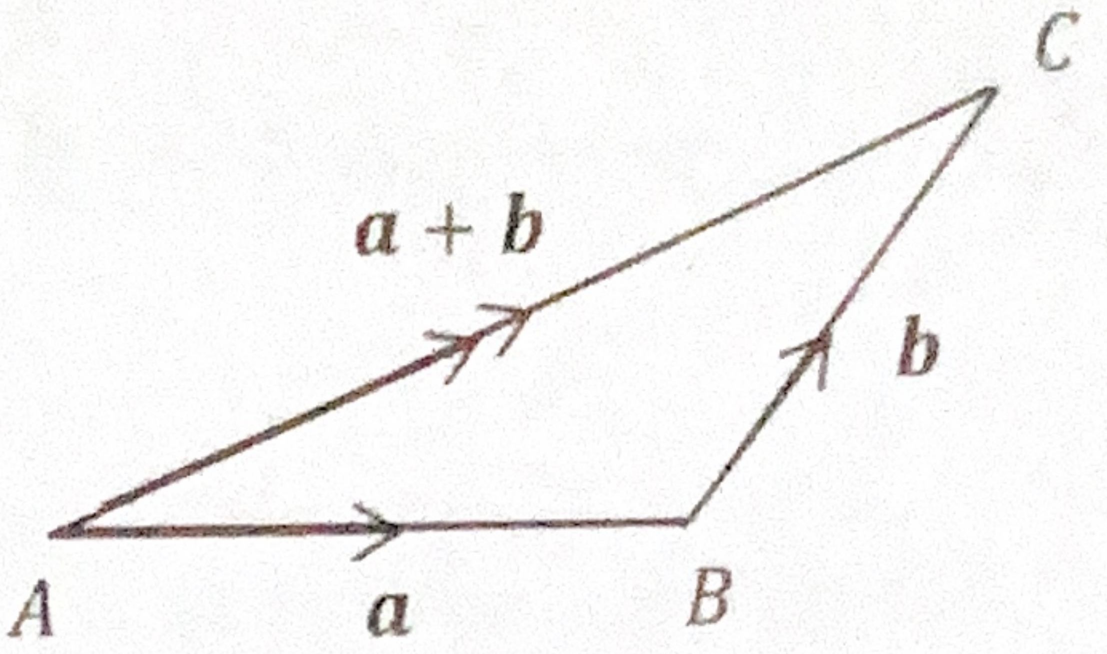


- **Zero/ Null vectors** are vectors whose magnitude is 0. It takes place when two vectors are equal in magnitude but in opposite direction are added, where the resultant vector is a zero vector, 0.
  - For example, $\overrightarrow{AB} + \overrightarrow{EF} = \boldsymbol{0}$ or $\boldsymbol{a} + \boldsymbol{c} = \boldsymbol{0}$

Note: $\boldsymbol{a}$ and $\boldsymbol{d}$ have different magnitudes but same direction. It will be covered in **Scalar Multiples of a Vector**.

4. Addition of Vectors

- **Triangle Law of Vector Addition**:

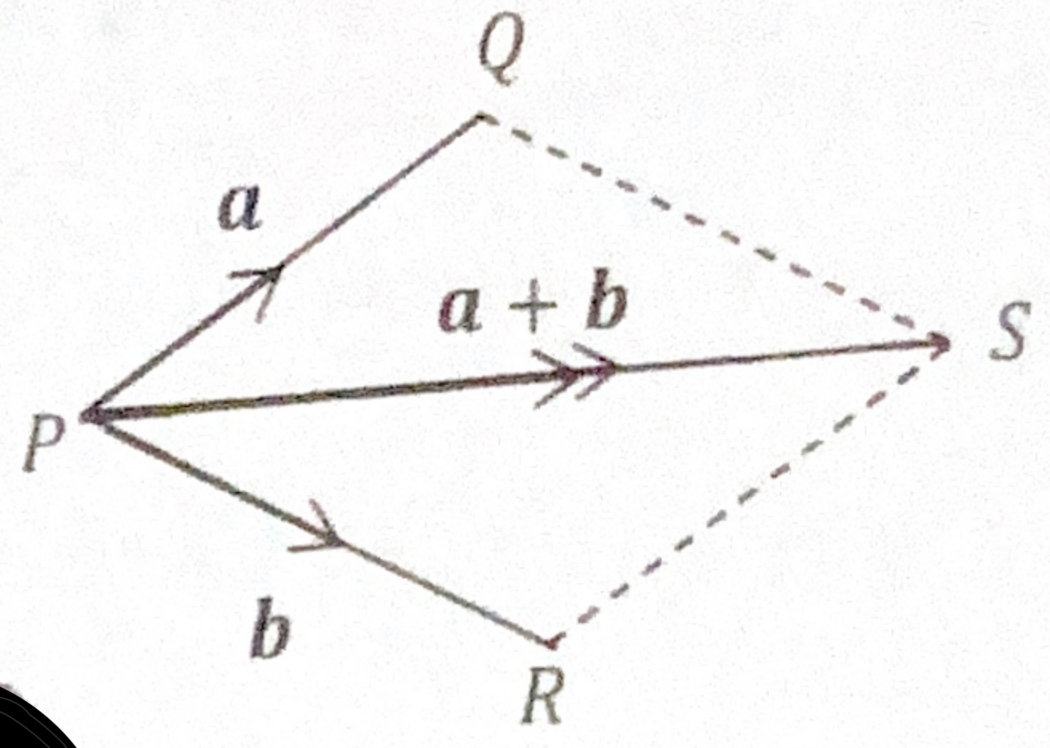


$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

Start, Common point, End, Start, End

  - In Triangle Law of Vector Addition, the ending point of the first vector must be the starting point of the second vector. In the example above, there is a common point B.

- **Parallelogram Law of Vector Addition**:



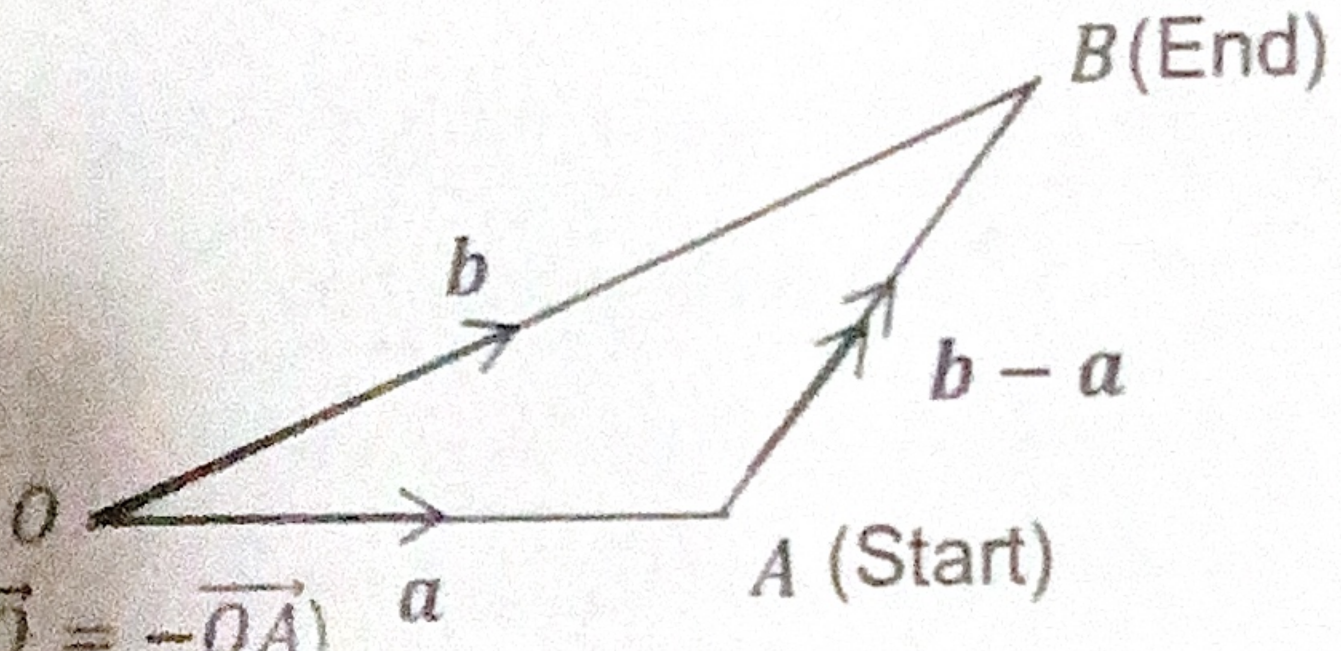
  - As $\overrightarrow{QS} = \overrightarrow{PR} = \boldsymbol{b}$, $\overrightarrow{PQ} + \overrightarrow{PR} = \overrightarrow{PS}$
  - In Parallelogram Law of Vector Addition, both vectors ($\overrightarrow{PQ}, \overrightarrow{PR}$),and the resultant vector ($\overrightarrow{PS}$), starts from the same point, P.

- If $\boldsymbol{a} = \begin{pmatrix} p \\ q \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} r \\ s \end{pmatrix}$, then, in column vectors,
  $\boldsymbol{a} + \boldsymbol{b} = \begin{pmatrix} p \\ q \end{pmatrix} + \begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} p + r \\ q + s \end{pmatrix}$.

[redacted] Note that $|\boldsymbol{a} + \boldsymbol{b}| \neq |\boldsymbol{a}| + |\boldsymbol{b}|$.

5. Vector Subtraction

- **Triangle Law of Vector Subtraction**:



$$\overrightarrow{OB} - \overrightarrow{OA} = \overrightarrow{AB}$$

End, Common point, Start, Start, End

  - $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$ (using addition of vector)
    $= -\overrightarrow{OA} + \overrightarrow{OB}$ (using negative vector where $\overrightarrow{AO} = -\overrightarrow{OA}$)
    $= \overrightarrow{OB} - \overrightarrow{OA}$
  - In Triangle Law of Vector Subtraction, both vectors ($\overrightarrow{OB}, \overrightarrow{OA}$) must start from the same point.

[redacted] To determine the direction in vector subtraction, use "end minus start" [redacted]

- If $\boldsymbol{a} = \begin{pmatrix} p \\ q \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} r \\ s \end{pmatrix}$, then, in column vectors,
  $\boldsymbol{a} - \boldsymbol{b} = \begin{pmatrix} p \\ q \end{pmatrix} - \begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} p - r \\ q - s \end{pmatrix}$.

Note that $|\boldsymbol{a} - \boldsymbol{b}| \neq |\boldsymbol{a}| - |\boldsymbol{b}|$.

6. Scalar Multiples of a Vector and Parallel Vectors

- If $\boldsymbol{a}$ and $\boldsymbol{b}$ are two **non-zero**, **parallel** vectors, and $\boldsymbol{a}$ has a magnitude $|k|$ times of $\boldsymbol{b}$, then $\boldsymbol{a} = k\boldsymbol{b}$, where k is a scalar and $k \neq 0$.
- If k is positive, $\boldsymbol{a}$ and $\boldsymbol{b}$ are in the same direction.
  If k is negative, $\boldsymbol{a}$ and $\boldsymbol{b}$ are in the opposite direction.
- In general, if $\boldsymbol{a} = \begin{pmatrix} x \\ y \end{pmatrix}$, then $k\boldsymbol{a} = \begin{pmatrix} kx \\ ky \end{pmatrix}$, and $|k\boldsymbol{a}| = |k||\boldsymbol{a}|$ for any real number of k.
- Points A, B and C are **collinear** if $\overrightarrow{AB} = k\overrightarrow{BC}, \overrightarrow{AB} = m\overrightarrow{AC}$ or $\overrightarrow{BC} = n\overrightarrow{AC}$.

7. Expressing a Vector in Terms of Two Other Vectors

- For **non-parallel** vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, and p, q, r and s are scalars,
  - If $p\boldsymbol{a} = q\boldsymbol{b}$, then $p = 0$ and $q = 0$.
  - If $r\boldsymbol{a} + s\boldsymbol{b} = p\boldsymbol{a} + q\boldsymbol{b}$, then $r = p$ and $s = q$.
- In general, $\overrightarrow{AB}$ can be expressed uniquely in terms of two other **non-zero** and **non-parallel** vectors, $\boldsymbol{u}$ and $\boldsymbol{v}$, that lies on the same plane.
  - For example: $\overrightarrow{AB} = m\boldsymbol{u} + n\boldsymbol{v}$, where m and n are scalars.

8. **Position Vector and Coordinates of a Point on a Cartesian Plane**
   - **Position vector** of a point must have a fixed starting point/ reference point, where it is usually the origin O on a Cartesian plane.
   - A directed line segment $\overrightarrow{OP}$ indicates the position of a point P that has taken with reference to vectors with the origin O as the initial point.
   - In general, the position vector of Point $P(x, y)$ is $\overrightarrow{OP} = \begin{pmatrix} x \\ y \end{pmatrix}$.
   - A vector $\overrightarrow{PQ}$ on a Cartesian plane can be expressed in terms of position vectors.
     - For example: $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$ (Note: "end minus start" – vector subtraction)

9. **Translation of a Point using Column Vector**
   - Vector $\overrightarrow{PQ}$ can be regarded as movement from P to Q, which is a **translation** from P to Q. $\overrightarrow{PQ}$ becomes a **translation vector** which describe the movement.
   - A translation can be represented as a column vector $\begin{pmatrix} h \\ k \end{pmatrix}$. This refers to a movement of h units along the x –axis and k units along the y –axis.
     - For example, Point $P(a, b)$, which can be expressed as $\overrightarrow{OP} = \begin{pmatrix} a \\ b \end{pmatrix}$, undergoes a translation $T = \begin{pmatrix} h \\ k \end{pmatrix}$ to Point Q, then Q can be found by $\overrightarrow{OQ} = \overrightarrow{OP} + \begin{pmatrix} h \\ k \end{pmatrix}$.
       Therefore, $\overrightarrow{OQ} = \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} h \\ k \end{pmatrix} = \begin{pmatrix} a + h \\ b + k \end{pmatrix}$.