

Secondary 3 WA3 Practice Paper

Name : _____

Date : _____

ADDITIONAL MATHEMATICS

Practice Paper

1 hour and 57 minutes

READ THESE INSTRUCTIONS FIRST:

Write your name and centre in the spaces at the top of this page.

Write in dark blue or black pen. You may use a HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 78.

List of Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

Points are awarded according to the question's allocated mark boundary. Expect 1 mark per essential working. However, points are deducted when a question fails to be answered correctly and where mistakes lie.

For examiner's use :

Question:	1	2	3	4	5	6	7	8	9	10	Total
Points:	6	6	10	10	6	9	9	10	7	5	78
Score:											

Presentation points : Excellent -> 3, Great -> 2, Acceptable -> 1, Poor -> 0

Total Points	Deductions	Final score

1 Solve the simultaneous equations

$$x^2 + y^2 - 4x - 6y + 9 = 0$$

$$\frac{10^y}{2^x} = 2^{x+1}(5^y)$$

Ans: _____ [6]

2 (a) By using the substitution of $y = \sqrt{x}$ (where $y \geq 0$), find the exact real value of x which satisfies the equation

$$x + 3\sqrt{x} = \frac{1}{2}$$

Ans: _____ [4]

2 (b) Hence using this value of x , show that

$$\frac{1}{x^2} + y^2 - 1 = 805 + 237\sqrt{11}$$

Ans: _____ [3]

3 In wireless communications, the further a receiver is located from the source, the weaker its reception will become, and thus its lower Signal-to-Noise Ratio (SNR) calculated in units of decibel.

In particular, suppose the receiver is located at a reference point x_0 metres away from the source and measures an SNR of SNR_0 . As the receiver's distance from the source increases to x metres, its SNR will drop to SNR_x , using the formula

$$SNR_x = SNR_0 - 10 \lg\left(\frac{x}{x_0}\right)^2$$

where x and SNR_x are obtained via measurements, x_0 and SNR_0 are the reference values.

(a) Show that whenever the receiver doubles its distance from the source, the SNR will decrease by about 6 decibels.

Ans: _____ [3]

- (b) A receiver is set at a distance of 20 metres away from the source, and measures an SNR of -5 decibels. If a minimum SNR of 15 decibels is required for reasonable reception quality, how close must the receiver move towards the source?

Ans: _____ [2]

- (c) Find the exact value of x such that

$$\log_9(4x^2 + 3x + 5) - \log_3(x + 1) = \frac{1}{2}$$

Ans: _____ [5]

4 (a) Find the value of n given that

$$\left(\frac{3}{x} - x\right)^5 - (8 - x^3)^n = \dots + 207x + \dots$$

Ans: _____ [4]

4 (b) Given that the coefficient of x^2 is -120 in the expansion of $(3 - x)^2(2 + \frac{ax}{3})^8$.

Find the possible value(s) of a .

Ans: _____ [6]

- 5** Express the following as partial fractions

$$\frac{2x^2 - 18x + 44 - 6x^3}{(x^2 + 4)(2 - 3x)}$$

Ans: _____ [6]

6 (a) The variables x and y are such that when the values of y^2 are plotted against $\frac{1}{\sqrt{x}}$ a straight line is obtained. It is given that $y = \frac{1}{2\sqrt{2}}$ when $x = \frac{4}{9}$ and that $y = \frac{1}{2}$ when $x = 1$. Find the value of y^2 when $x = \frac{1}{4}$.

Ans: _____ [5]

6 (b) A manufacturer produces an anti-bacterial spray that is said to reduce 21% of the number of bacteria, N , per minute. Explain why after n minutes, the number of bacteria is represented by the equation $N = (0.79)^n N_0$ where N_0 is the initial bacteria count.

Ans: _____ [2]

6 (c) Given that the number of bacteria expected to be alive after n minutes can be expressed as Ne^{kn} . Find the value of the constant k .

Ans: _____ [2]

7 $ABCD$ forms a square where A lies on the y -axis and D is a point below the x -axis. The gradient of AC is $\frac{1}{4}$ and the coordinates of the midpoint of AC are $(4, 3)$. Given that $(2, \frac{4}{5})$ lies on the line AD . Find the coordinates of D .

Ans: _____ [9]

- 8** AB is a chord of the circle, C , and $M(\frac{4}{5}, 2\frac{2}{5})$ is the midpoint of AB .

$$C : x^2 + y^2 - 8x - 2y - 3 = 0$$

- (a)** Find the radius and the coordinates of the centre of the circle

Ans: _____ [3]

- (b)** Find the equation of chord AB

Ans: _____ [3]

If P is a variable point on the circle,

(c) find the maximum area of $\triangle ABP$

Ans: _____ [4]

9 The function $f(x)$ is defined as $f(x) = x^2 + px - x + p^2 + 2$ where p is a constant.

(a) Show that $f(x) > 0$ for all real values of x

Ans: _____ [3]

(b) Hence, find the range of values of x for which

$$\frac{3x^2 + x - 2}{f(x)} > 0$$

Ans: _____ [4]

10 Given that $A(2, 2)$ is a point on the circle, find the equation of the tangent to the circle at the point A . The equation of a circle is

$$x^2 + 2x + 4y = 20 - y^2$$

Ans: _____ [5]